We will spend the first half of this quarter studying infinite series, arguably the single most difficult concept in the Calculus sequence. To help us understand the need to develop this concept we will bring all of our Math 1B skills to bear on the indefinite integral

$$\int e^{x^2} dx$$

I. Integration by Parts

Integration by Parts is considered by many mathematicians to be the single most useful tool in our integration toolkit. We will explore whether or not it can help with this indefinite integral.

a) Remembering that any function can be thought of as a function multiplied by the constant function 1, apply integration by parts to the integral one time. (5 points)

b) The resultant integral can also be tackled using integration by parts. I can think of 4 different ways you might separate the integral into the product of two functions. Try two of them. (10 points)

c) If you continued to apply integration by parts to this integral 5 times, without ever making a choice that would undo your work, what would you expect to end up with? (I am looking for a qualitative description of the result, IN WORDS; I do NOT want you to actually apply integration by parts 5 times.) (5 points)

II. Substitution

If there is another tool that could challenge integration by parts in terms of usefulness, it would be the substitution rule. We will explore how this tool might help us with our integral.

a) Use the substitution $u = x^2$ to rewrite the integral in terms of u. You may assume that $x > 0$. Be sure that no x's are left in your integral. What technique suggests itself now? Try it once. Explain WHY you believe this won't get you anywhere. (10 points)

b) Use the substitution $w = e^{x^2}$ to rewrite the integral in terms of w. You may continue to assume that $x > 0$. Give a reasonable DESCRIPTION of how you might proceed from here. Do you think these tactics will work? Why or why not? (10 points)

III. Other techniques of integration

Name two other techniques you learned in Math 1B and explain why you believe they cannot apply to this integral. (10 points)

IV. What do we know about this function?

a) Sketch a graph of $f(x) = e^{x^2}$. (5 points)

b) Is $f(x)$ differentiable? If so, find $f'(x)$. (5 points)

c) Is $f(x)$ integrable? Be careful to justify your answer. Write down an anti-derivative for $f(x)$. In light of parts I-III of this lab, why is this anti-derivative impractical? (You may find it useful to re-read sections 5.2 and 5.3 in your textbook.) (10 points)

V. Where to next?

In order for us to find a way to integrate $f(x) = e^{x^2}$, we will need to invest a lot of energy into studying the tools available to us in Chapter 11. Unfortunately, series seem particularly alien to us when we begin studying them. To help us come to grips with these new tools we will need to rely on one another. Toward that end you need to get to know the people in this class, so that we can help one another. We will begin with your lab group.

Write a paragraph describing each member of your group. Include the following information for each person: the name they like to be called, their intended major, where they would like to transfer to, something they like to do, and at least one other additional piece of information about them that I did not ask you for. You will be graded on your paragraphs independently, and they should be written as paragraphs, not lists of facts. They should be written in your own words (so two different members of your group writing about a third member should NOT have the same exact paragraphs). You do not need to include a paragraph about yourself. (30 points)