Ch 10: Hypothesis Tests of 2 Means or Proportions

Used when we want to compare 2 groups

Examples:
- Medicine v. placebo
- Comparing 2 weight loss programs
- Pre & post testing

Types of Tests:

I. Test of Independent Groups: Means

- Samples chosen independently
- Either both popplations have a normal distr. -or- $n \geq 30$ for both samples

ex) Time spent by girls v. boys playing sports

RV: $\bar{X}_1 - \bar{X}_2$

Distr: If σ unknown $T \sim t_{df}$ *Calculator finds df

II. Test of Two Proportions

- Samples chosen indepently
- Number of successes & number of failures in each sample must be ≥ 5

ex) % of patients who improved w/ Drug A v. Drug B

RV = $P_1' - P_2'$

Distr = normal

III. Dependent Groups / Paired Samples → Matched Pairs

- Samples are not independently chosen
- 2 measurements (i.e. the samples) are drawn from the same individuals
- We compare the 2 measurements by subtracting them for each individual
- Either pop. is normally distributed -or- $n \geq 30$

RV = $\bar{X}_d$ (d stands for difference)

Distr: $T \sim t_{df}$, $df = n - 1$

ex) Pre & post test

Example:

* A. Independent group means, σ known ← rare
* B. Independent group means, σ unknown
* C. Matched or paired samples
* D. Single mean → σ known or σ unknown
* E. 2 proportions
* F. Single proportion

1. Two chocolate bars are taste-tested on consumers. Of interest is whether a larger percentage of consumers will prefer one bar over the other. F

2. A football league reported that the average number of touchdowns per game was 5. A study is done to determine if the average number of touchdowns has decreased. D

3. According to a YWCA Rape Crisis Center newsletter, 75% of rape victims know their attackers. A study is done to verify this. F

4. A recent drug survey showed an increase in use of drugs and alcohol among local high school students as compared to the national percent. Suppose that a survey of 100 local youths and 100 national youths is conducted to see if the percentage of drug and alcohol use is higher locally than nationally. E

5. University of Michigan researchers reported in the Journal of the National Cancer Institute that quitting smoking is especially beneficial for those under age 49. In this American Cancer Society study, the risk (probability) of dying of lung cancer was about the same as for those who had never smoked. E

What kind of test should you use?

- **If you are comparing to a single value**

  1-mean or 1-prop test

  (Ch. 9)

- **If you are comparing two independent groups to each other**

  2-mean or 2-prop test

  $\bar{x}_1 - \bar{x}_2$ $\quad$ $p'_1 - p'_2$

- **If you are pairing up data values**

  1-mean hyp test $(\bar{x}_d)$

Observational Studies and Designed Experiments

Once we have picked a subject for study and identified our population, we need to collect data for the study. There are two primary ways that studies can be conducted: **observational studies** and **designed experiments**.

In an **observational study**, the researcher observes the behavior of the individuals in the study without trying to influence the outcome.

In a **designed experiment**, the researcher intentionally manipulates one of the variables and then monitors how the manipulation influences other variables of interest.

Example 1.

Two Math 10 students conducted a study of 50 male patients in a treatment clinic who were receiving treatment for male pattern baldness using minoxidil. Patients were asked whether they were using a 2% topical minoxidil solution or a 3% minoxodil solution. The patients were also asked whether they had noticed significant increase in hair count in the last 12 months. The percent of men who stated that they had noticed significant increase in hair count in the last 12 months were compared for the 3% solution vs. the 2% solution. (Math 10 Project 3, Fall 1999)

Example 2:

A double-blind trial was conducted to compare 2% topical minoxidil with 3% topical minoxodil and placebo in the treatment of men with male pattern baldness. 150 men with male pattern baldness were randomly assigned to treatment with either 2% topical minoxidil, 3% topical minoxidil or placebo. Hair counts were conducted at monthly intervals and by subjective patient evaluations at 12 months. The percents of men who showed new hair growth at 12 months were compared for the three groups. (Journal of the American Academy of Dermatology, March 1987)

Observational studies do not allow a researcher to claim causation, only association. That is because of the possible presence of *lurking variables*. A lurking variable is a variable that was not considered in a study, but affects the results of the study.

A widely reported study in May, 2012 looked at the association of coffee-drinking with mortality. The study followed over 400,000 men and women ages 50 – 71 years of age at baseline. Participants with cancer, heart disease, and stroke were excluded. Participants were asked their level of coffee consumption at the beginning of the study and then were monitored for mortality between 1995 and 2008. After adjusting for age, tobacco smoking, body weight and other factors, it was observed that those who drank 4 or 5 cups of coffee per day had lower mortality rates than those who drank less than 1 cup of coffee per day. (Neal D. Freedman, Ph.D., Yikyung Park, Sc.D., Christian C. Abnet, Ph.D., Albert R. Hollenbeck, Ph.D., and Rashmi Sinha, Ph.D. N Engl J Med 2012; 366:1891-1904 May 17, 2012 DOI: 10.1056/NEJMoa1112010)

- All factors affecting health may not have been completely filtered out. For example, participants were not asked whether they had high blood pressure or high cholesterol levels. The authors of the survey acknowledged that if no adjustment was made for such factors, coffee drinkers actually tended to have a higher risk of dying than those who abstained.
- The participants only supplied information about their coffee habits once, with no updates if they changed over the next 13 years of the survey.
- People in poor health might drink less coffee.

I. Independent Groups: Means

- **Samples are chosen independently of each other**
- **The two populations are normally distributed unless the samples sizes are >30**

Random Variable: $\bar{X}_1 - \bar{X}_2$

Distribution:

- **If σ is known** : normal
- **If σ is unknown** $T \sim t_{df}$

* Calculator finds df

$n_M = 29 \quad \bar{x}_M = 3 \quad s_M = 0.8$

$n_F = 16 \quad \bar{x}_F = 4 \quad s_F = 1.0$

Example: The average number of English courses taken in a two-year time period by male and female college students is believed to be about the same. Data is collected from 29 males and 16 females. The males took an average of 3 English courses with a standard deviation of 0.8. The females took an average of 4 English courses with a standard deviation of 1.0. Are the averages statistically the same?

*Use meaningful subscripts instead of 1 & 2

SOLUTION SHEET: Hypothesis Testing for Two Means, Paired Data, and Two Proportions

*In 2-mean & 2-prop test H_0 is $=$

a. H_0: $\mu_M = \mu_F$

H_a: $\mu_M \neq \mu_F$

(b) H_0: On avg., males & females take the same # of Eng. classes

H_a: On avg., males & females don't take the same # of Eng. classes

c. In words, CLEARLY state what your random variable X1 - X2, P1'- P2' or Xd represents.

$\bar{X}_M - \bar{X}_F$ = the difference in the avg. # of Eng. classes taken by males and females

d. State the distribution to use for the test.

$T \sim t_{df}$ $\quad T \sim t_{25.74}$

STAT → TESTS → 2-SampTTest → STATS

e. Skip

f. p-value = 0.0020

g. Use the previous information to sketch a picture of this situation. CLEARLY, label and scale the horizontal axis and shade the region(s) corresponding to the p-value.

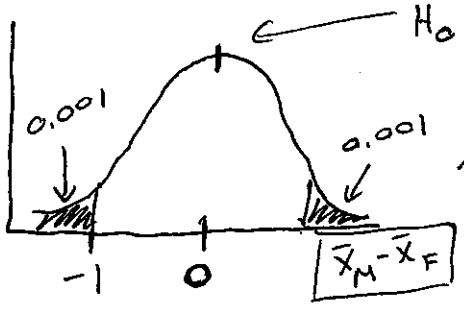
h. Indicate the correct decision ("reject" or "do not reject" the null hypothesis) and write appropriate conclusions, using COMPLETE SENTENCES.

alpha	**decision**	**reason for decision**
0.05	0.05 > 0.002	Reject H_o & we conclude
	α > p-value	on avg. males & females do not take the same number of Eng. classes

Conclusion:

i. In complete sentences, explain how you determined which distribution to use.

g)



$H_o: \mu_M = \mu_F$ RV: $\bar{x}_M - \bar{x}_F$

$-\mu_F \quad -\mu_F$

$\mu_M - \mu_F = 0$

$H_a: \mu_M \neq M_F \Rightarrow$ 2-tails

Data: $\bar{x}_M - \bar{x}_F = 3 - 4 = -1$

p-value = 0.002 = total area

II. Proportions

- **Two samples are chosen independently of each other**

Random Variable: $P_1' - P_2'$

Distribution: normal

$n_M = 67$
$x_M = 24$
$n_F = 24$
$x_F = 8$
$\alpha = 0.05$

Example: A study was done to determine whether the percent of men who enjoy shopping for electronic equipment is higher than the percent of women who enjoy shopping for electronic equipment. The population was Saturday afternoon shoppers. Out of 67 men, 24 said they enjoyed shopping for electronic equipment. 8 out of 24 women surveyed claimed to enjoy the activity. Test at the 5% level.

SOLUTION SHEET: Hypothesis Testing for Two Means, Paired Data, and Two Proportions

a. H_0: _____

b. H_a: _____

c. In words, CLEARLY state what your random variable X1 - X2 , P1' - P2' or Xd represents.

_____

_____

d. State the distribution to use for the test.

_____

Ch. 9 Solution Sheet (for HW #5, 7, 9, 11, 13, 25)

a) H_0: $P'_M = P'_F$ H_a: $P'_M > P'_F$

b) In words:
H_0: The proportion of men who enjoy shopping for elec. is equal to the proportion of women
H_a: The proportion of men who enjoy shopping for elec. is greater than the prop. of women.

c) In words, clearly state what your random variable ~~(x-bar)~~ represents:

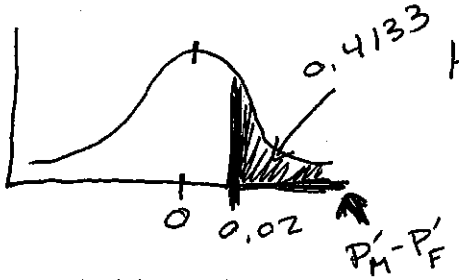
$P'_M - P'_F$ = the difference in the prop. of men & prop. of women who enjoy shopping for elec.

d) State the distribution of the test: _____~_____
normal

e) Test Statistic: t or z = z (or SKIP)

f) p-value= 0.4133 STATS → TEST → 2Prop Z Test

g) Sketch a picture of the situation.



H_0: $P'_M - P'_F = 0$
Data: $P'_M - P'_F = \frac{24}{67} - \frac{8}{24} = 0.02$

h) Indicate the correct decision and conclusion:

alpha	decision	conclusion
0.05	0.05 < 0.4133	We do not reject H_0
	$\alpha <$ p-value	& we accept that the percents are equal

~~i) Construct a confidence interval.~~

* Observational study
* Study flaws?
  → Convenience sampling
  → Biased sampling - shoppers
  → More men & few women were asked
  → Possible lurking variables
  • income, location, race, age

III. Dependent Groups / Paired Samples

- **Samples are NOT independently chosen**
- **Two measurements chosen from same pair of individuals or related subjects**
- **Population from which samples taken are normally distributed or the sample size is >30**

Random Variable: $\overline{X}_d$ ← d stands for difference

Distribution: $T \sim t_{df}$ ← once again $df = n-1$

$n=10$

Example: Ten individuals went on a low-fat diet for 12 weeks to lower their cholesterol. Evaluate the data below. Do you think that their cholesterol levels were significantly lowered?

Starting cholesterol level x_1	Ending cholesterol level x_2	end − start $x_d = x_2 - x_1$
140	140	0
220	230	10
110	120	10
240	220	-20
200	190	-10
180	150	-30
190	200	10
360	300	-60
280	300	20
260	240	-20

enter into L1

RV: $\bar{x}_d$ is the difference in chol. levels

$H_0: \mu_d = 0$ $\quad H_a: \mu_d < 0$

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2 options

① STAT → TESTS → T-Test → DATA

$\mu_0 = 0$

L1

p-value = 0.1353

② 1-var stats → $\bar{x} = -9$

$S_x = 24.2441$

STAT → TESTS → T-test → STATS

SOLUTION SHEET: Hypothesis Testing for Two Means, Paired Data, and Two Proportions

a. H_0: $\mu_d = 0$

b) H_0: There is no difference in the cholesterol levels before & after

~~b~~. H_a: $\mu_d < 0$

H_a: The cholesterol level, or avg, is lower after the diet

c. In words, CLEARLY state what your random variable X1 - X2 , P1'- P2' or Xd represents.

$\bar{X}_d$ = the difference in avg. cholesterol level before & after the diet (after - before)

d. State the distribution to use for the test.

$T \sim t_q$

e. Skip T

f. p-value = 0.1353

g. Use the previous information to sketch a picture of this situation. CLEARLY, label and scale the horizontal axis and shade the region(s) corresponding to the p-value.

h. Indicate the correct decision ("reject" or "do not reject" the null hypothesis) and write appropriate conclusions, using COMPLETE SENTENCES.

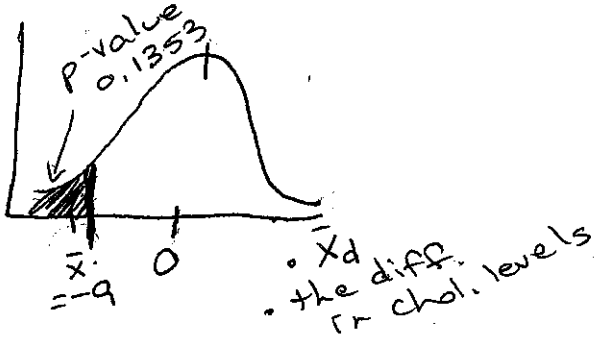
alpha	**decision**	**reason for decision**
$\alpha = 0.05$	Do not reject H_0	$\alpha <$ p-value

Conclusion:

We do not have enough evidence to show the cholesterol levels have changed.

i. In complete sentences, explain how you determined which distribution to use.

g)



$H_a: \mu_d < 0$
$\Rightarrow$ L-tailed test