Ch 9: Hypothesis Testing

A hypothesis test is a procedure to verify a claim about a population parameter based on sample data.

Examples:

- GE claims that its 60 W light bulbs last a minimum of 1000 hours
- Planter's claims that a can of its Mixed Nuts contains less than 50% peanuts
- A political party claims that at least 50% of voters will vote for their candidate.

P' = proportion of peanuts

$H_0: P' \leq 0.5$

$H_a: P' > 0.5$

Note: We will always put the "=" with H_0

Chapter 9 Practice 0: Writing Hypotheses

Write the null hypothesis H_0 and the alternative hypothesis H_a for each situation.

1. General Electric claims that the average life of its 60 WATT soft-white light bulbs is at least 1000 hours.

   μ or P' μ = # of hours a light bulb last on average

In Symbols:	In Words
H_0: $\mu \geq 1000$	The avg. light bulb lasts at least 1,000 hrs.
H_a: $\mu < 1{,}000$	The avg. light bulb lasts less than 1,000 hrs

2. An article in the San Jose Mercury News states that 75% of new teachers feel that they are underpaid.

   P' = proportion of teachers who feel underpaid

   H_0: $P' = 0.75$ 75% of teachers feel underpaid

   H_a: $P' \neq 0.75$ Either more or less than 75% of teachers feel underpaid

3. A can of Planter's Mixed Nuts claims that it contains less than 50% peanuts.

   P' = proportion of peanuts

   H_0: $P' \leq 0.5$ Proportion of peanuts is less than 50%

   H_a: $P' > 0.5$ Proportion of peanuts is more than 50%

4. A small bag of M&M plain candies contains a average of 40 calories

   μ = # of a cal

   H_0: $\mu = 40$

   H_a: $\mu \neq 40$

5. It takes a college student an average of over 5 years to obtain a B.A. degree.

   μ = # of yrs.

   H_0: $\mu \geq 5$

   H_a: $\mu < 5$

Steps to HYPOTHESIS TEST

step 1. Write 2 contradictory hypotheses
- Null Hypothesis H_0
- Alternate Hypothesis H_a

H_0	$\leq$	$\geq$	$=$
H_a	$>$	$<$	$\neq$ (> or <)

- Both hypotheses involve a single population parameter i.e. the same parameter

  μ Test of a Single Mean

  p Test of a Single Proportion

$x = 219.5$ peanuts
$n = 324.5$ nuts

Step 2. Collect Data from a sample of the population

3. Examine Sample Data

A. Determine the distribution to use

- Test of a Single Mean:

  1. If σ is known

  $$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

  * μ is from H_0
  * n are from data

  2. If σ is unknown

  $$T \sim t_{df} \quad (df = n-1)$$

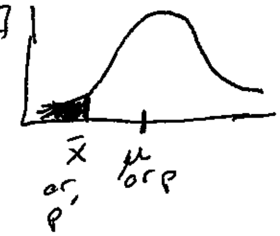
- Test of a Single Proportion

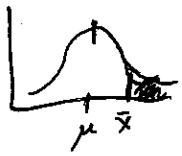
  $$P' \sim N\left(p, \sqrt{\frac{pq}{n}}\right)$$

  * p is H_0
  * n are from the data

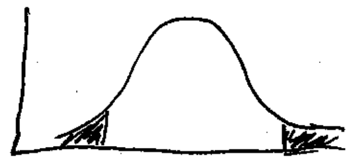
B. Determine the type of test: *This comes from H_a

- Left Tailed Test : If H_a: $\mu < \square$
  $p < \square$





- Right Tailed Test : If H_a: $\mu > \square$
  $p > \square$
- Two Tailed Test If H_a: $\neq$



C. Do Calculations (Calculator will do them) → p.133

- Test Statistic a z-score or t-score
  If it's a mean w/ σ known → z-score
  If it's a mean w/ σ unknown → t-score
  If it's a prob. → z-score
- p-value
  It is the probability that $\bar{x}$ or p' occurs by chance if you assume H_0 is true
- Pick a preconceived α In gen., $\alpha = 0.05$
  This is the cut-off value for a decision

2. Make a Decision:

- Reject H_0 We decide H_0 is false
- Do not Reject H_0 We Decide ~~H_0 is not false~~ there is not enough evidence to say H_0 is false

We Make our decision based on α:

- * If α>p-value Reject H_0 because the prob. of getting the data we got is too small
- * If α≤p-value Do not Reject H_0
- ~~If α≈p-value Test is Inconclusive~~

Important: We did not prove H_0 is true, only that it is reasonable

Statistics Calculator Instructions for TI83/84

Chapter 9 Hypothesis Testing Single Mean/Prop

A. Test of a Single Mean σ <u>known</u>	1: Z-Test
B. Test of a Single Mean σ <u>unknown</u>	2: T-Test
C. Test of a Single Proportion	5: 1-PropZTest

Chapter 10 Hypothesis Testing Two Means/Props

D. Test of Two Means σ <u>known</u>	3: 2-SampZTest
E. Test of two Means σ <u>unknown</u>	4: 2-SampTTest
F. Test of two Proportions	6: 2-PropZTest
G. Means: Dependent Groups/Paired Samples	2: T-Test

Points to remember:

- If you have data entered in L1 and/or L2, you can highlight **DATA.**
- If you have a book problem for which the mean and standard deviation are given, highlight **STATS**.

Mean z-score

STAT → TESTS → Z-test → STATS

μ_0 = null hyp.

σ = pop. std. dev.

$\bar{x}$ = sample avg. (from data)

n = sample size

μ: ... = from H_a

Calculate → p-value

Draw → graph (L, R or 2 tail test)

Mean t-score

STAT → TESTS → T-test → STATS

Proportion z-score

STAT → TESTS → 1-PropZTest

p_0 = null hyp.

x = # of successes

n = # of trials

prop: ... = from H_a

Ch. 9 Solution Sheet (for HW #5, 7, 9, 11, 13, 25)

a) H_0: $p \le 0.5$ H_a: $p > 0.5$

b) In words:

H_0:

H_a:

c) In words, clearly state what your random variable $\overline{X}$ or P' represents:

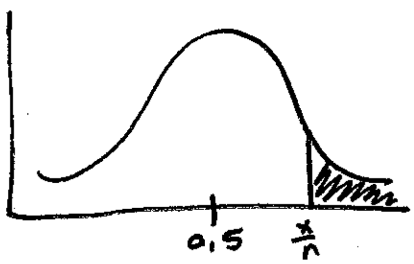
P' = proportion of peanuts

d) State the distribution of the test: $P' \sim N\left(p, \sqrt{\frac{pq}{n}}\right)$ Note: $p = 0.5$, $q = 0.5$, $n =$

e) Test Statistic: t or z = z

f) p-value= 1-PropZTest ≷ $p_0 = 0.5$, x = # peanuts, n = # total nuts, Calc $> p_0$, Draw

g) Sketch a picture of the situation.



h) Indicate the correct decision and conclusion:

alpha	decision	conclusion
0.05	$\alpha \overset{?}{<}$ p-value 0.05 < ...	We do not reject the null hyp. that the can is less than 50% peanuts

i) Construct a confidence interval.

1-PropZInt ($p' \cdot n$, n, 0.95) ← CL = 1 − α

Planter's nuts ex.

Ch. 9 Solution Sheet (for HW #5, 7, 9, 11, 13, 25)

1 a) H_0: $p' \leq 0.5$ H_a: $p' > 0.5$

b) In words:

H_0: A can of nuts is less than 50% peanuts

H_a: A can of nuts is more than 50% peanuts

c) In words, clearly state what your random variable $\overline{X}$ or P' represents:

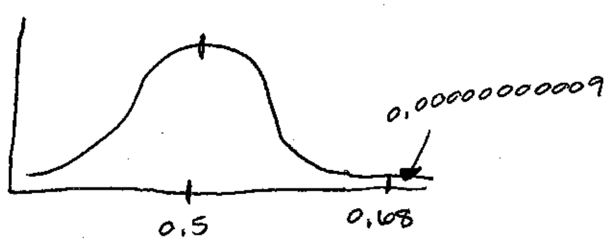
P' = proportion of peanuts

d) State the distribution of the test: $P' \sim N\left(0.5, \sqrt{\frac{(0.5)(0.5)}{324.5}}\right)$

e) Test Statistic: t or z = Z

f) p-value= 0.00000000009

g) Sketch a picture of the situation.



x = 219.5 peanuts

n = 324.5 total nuts

p = 0.5, q = 0.5

1-PropZTest

p_0 = 0.5

x = 220

n = 325

prop. > p_0

h) Indicate the correct decision and conclusion:

alpha	decision	conclusion
$\alpha = 0.05$	$\alpha >$ p-value	We reject H_o, and it is false that the avg. can is less than 50% peanuts

i) Construct a confidence interval.

Example: Statistics instructors believe that statistics students $\alpha = 0.05$ spend an average of 10 hours per week outside of class on study and assignments.

class decision

a. H_0: $\mu = 10$ H_a: $\mu < 10$

b. In words:

H_0: On avg. students spend 10 hrs working outside of class.

H_a: On avg. students spend less than 10 hrs working outside of class.

c. In words, clearly state what your random variable $\overline{X}$ or P' represents:

$\overline{X}$ = avg. # of hrs spent working outside of class

d. State the distribution of the test: $T \sim t_{13}$

(b/c we don't know σ)

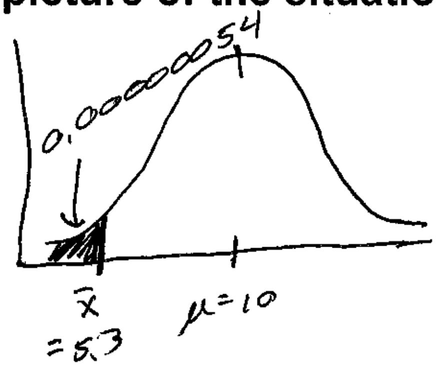
$t_{\alpha/2}$ or $z_{\alpha/2}$

e. **Test Statistic: t or z =** $t_{0.025}$

T-test

f. **p-value=** 5.4×10^{-7} = 0.00000054

g. **Sketch a picture of the situation.**



h. **Indicate the correct decision and conclusion:**

alpha	decision	conclusion
.05 >	p-value	We reject H_0
or if .05 <	p-value	We have no evidence to reject H_0

i. **Construct a 95% confidence interval.**

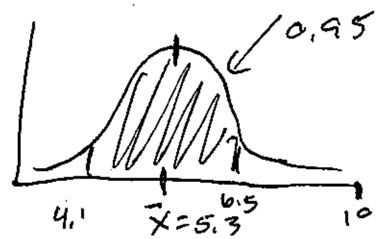
Ch.8

T Int.: $\bar{x} = 5.3$

$s_x = 2.06$

$n = 14$

Given our data, we are 95% confident that μ is between 4.1 & 6.5. hrs. working outside class.



Type I and Type II Errors

If our decision is:	If H_0 is actually	
	True	False
Reject H_0	Type I error	correct :)
Do not reject H_0	correct :)	Type II error

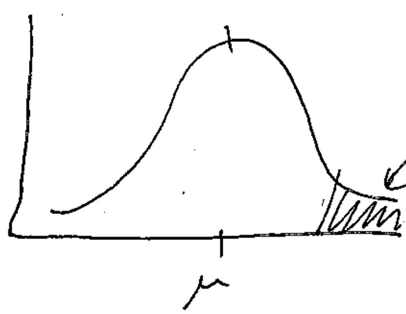
Type I error: We reject H_0 when, in fact, it's true.

Type II error: We do not reject H_0 when, in fact, it's false

α = probability of a Type I error

β = probability of a Type II error

- If $\alpha >$ p-value we reject H_0



There's an α% chance that H_0 is true & $\bar{x}$ lives in here (i.e. in the tail)

Example: Suppose the null hypothesis H_0 is: The parachute will open. You are about to take your first jump out of an airplane.

H_0: The parachute will open
H_a: The chute won't open

What is the Type I error?

The parachute will open but we conclude that it won't

What is the Type II error?

The parachute won't open, but we conclude that it will

Which has the worse consequence?

Type II

Example: GE claims the average life of a 60 watt bulb is at least 1000 hours.

H_0: $\mu \geq 1000$ H_a: $\mu < 1000$

Type I error:

The bulbs last on avg. at least 1,000 hrs, but ~~we~~ the hypothesis test concludes that they don't

Type II error:

The avg. life of a bulb is less than 1000 hrs, but ~~we~~ the hypothesis test concludes that it's more

Worse consequence for consumer:

Type II

Chapter 9 Practice 2
Type I and Type II Errors Worksheet

For each problem below:

- **Write the null and alternate hypotheses**
- **State the Type I and Type II Errors in terms of the problem**
- **State which error has the worse consequence and why**

1. The average time it takes for a student to graduate from college is at least 6.5 years.

H0: $\mu \geq 6.5$ yrs Ha: $\mu < 6.5$

Type I error: We wrongly conclude it takes less than 6.5 yrs. even though it actually takes longer

Type II error: We wrongly conclude it takes at least 6.5 yrs. when it actually takes less time

Worse consequence:

2. The average length of time needed for a car to accelerate from 0 to 60 is less than 10.3 seconds.

H0: $\mu \leq 10.3$ Ha: $\mu > 10.3$

Type I error: We incorrectly concluded the avg. time is more than 10.3 sec, when, in fact it's ~~the~~ true that the avg. time is less than or equal to 10.3 sec.

Type II error:
We incorrectly conclude the avg. time is less than 10.3 sec. when, in fact, it's ~~true~~ that it's more

Worse consequence:

3. More than 85% of computer crimes go unpunished.

H0: $p' \geq 0.85$ Ha: $p' < 0.85$

Type I error: We incorrectly conclude less than 85% go unpunished, when, in fact, the truth more than 85% go unpunished.

Type II error:

Worse consequence:

#7

Ch. 9 Solution Sheet

a) H_0: $\mu = 4.5$ H_a: $\mu > 4.5$

b) In words:

H_0: On avg. Cal. St. students take 4.5 yrs to grad.

H_a: On avg. Cal St. students take more than 4.5 yrs to grad.

c) In words, clearly state what your random variable $\overline{X}$ or P' represents: mean proportion

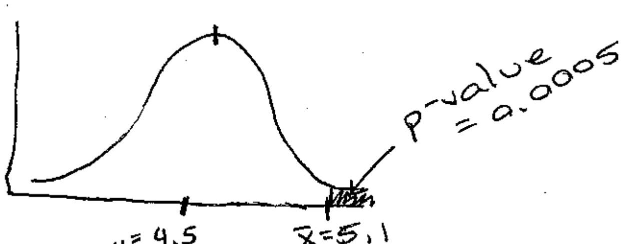
$\overline{X}$ = avg. time it takes 49 CSU students to grad.

d) State the distribution of the test: $T \sim t_{48}$

e) Test Statistic: t or z = t

f) p-value= 0.0005

g) Sketch a picture of the situation.

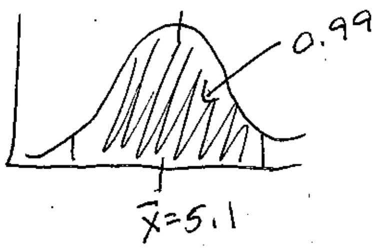


h) Indicate the correct decision and conclusion:

alpha	decision	conclusion
0.01	α > p-value 0.01 > 0.0005	We reject H_0 & we assume the true avg. is more than 4.5 yrs.

i) Construct a confidence interval.

Given $\bar{x} = 5.1$, we want a 99% CL CI



CI = (4.64, 5.56)

We're 99% conf. that the true avg. μ is b/t 4.64 yrs & 5.56 yrs.

$X = \#$ of successes $= 11$

$n = 84$

Example: A statistics instructor believes that fewer than 20% of Evergreen Valley College (EVC) students attended the opening midnight shows of the latest Harry Potter movie. She surveys 84 of her students and finds that 11 of them attended the midnight showing. Conduct a hypothesis test to see if the instructor was correct.

a) **H₀:** $p' \le 0.2$ **Hₐ:** $\cancel{p}' > 0.2$

b) ~~In words:~~

~~H_0:~~

~~H_a:~~

Type I error: The proportion of EVC students who attended the show is less than 20% but the hyp. test incorrectly concluded it was more than 20%

i.e. We incorrectly concluded ~~more~~ more than 20% attended when, in fact, it was less.

c) **In words, clearly state what your random variable $\overline{X}$ or P' represents:**

P' = the proportion of EVC students who attended the midnight showing

d) **State the distribution of the test:**

$P' \sim N\left(0.2, \sqrt{\frac{(0.2)(0.8)}{84}}\right)$

e) Test Statistic: ~~t~~ or z = z

f) p-value= 0.94

g) Sketch a picture of the situation.

p-value = 0.94

$p' = \frac{11}{84} = 0.13$

h) Indicate the correct decision and conclusion:

alpha	decision	conclusion
.05	$0.05 < 0.94$	We do not reject H_0 &
	$\alpha <$ p-value	we accept that less than 20% of students went

i) Construct a 95% confidence interval.

1-PropZInt → CI = (0.06, 0.20)