Much of this course pack is edited from:

Introductory Algebra Student Workbook

Sixth Edition

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KEY TERMS AND CONCEPTS Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms *in your own words*. Provide examples that are not identical to those in the Media Lesson.	
Inequality Symbols	
Algebraic Inequality	
Interval Notation	
Solution to an Algebraic Inequality	
Solution Set	

Unit 4: Main Lesson

Section 4.1: Inequalities

Symbol	In words	Examples
$<$		
$\leq$		
$>$		
$\geq$		
$\neq$		

Definitions
An **algebraic inequality** is a mathematical sentence connecting an expression to a value, variable, or another expression with an inequality sign. A **solution** to an inequality is a value that makes the inequality true.

Example 1: Determine whether the number 4 is a solution to the following inequalities.

$x > 1$ $x < 1$ $x \leq 9$ $x > 4$ $x \geq 4$

THE SOLUTION SET OF A LINEAR INEQUALITY

Inequality	Graph	Interval Notation
$x > 2$	$-\infty$ ∞ $-\infty$ ∞	
$x \geq 2$	$-\infty$ ∞ $-\infty$ ∞	
$x < 2$	$-\infty$ ∞ $-\infty$ ∞	
$x \leq 2$	$-\infty$ ∞ $-\infty$ ∞	

Translate a statement into an inequality

Example 2: Write an inequality to represent the following situation. Clearly indicate what the variable represents.

a. In order to go on the ride, a child must be more than 48 inches tall.

b. Jordan can spend at most $10 on lunch.

Section 4.1 – You Try

Complete the following problems.

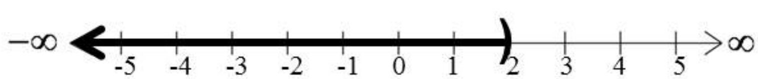
a. Which of the following values are in the solution set for $n < 5$?

$n = -3$ $n = 0$ $n = 4.99$ $n = 5$ $n = 12$

b. Translate the statement into an inequality. Let a represent the age of a child.

Children age 2 and under are free at Disneyland

c. Complete the table below:

Inequality	Graph	Interval Notation
$x \geq -3$		
		$(-\infty, 11]$
		

Section 4.2: Solving Linear Inequalities

STEPS FOR SOLVING A LINEAR INEQUALITY
1. Simplify each side of the inequality. Remove parenthesis if necessary. Collect like terms. 2. Add or subtract terms on each side of the inequality so that all terms containing the variable are on one side and all constant terms are on the other side. 3. Simplify each side of the inequality by combining like terms. *4.* Multiply or divide on both sides to isolate the variable. CAUTION*!!! If you multiply or divide both sides of an inequality by a* ***negative number****, you have to reverse the inequality sign.* 5. Check by substituting the solution (*endpoint and a value from the solution set*) into the original inequality.

Solve the inequality, check your answer, and graph the solution on a number line.

Example 1: Solve the inequality, check your answer, and graph the solution on a number line.

$3x > x + 6$

Graph:

$-\infty \longleftrightarrow \infty$

Interval Notation: ________________

Example 2: Solve the inequality and graph the solution on a number line.

$3 - 5a \leq 2(a + 5)$

Graph:

$-\infty \longleftrightarrow \infty$

Interval Notation: ________________

Example 3: Solve the inequality and graph the solution on a number line.

$-5(x + 2) \geq -3(x + 4)$

Graph:

$-\infty \longleftrightarrow \infty$

Interval Notation: _________________

Section 4.2 – You Try

Solve the inequality, check your answer, and graph the solution on a number line. Show all steps.

a. $7 - 4x \geq -5$

Graph:

$-\infty \longleftrightarrow \infty$

Interval Notation: _________________

.

b. $6x + 13 < 5(2x - 3)$

Graph:

$-\infty \longleftrightarrow \infty$

Interval Notation: _________________

Section 4.3: Solving Inequalities – Applications

For each problem, underline the **givens** and circle the **goal**. Form a **strategy**, **solve**, and **check**. Write your answer in a complete sentence.

Example 1: The cost of tuition is \$76 per credit hour. Write an *inequality* that can be used to determine the number of credit hours a student can take for under \$1000. Solve the inequality, and write your answer in a complete sentence.

Example 2: Sean owns a business that builds computers. The fixed operating costs for his business are \$2,700 per week. In addition to fixed operating costs, each computer costs \$600 to produce. Each computer sells for \$1,500. Write an *inequality* that can be used to determine the number of computers Sean needs to sell in order make a profit each week. Solve the inequality, and write your answer in a complete sentence.

Section 4.3 – YOU TRY

Gasoline costs \$3.79 per gallon.

a. Write an *inequality* that can be used to determine how many gallons of fuel can be purchased for under \$20. Clearly indicate what the variable represents.

b. Solve the inequality in part a, and write your answer in a complete sentence. Show all steps.

Unit 4: Practice Problems

Skills Practice

1. For each of the following, circle *all* correct answers.

   a. Which of the given values are in the solution set for $x < 3$?

   $x = 0$ $x = -1$ $x = -5$ $x = 3$ $x = 5$ $x = -\frac{5}{3}$

   b. Which of the given values are in the solution set for $x \geq -1$?

   $x = 0$ $x = -1$ $x = -5$ $x = 3$ $x = 5$ $x = -\frac{5}{3}$

   c. Which of the given values are in the interval $[-2, \infty)$?

   $x = 0$ $x = -1$ $x = -5$ $x = 3$ $x = 5$ $x = -\frac{5}{3}$

   d. Which of the given values are in the interval $(-\infty, -1)$?

   $x = 0$ $x = -1$ $x = -5$ $x = 3$ $x = 5$ $x = -\frac{5}{3}$

   e. Which of the given values are in the interval $(-1, 5]$?

   $x = 0$ $x = -1$ $x = -5$ $x = 3$ $x = 5$ $x = -\frac{5}{3}$

   f. Which of the given values are in the interval $-5 < x \leq 3$?

   $x = 0$ $x = -1$ $x = -5$ $x = 3$ $x = 5$ $x = -\frac{5}{3}$

2. Complete the table below:

Inequality	Graph	Interval Notation
$x > 8$	$-\infty$  ∞	
$x \leq -1$	$-\infty$  ∞	
	$-\infty$  ∞	$(-2, \infty)$
	$-\infty$  ∞	$(-\infty, 6]$

3. Solve the inequality, showing all steps. Write your answer as an inequality *and* in interval notation, then graph the solution set on the number line.

$4x \le 2x + 12$

Interval Notation: ________________

Graph:

$-\infty \longleftrightarrow \infty$

4. Solve the inequality, showing all steps. Write your answer as an inequality *and* in interval notation, then graph the solution set on the number line.

$14m + 8 > 6m - 8$

Interval Notation: ________________

Graph:

$-\infty \longleftrightarrow \infty$

5. Solve the inequality, showing all steps. Write your answer as an inequality *and* in interval notation, then graph the solution set on the number line.

$5(-2a - 8) \le -9a + 4$

Interval Notation: ________________

Graph:

$-\infty \longleftrightarrow \infty$

Applications

6. Translate each of the given statements into an algebraic inequality.

   a. You must be at least 13 years of age in order to view a PG-13 movie. Let *a* represent your age.

   b. Your car's gas tank can hold up to 25 gallons of gas. Let *g* represent the number of gallons in your gas tank.

   c. A company must sell more than 850 items in order to make a positive profit. Let *n* represent the number of items sold.

7. You have $1200 for your trip to the beach. You estimate that it will cost $160 a day for food, entertainment and hotel, plus $230 round trip air fair.

   a. Write an *inequality* that can be used to determine the maximum number of days you can stay at the beach. Clearly indicate what the variable represents.

   b. Solve the inequality, and interpret your answer in a complete sentence.

8. Let *p* represent the marked price of an item at Toys R Us. Bella's aunt gave her a $100 gift card to Toys R Us for her birthday.
   a. If sales tax is currently 9%, set up an algebraic *inequality* to express how much she can spend using her gift card. Clearly indicate what the variable represents.

   b. Solve the inequality, and interpret your answer in a complete sentence.

9. Your car is worth $1000 at most. It is old. You find out that it needs repairs to pass inspection. The auto shop tells you that the parts cost a total of $520, and the labor cost is $68 per hour. If the repairs are more than the car is worth, you are going to donate the car to charity.
   a. Write an *inequality* that can be used to determine the maximum number of hours the mechanic can spend working on your car to help you decide to repair it or donate it. Clearly indicate what the variable represents.

   b. Solve the inequality, and interpret your answer in a complete sentence.

10. You own a bike repair shop, and it costs you \$3020 per month to run your repair shop. You charge \$20 per hour for labor.
    a. Write an *inequality* that can be used to determine the minimum number of hours you need to spend working per month in order make a profit. Clearly indicate what the variable represents.

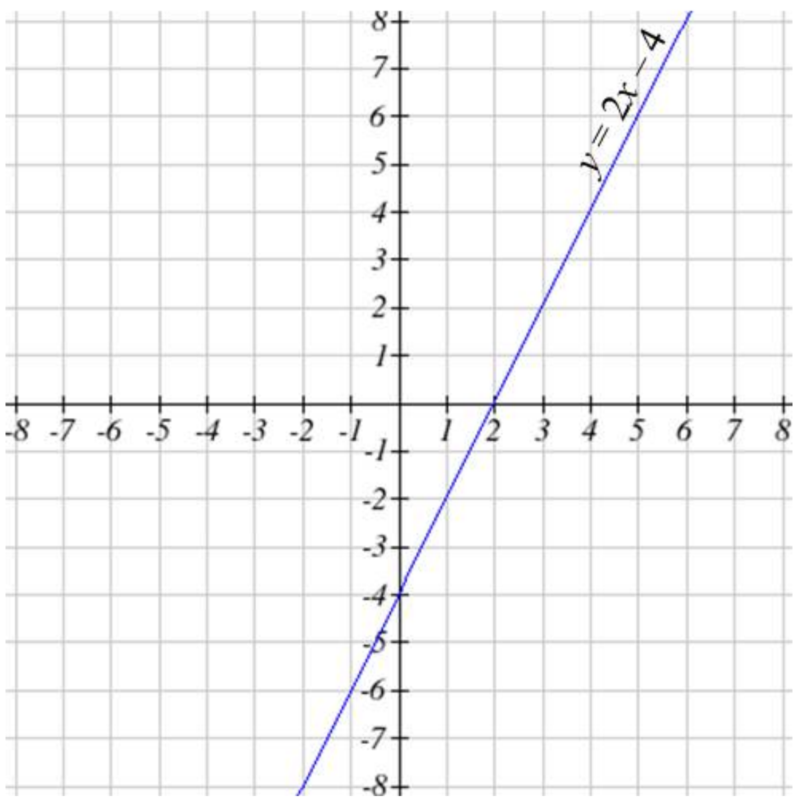
    b. Solve the inequality, and interpret your answer in a complete sentence.

Extension

11. Which of the following ordered pairs satisfy the **inequality** $y < 2x - 4$? Select all that apply and plot the selected points on the graph below.

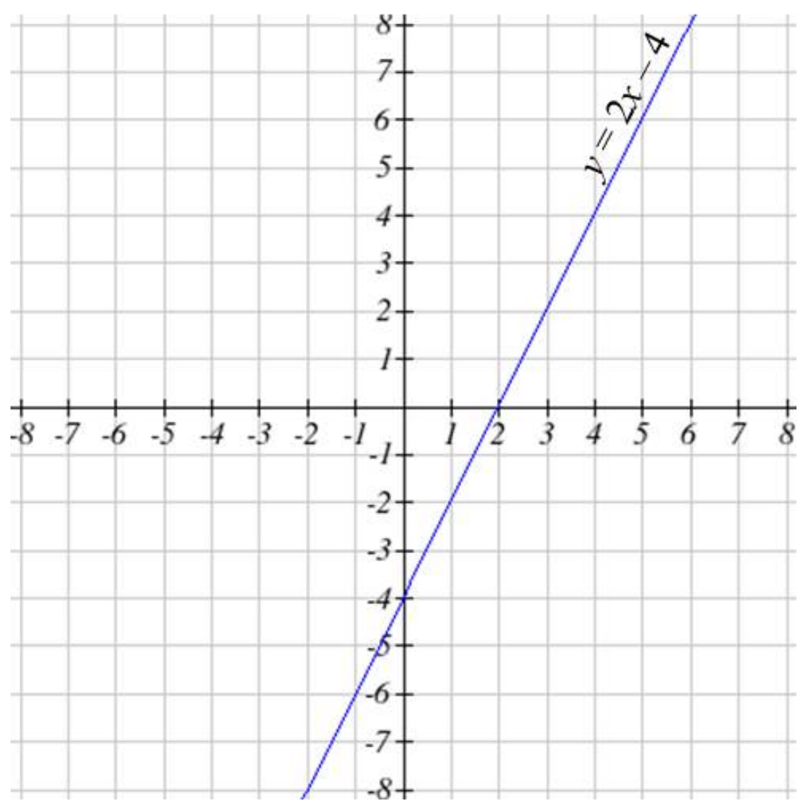
(–5, 2) (4, 1) (3, –6) (0, 0) (6, 4)

(7, 0) (1, –8) (–5, 6) (2, 0) (7, –5)



12. Which of the following ordered pairs satisfy the **inequality** $y \geq 2x - 4$? Select all that apply and plot the selected points on the graph below.

(–5, 2)	(4, 1)	(3, –6)	(0, 0)	(6, 4)
(7, 0)	(1, –8)	(–5, 6)	(2, 0)	(7, –5)



Unit 4: Review

1. Which of the given values are in the interval $(-1, \infty)$? Circle all that apply.

   $x = 0$ $x = -1$ $x = -5$ $x = 3$

2. Which of the given values are in the interval $(-3, 5]$? Circle all that apply.

   $x = 8$ $x = -2$ $x = -3$ $x = 5$

3. You have \$1400 for your trip to the beach. You estimate that it will cost \$250 a day for food, entertainment and hotel, plus \$198 for round trip air fair.

   a. Write an *inequality* that can be used to determine the maximum number of full days you can stay at the beach. Clearly indicate what the variable represents.

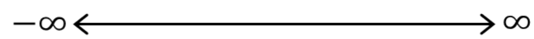
   b. Solve the inequality, and interpret your answer in a complete sentence.

4. Solve the inequality, showing all steps. Write your answer as an inequality *and* in interval notation, then graph the solution set on the number line.

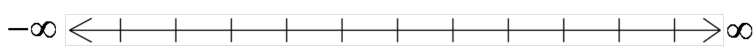
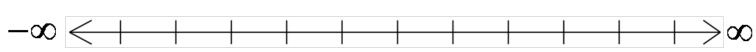
$1 - 3x > 14 - (4 - 6x)$

Interval Notation: ________________

Graph:



5. Complete the table below.

Inequality	Graph	Interval Notation
$x < 0$		
		$[-3, \infty)$

Unit 7: Introduction to Functions

Section 7.1: Relations and Functions

Section 7.2: Function Notation

Section 7.3: Domain and Range

Section 7.4: Practical Domain and Range

Section 7.5: Applications

KEY TERMS AND CONCEPTS Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms *in your own words*. Provide examples that are not identical to those in the Media Lesson.	
Relation	
Function	
Vertical Line Test	
Dependent Variable	
Independent Variable	

Behavior of Functions	
Function Notation	
Compare: Find $f(4)$ Find x when $f(x) = 4$	
Domain	
Range	
Practical Domain	
Practical Range	

Unit 7: Main Lesson

Section 7.1: Relations and Functions

Definitions

A **RELATION** is any set of ordered pairs.

A **FUNCTION** is a relation in which **every** input value is paired with **exactly one** output value

Table of Values

One way to represent the relationship between the input and output variables in a relation or function is by means of a table of values.

Example 1: Which of the following tables represent functions?

Input	Output
1	5
2	5
3	5
4	5

Yes No

Input	Output
1	8
2	–9
3	7
3	12

Yes No

Input	Output
2	4
1	–5
4	10
–3	–87

Yes No

Ordered Pairs

A relations and functions can also be represented as a set of points or ordered pairs.

Example 2: Which of the following sets of ordered pairs represent functions?

A = {(0, –2), (1,4), (–3,3), (5,0)}

B = {(–4,0), (2, –3), (2, –5)}

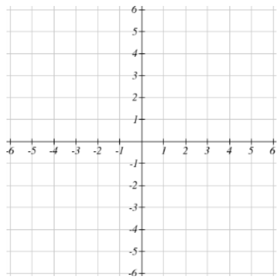
C = {(–5,1), (2,1), (–3,1), (0,1)}

D = {(3, –4), (3, –2), (0, 1), (2, –1)}

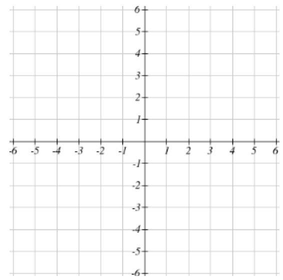
E = {(1,3)}

Example 3: On the graphs below, plot the points for A, B, C, and D from Example 2, then circle the "problem points"

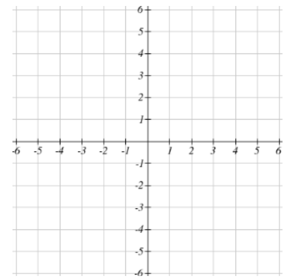
A



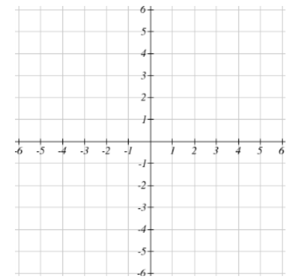
B



C



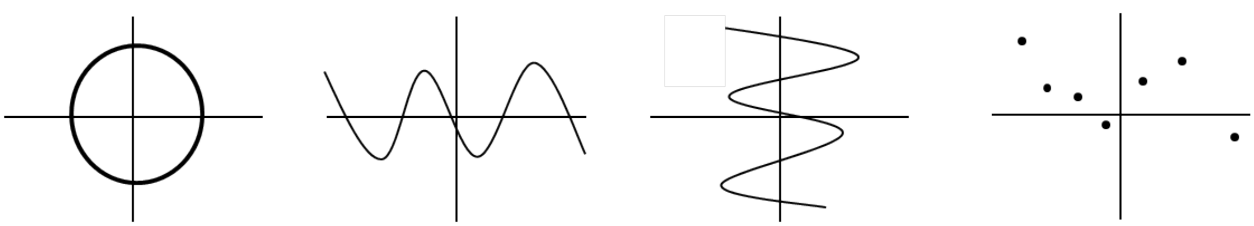
D



The Vertical Line Test

- If all vertical lines intersect the graph of a relation at no more than one point, the relation *is* also a function. One and only one output value exists for each input value.
- If any vertical line intersects the graph of a relation at more than one point, the relation "fails" the test and is NOT a function. More than one output value exists for some (or all) input value(s).

Example 4: Use the Vertical Line Test to determine which of the following graphs are functions.



Behavior of Graphs

Increasing	Decreasing	Constant

Dependent and Independent Variables

In general, we say that the output **depends** on the input.

Output variable = **Dependent Variable**

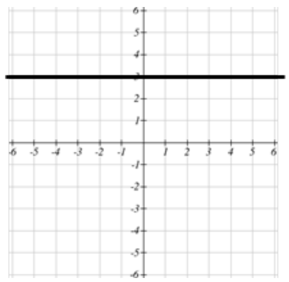
Input Variable = **Independent Variable**

If the relation is a function, then we say that the output **is a function of** the input.

Section 7.1 – You Try

Is it a function? Circle "Yes" or "No" for each of the following.

Yes or No



Yes or No

Input	Output
4	12
6	14
8	14
10	16

Yes or No

(2, –3), (–5, 2), (–3, 1)

Section 7.2: Function Notation: f(input) = output

If a relation is a function, we say that the *output is a function of the input.*

Function Notation: f(input) = output

Example: If y is a function of x, then we can write $f(x) = y$.

Example 1: The function V(m) represents value of an investment (in thousands of dollars) after m months. Explain the meaning of V(36) = 17.4.

Ordered Pairs

Example 2:

Ordered Pair (input, output)	Function Notation f(input) = output
(2, 3)	$f(2) = 3$
(–4, 6)	f(____) = _____
(_____ , _____)	$f(5) = -1$

Example 3: Consider the function: $f = \{(2, -4), (5, 7), (8, 0), (11, 23)\}$

$f(5)$ = _________ f(______) = 0

Table of Values

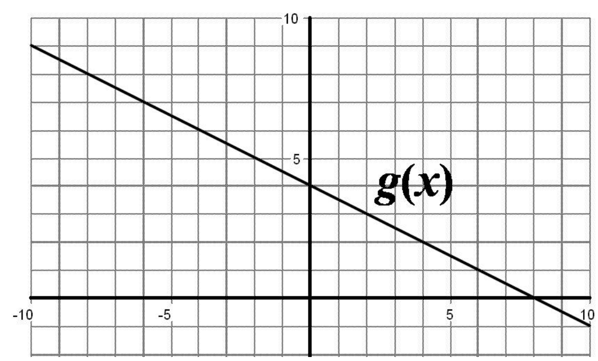
Example 4: The function $B(t)$ is defined by the table below.

t	1	3	12	18	22	31
$B(t)$	70	64	50	39	25	18

$B(12) =$ ______________________ $B(t) = 18$ when $t =$ ____________________

Graph

Example 5: Consider the graph $g(x)$ of shown below



$g(2) =$ ________

Ordered pair:__________

$g($______$) = 2$

Ordered pair:___________

$g(0) =$ ________

Ordered pair:__________

$g($______$) = 1$

Ordered pair:___________

Section 7.2 –You Try

Complete the problems below.

a. Complete the table.

Ordered Pair	Function Notation
(8, 1)	f(____) = _____
(_____ , _____)	$f(0) = 11$

b. The function $k(x)$ is defined by the following table

x	–2	–1	0	1	2	3	4
$k(x)$	8	2	–9	4	6	1	0

$k(2)$ = _________

Ordered Pair: ___________

$k(x) = 1$ when x = _______

Ordered Pair: ___________

c. At an ice cream factory, the total cost production is a function of the number of gallons of ice cream produced. The function C(g), gives the cost, in dollars, to produce g gallons of ice cream. Explain the meaning of $C(580)=126$ in terms of ice cream production.

Section 7.3: Domain and Range

DEFINITIONS
The **DOMAIN** of a function is the set of all possible values for the **input** variable. The **RANGE** of a function is the set of all possible values for the **output** variable.

DOMAIN AND RANGE

Example 1: Consider the function below

x	–2	0	2	4	6
$k(x)$	3	–7	11	3	8

Input values ______________________________________

Domain: {______________________________}

Output values:_____________________________________

Range: {______________________________}

Example 2: Consider the function: $B = \{(2, -4), (5, 7), (8, 0), (11, 23)\}$

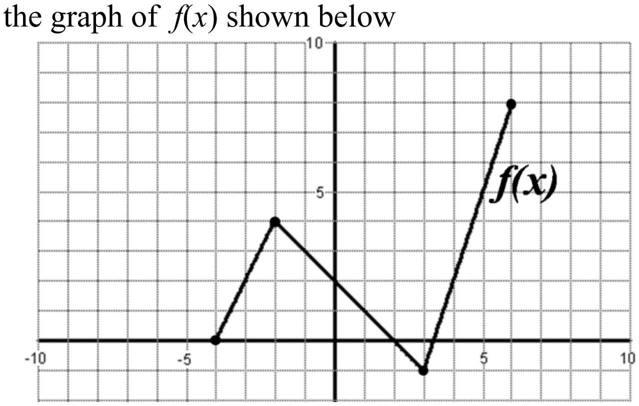
Input values ______________________________________

Domain: {______________________________}

Output values:_____________________________________

Range: {______________________________}

Example 3: Consider the graph of $f(x)$ shown below

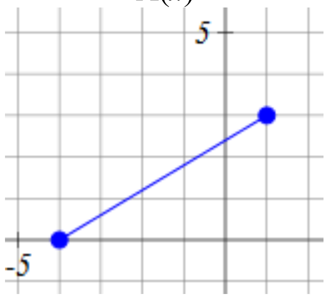


Domain: _______________ $\leq$ x $\leq$ ________________

Range: _______________ $\leq$ $f(x)$ $\leq$ ________________

Example 4: Determine the Domain and Range of each of the following graphs:

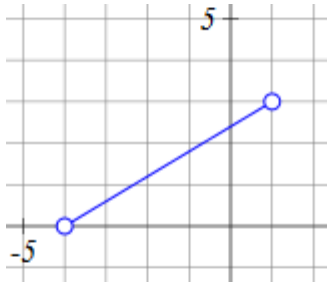
A(x)



Domain

Range

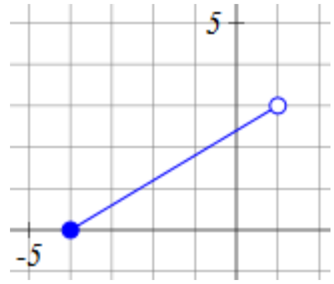
B(x)



Domain

Range

C(x)



Domain

Range

SECTION 7.3 – YOU TRY

Determine the Domain and Range of the functions below.

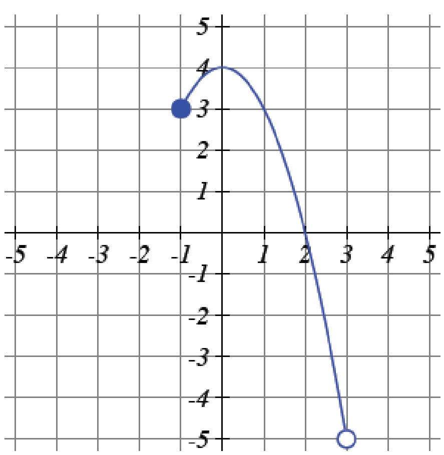
a.

Input	Output
4	12
6	12
8	12
10	12

Domain:

Range:

b. The graph of $f(x)$ is shown below



Domain:

Range:

Section 7.4: Practical Domain and Range

Definitions
The **Practical Domain** of a function is the set of all possible values for the input variable *that make sense* in a given situation. The **Practical Range** of a function is the set of all possible values for the output variable *that make sense* in a given situation.

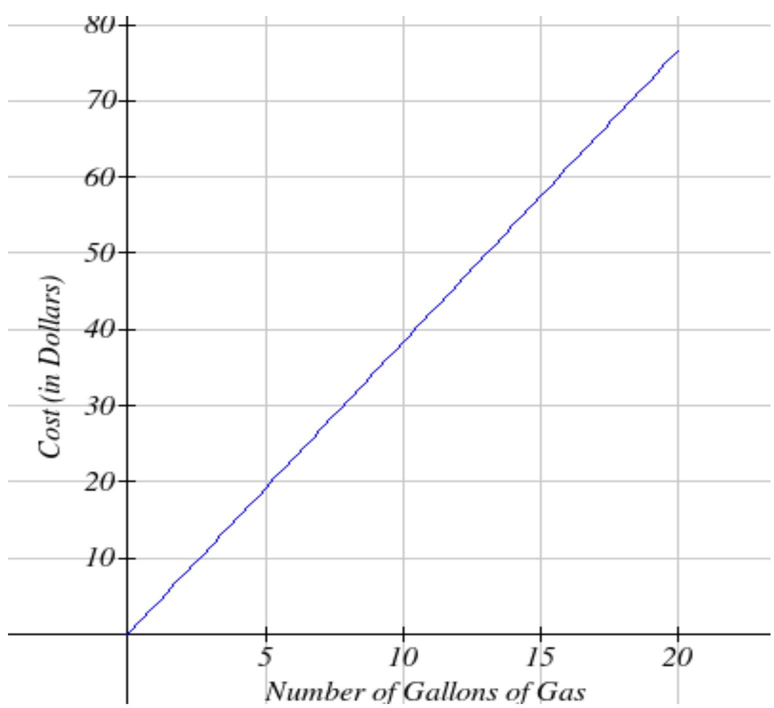
Example 1: The gas station is currently charging $3.83 per gallon for gas. The cost, C(n), in dollars, to fill up your car depends on the number of gallons, n, that you pump. Your car's tank can hold a maximum of 20 gallons of gas.

a. In this situation, the input variable is _______________________________________.

b. The *practical* domain of this function is _____________________________________.

c. The output variable in this situation is ______________________________________.

d. The *practical* range of this function is ______________________________________.



Section 7.4 – You Try

The platform for the high dive is 35 feet above the water. A diver jumps from the platform and lands in the water after 1.5 seconds. The function H(*s*) represents the height of the diver after *s* seconds.

a. In this situation, the input variable is ______________________________________.

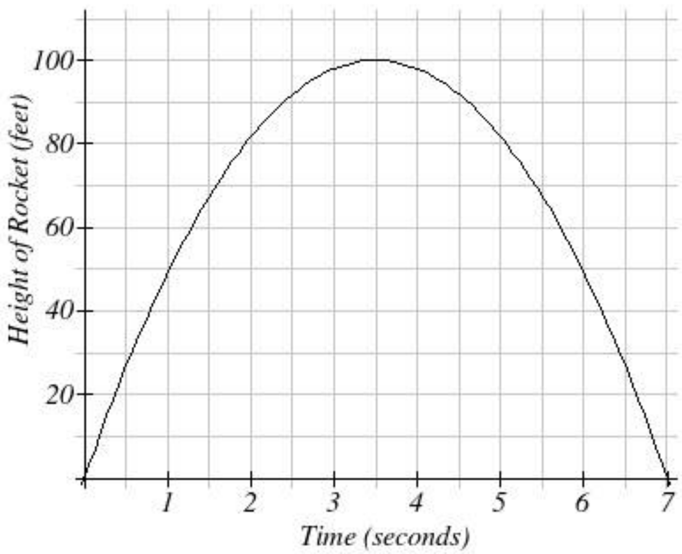
b. The *practical* domain of this function is ____________________________________.

c. The output variable in this situation is _____________________________________.

d. The *practical* range of this function is _____________________________________.

Section 7.5: Applications

Example 1: Consider the graph of the function H(*t*) shown below.



Input Variable: ________________

Units of Input Variable: __________

Output Variable: _______________

Units of Output Variable: __________

a. Interpret the meaning of the statement H(5)=82.

b. Determine H(7). Write it as an ordered pair and interpret its meaning in a complete sentence.

c. Determine t when $H(t) = 50$. Write it as an ordered pair and interpret its meaning in a complete sentence.

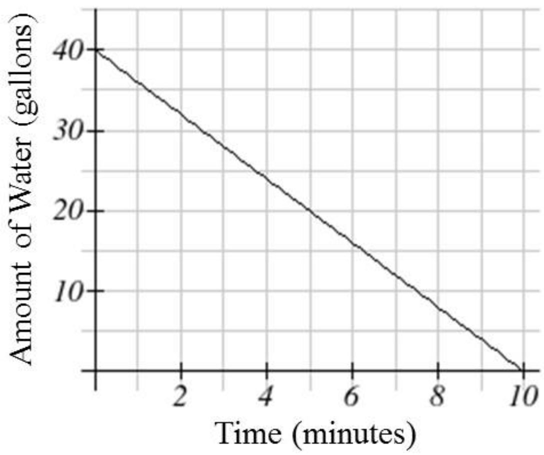
d. Determine the maximum height of the rocket.

e. Determine the practical domain for H(*t*).

f. Determine the practical range for H(*t*).

Section 7.5 – You Try

The graph of A(*m*) below shows the amount of water in a play pool.



Input Variable: ______________________

Units of Input Variable: _______________

Output Variable: ______________________

Units of Output Variable: _______________

a. Interpret the meaning of the statement A(3)=28.

b. Determine A(5). Write it as an ordered pair and interpret its meaning in a complete sentence.

c. Determine t when A(*m*) = 0. Write it as an ordered pair and interpret its meaning in a complete sentence.

d. Describe what is happening to the water in the pool. (Is the pool being filled or drained?)

e. Determine the practical domain for A(*m*). Use inequality notation and include units.

f. Determine the practical range for A(*m*). Use inequality notation and include units.

Unit 7: Practice Problems

Skills Practice

1. Are these functions? Circle yes or no.

Input	Output
3	12
7	12
4	12
2	12

Yes No

Input	Output
1	8
2	–9
3	7
3	12

Yes No

Input	Output
2	4
1	–5
4	10
–3	–87

Yes No

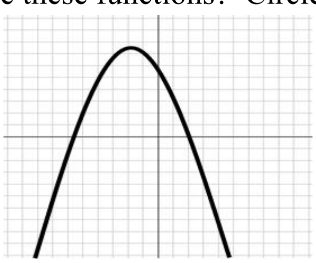
2. Are these functions? Circle yes or no.

   a. {(2, –4), (6, –4), (0, 0), (5, 0)} Yes No

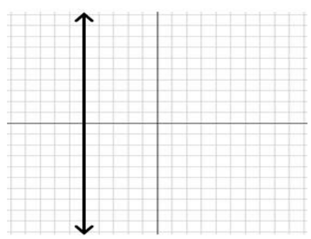
   b. {(1, 1), (2, 2), (3, 3), (4, 4)} Yes No

   c. {(1, –8), (5, 2), (1, 6), (7, –3)} Yes No

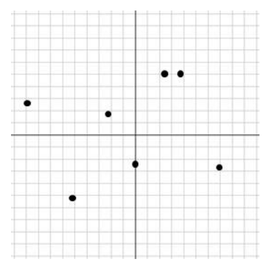
3. Are these functions? Circle yes or no.



Yes **No**



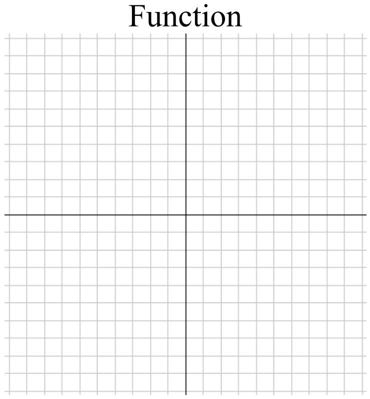
Yes **No**



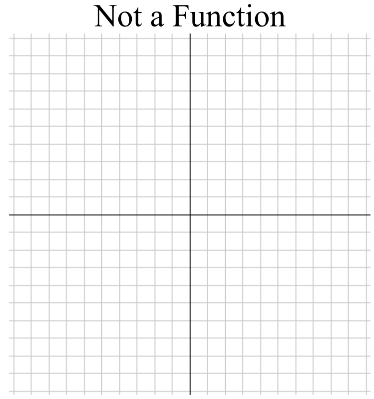
Yes **No**

4. In the space below, draw a graph that represents a function, and a graph that does NOT represent a function.

Function



Not a Function



5. The function $r(x)$ is defined by the following table of values.

x	3	5	6	9	13
$r(x)$	–9	3	2	2	1

a. $r(9)$ = ___________ b. $r(3)$ = _____________

c. $r($ _________ $) = 1$ d. $r($ ___________ $) = 3$

e. The domain of $r(x)$ is { ____ }

f. The range of $r(x)$ is { ____ }

6. Consider the function $g = \{(2, 5), \ (0, 6), \ (5, 8), \ (-3, 7)\}$

a. $g(0)$ = ___________ b. $g(5)$ = _____________

c. $g($ _________ $) = 7$ d. $g($ ___________ $) = 5$

e. The domain of g is { ____ }

f. The range of g is { ____ }

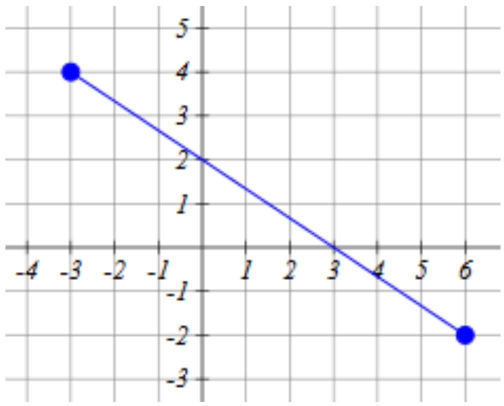
7. Given $f(4) = 8, f(3) = 11, f(0) = 6$

a. The domain of f is { ____ }

b. The range of f is { ____ }

c. Write the function f as a set of ordered pairs.

8. The graph of $g(r)$ is given below.



a. Domain: ________________________

b. Range ________________________

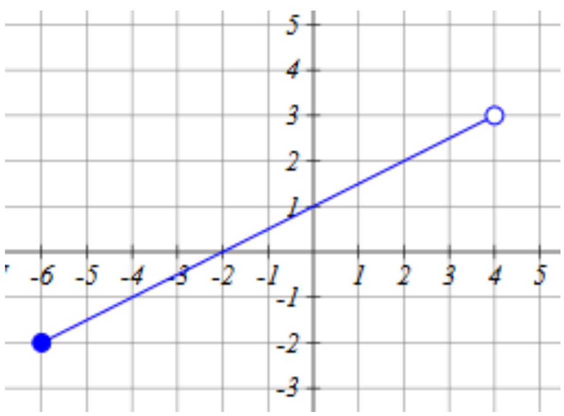
c. $g(-3) =$ _________

d. $g(0) =$ _________

e. $g(r) = 4$ when $r =$__________

f. $g(r) = 0$ when $r =$__________

9. The graph of $A(m)$ is given below.



a. Domain: ________________________

b. Range ________________________

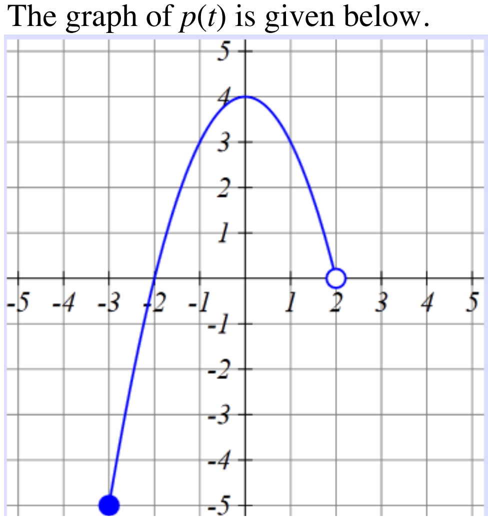
c. $A(-4) =$ _________

d. $A(0) =$ _________

e. $A(m) = -2$ when $m =$__________

f. $A(m) = 0$ when $m =$__________

10. The graph of $p(t)$ is given below.



a. Domain: ________________________

b. Range ________________________

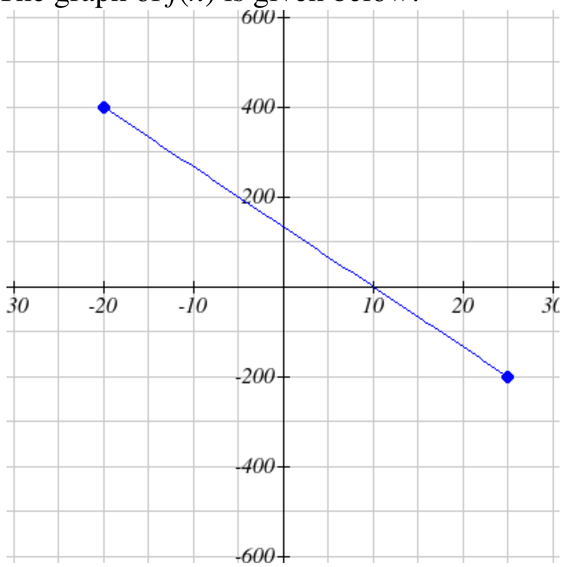
c. $p(-1) =$ _________

d. $p(0) =$ _________

e. $p(t) = -5$ when $t =$__________

f. $p(t) = 3$ when $t =$___________________

11. The graph of $f(n)$ is given below.



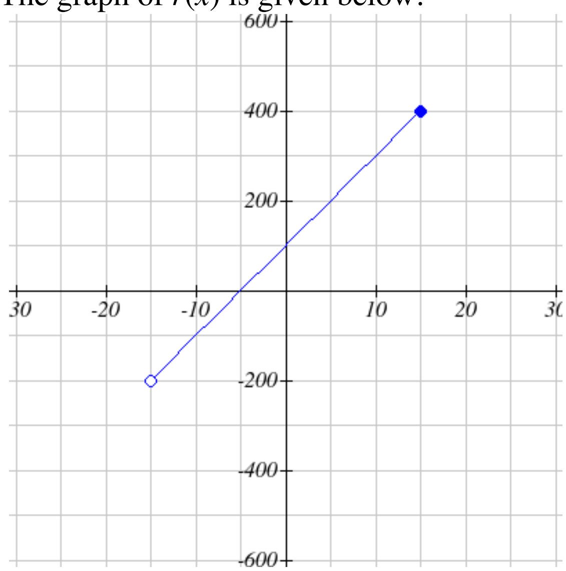
a. Domain: ________________________

b. Range ________________________

c. $f(-5)=$ _________

d. $f(n)=0$ when $n=$__________

12. The graph of $r(x)$ is given below.



a. Domain: ________________________

b. Range ________________________

c. $r(-10)=$ _________

d. $r(x)=300$ when $x=$__________

Applications

13. A rock is dropped from the top of a building. The function $H(t)$ gives the height (measured in meters) of the rock after t seconds. In a complete sentence, explain the meaning of the statement $H(2) = 35$. Your answer must include correct units.

14. The function $P(n)$ represents a computer manufacturer's profit, in dollars, when n computers are sold. In a complete sentence, explain the meaning of the statement $P(40) = 1680$. Your answer must include correct units.

15. The function $E(t)$ gives the surface elevation (in feet above sea level) of Lake Powell t years after 1999. In a complete sentence, explain the meaning of the statement $E(5) = 3675$. Your answer must include correct units.

16. The function $V(n)$ gives the value, in thousands of dollars, of an investment after n months. In a complete sentence, explain the meaning of the statement $V(24) = 18$. Your answer must include correct units.

17. The function $P(t)$ can be used to approximate the population of a town, in thousands of people, t years after 1980. In a complete sentence, explain the meaning of the statement $P(31) = 52$. Your answer must include correct units.

18. A candy company has a machine that produces candy canes. The table below is a partial list of the relationship between the number of minutes the machine is operating and the number of candy canes produced by the machine during that time period.

Minutes t	3	5	8	12	15
Candy Canes $C(t)$	12	20	32	48	60

a. Include units. $C(12) =$ ________________________________

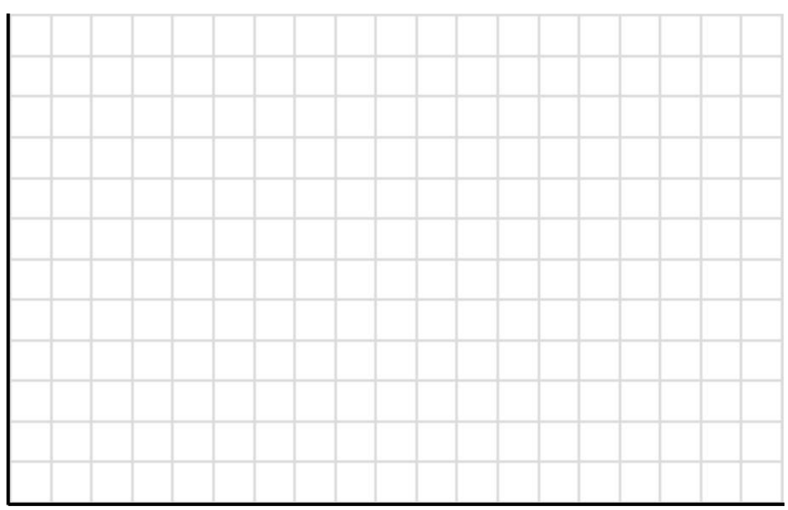
b. In a complete sentence and including all appropriate units, explain the meaning of your answer in part a.

c. Include units. $C(t) = 12$ when $t =$ ________________________________

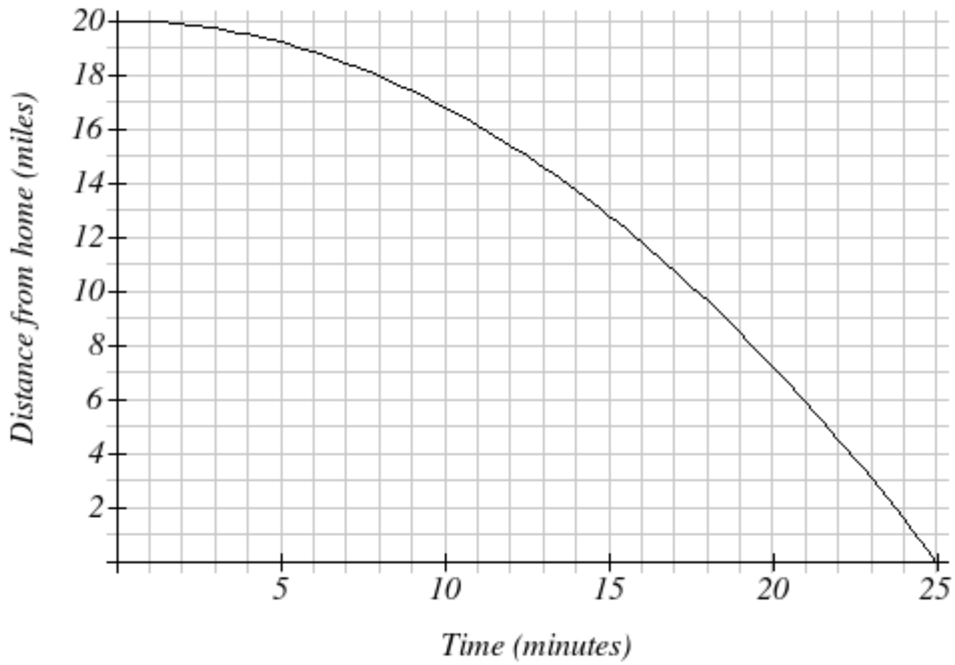
d. In a complete sentence and including all appropriate units, explain the meaning of your answer in part c.

e. This function is (circle one) **increasing** **decreasing**

f. Construct a properly scaled and labeled graph $C(t)$.



19. The function D(t) is shown below.



a. Determine $D(0)$ and interpret its meaning in a complete sentence.

b. Determine $D(8)$ and interpret its meaning in a complete sentence.

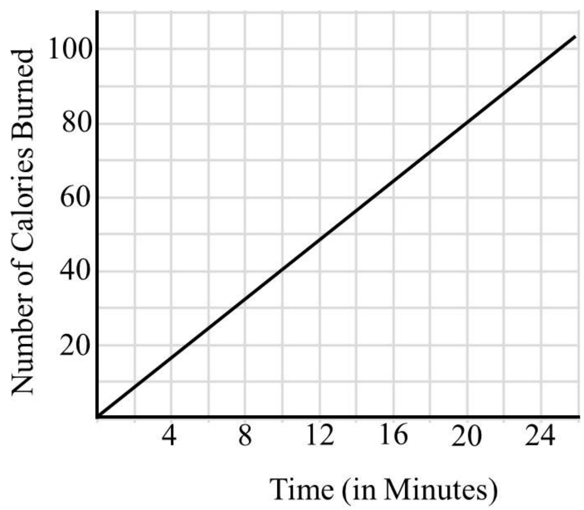
c. For what value of t is $D(t) = 3$? Write a sentence explaining the meaning of your answer.

d. For what value of t is $D(t) = 0$? Write a sentence explaining the meaning of your answer.

e. Determine the practical domain of $D(t)$.

f. Determine the practical range of $D(t)$.

20. The graph of the function $C(n)$ below shows the number of calories burned after riding a stationary bike for n minutes.



a. Is this function increasing or decreasing? ___________________________________

b. Interpret the meaning of the statement $C(8) = 32$.

c. Determine $C(10)$ and interpret its meaning in a complete sentence.

d. For what value of n is $C(n) = 80$? Write a sentence explaining the meaning of your answer.

Extension

21. Sort the following terms into the two groups below.

Dependent Variable

Domain

Horizontal Axis

Independent variable

Range

Vertical Axis

Input	Output

22. In a relation, we say that the output **depends** on the input. If the relation is a function, then we say that the output **is a function of** the input. For each of the following, identify the input variable and the output variable, and then determine if the relation is a function.

a. Is the outside temperature in Tempe, AZ a function of the time of day?

Input Variable: ____

Output Variable: ____

Function? Yes No

b. Is your letter grade a function of your numerical grade in the class?

Input Variable: ____

Output Variable: ____

Function? Yes No

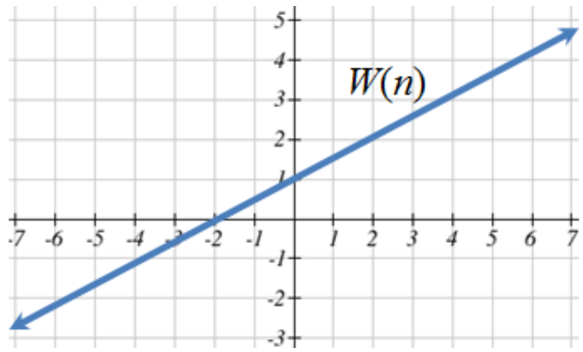
c. Is your numerical grade a function of your letter grade?

Input Variable: ____

Output Variable: ____

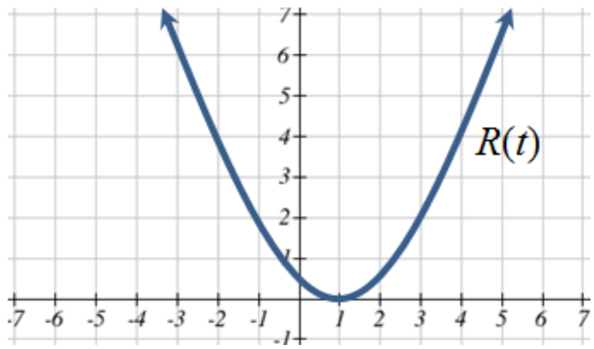
Function? Yes No

23. Determine the domain and range of each of the graphs shown below. Use correct notation.



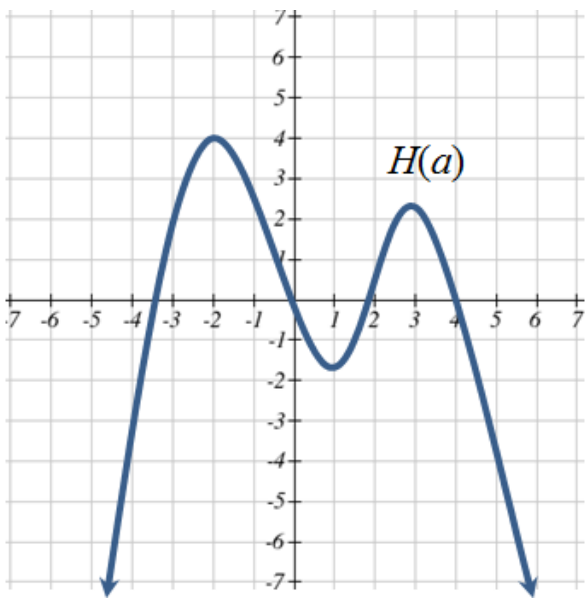
Domain: ________________________

Range: _________________________



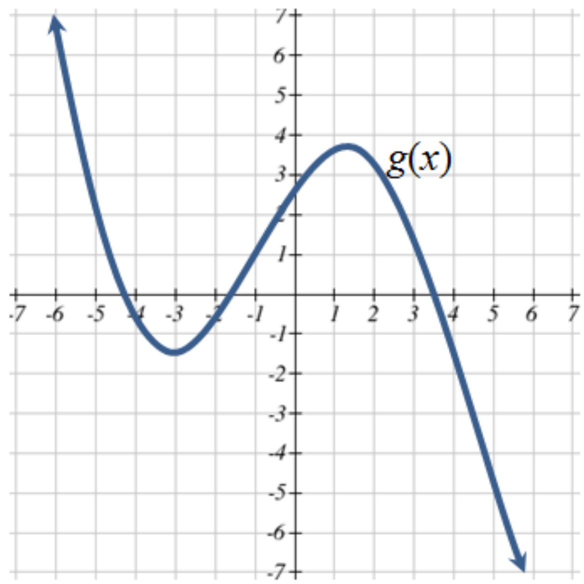
Domain: ________________________

Range: _________________________



Domain: ________________________

Range: _________________________



Domain: ________________________

Range: _________________________

24. Is "square root of *x*" a function? Let's answer a few other questions first.

a. Find 2^2

b. Find $(-2)^2$

c. We say *z* is **a square root** of *x* if $z^2 = x$. With this in mind, are 2 and -2 **both** square roots of 4? Explain your answer in a sentence.

d. We define $\sqrt{x}$ to be the **non-negative square root** of *x*. With this in mind, find $\sqrt{4}$. (Hint: You should only get one answer.)

e. Is $y = \sqrt{x}$ a function? Why or why not?

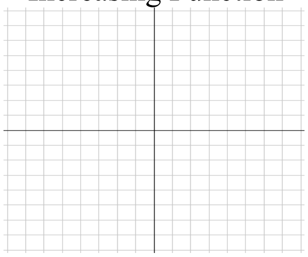
f. If *y* is any square root of *x*, we write $y = \pm\sqrt{x}$? Is $y = \pm\sqrt{x}$ a function? Why or why not?

g. Finally, let's answer our original question. Is "square root of *x*" a function? Justify your answer in 1-2 sentences.

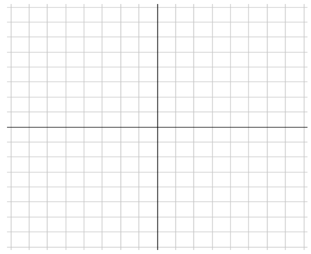
Unit 7: Review

1. In the space below, draw a graph that represents an increasing function, a constant function, and a graph that does NOT represent a function.

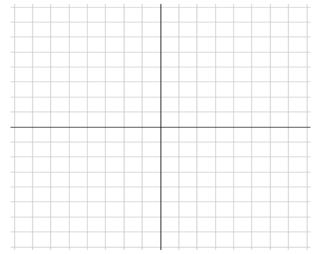
Increasing Function



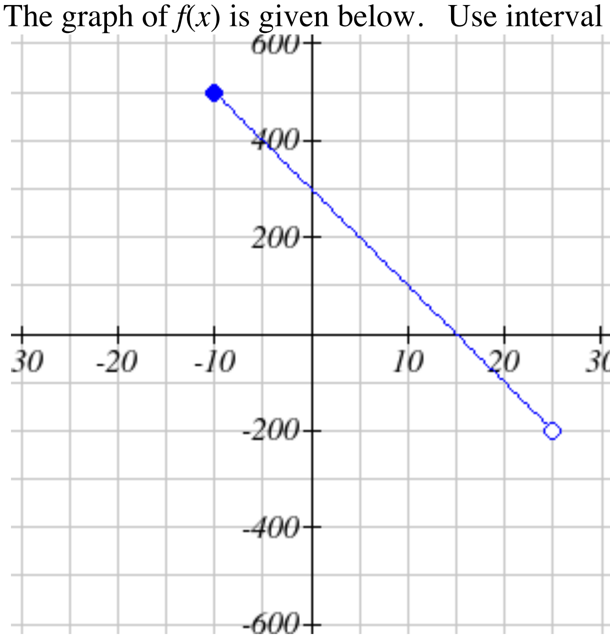
Constant Function



Not a Function



2. The graph of $f(x)$ is given below. Use interval notation for the domain and range.



a) Domain: _______________

b) Range ______________

c) $f(0) =$ ______

d) $f(x) = 0$ when $x =$_______

3. Consider the following table of values. Fill in the blanks below, and identify the corresponding ordered pairs.

x	–2	–1	0	1	2	3	4
g(x)	1	4	8	6	5	0	2

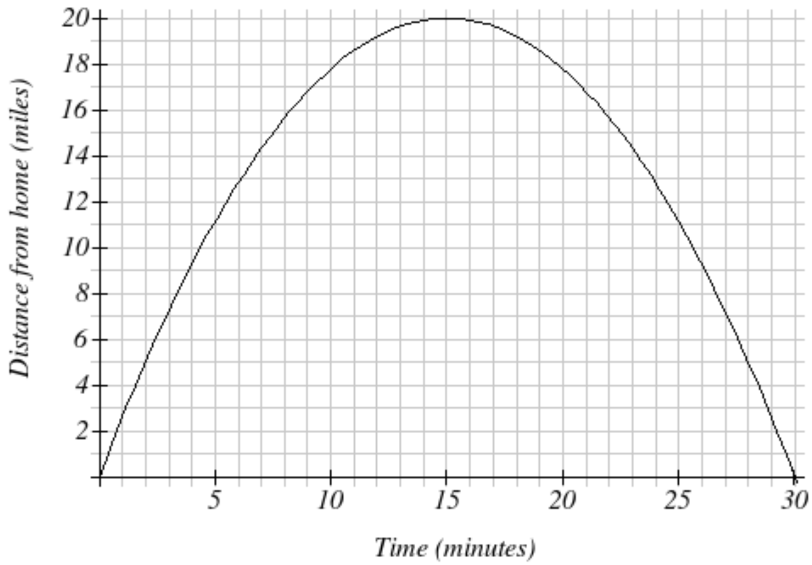
$g(1) =$ _______

Ordered pair: _______________

$g(x) = 1$ when $x =$ _________

Ordered Pair: _______________

4. The function D(*t*) shown below represents Sally's distance from home over a 30-minute time period.



a. Identify the vertical intercept of D(*t*). Write it as an ordered pair and explain its meaning in this situation.

b. Identify the horizontal intercepts of D(*t*). Write them as an ordered pairs and explain their meaning in this situation.

c. Determine *D*(15) and interpret its meaning in a complete sentence.

d. For what value of *t* is *D*(*t*) = 5? Write a sentence explaining the meaning of your answer.

e. Determine the practical domain of *D*(*t*). ______________________________________

f. Determine the practical range of *D*(*t*). ______________________________________

Unit 8: Formulas and Functions

Section 8.2: Formulas in Function Notation

Section 8.3: Formulas in Function Notation – Applications

Section 8.5: Connecting Representations

Section 8.6: Applications

KEY TERMS AND CONCEPTS Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms *in your own words*. Provide examples that are not identical to those in the Media Lesson.	
Input	
Output	
Function Notation	
Ordered Pair	

Formula	
Four Representations of a Function	
Compare: Find $f(4)$ Find x when $f(x) = 4$	

Unit 8: Main Lesson

Section 8.2: Formulas in Function Notation

Example 1: Let $f(x) = x^2 - 2x + 11$

a. Determine $f(-3)$

b. Determine $f(0)$

c. What is the domain of $f(x)$?

Example 2: Let $h(x) = 2x - 5$

a. Determine $h(4)$

b. For what value of x is $h(x) = 17$?

c. What is the domain of $h(x)$?

d. What is the range of $h(x)$?

Example 3: Let $g(x) = 71$

a. Determine $g(5)$.

b. Determine $g(-40)$.

c. What is the domain of $g(x)$?

d. What is the range of $g(x)$?

Example 4: Let $f(x) = \sqrt{x - 2}$

a. Determine $f(11)$.

b. Determine $f(1)$.

c. What is the domain of $f(x)$?

d. What is the range of $f(x)$?

Example 5: Let $g(x) = \frac{7}{x}$

a. Determine $g(7)$.

b. Determine $g(0)$.

c. What is the domain of $g(x)$?

d. What is the range of g(x)?

Section 8.2 – You Try

Let $r(a) = 4 - 5a$. Show all steps. Write each answer using function notation **and** as an ordered pair.

a. Determine $r(-2)$.

b. For what value of a is $r(a) = 19$?

Section 8.3: Formulas in Function Notation – Applications

Example 1: Grace is selling snow cones at a local carnival. Her profit, in dollars, from selling x snow cones is given by the function $P(x) = 2.5x - 30$.

a. Write a complete sentence to explain the meaning of $P(30) = 45$ in words.

b. Determine $P(10)$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________

c. Determine $P(0)$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________

d. Determine x when $P(x) = 100$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________

e. Determine x when $P(x) = 0$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________

Section 8.3 – You Try

The function $T(a) = 0.7(220 - a)$, gives the target heart rate, in beats per minute, for a person who is a years of age.

a. Write a complete sentence to explain the meaning of $T(30) = 133$ in words.

b. Determine $T(50)$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________

c. Determine a when $T(a) = 140$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________

Section 8.5: Connecting Representations

There are four main ways to **represent** a function.

1. Verbally in words
2. Table of values or list of ordered pairs
3. Formula
4. Graphically

Example 1: Identify the pattern from the **table**, and use that information to construct the **graph** and determine the **formula** for the function $g(x)$. Then use **words** to describe the relationship between the input and output variables.

x	$g(x)$	Ordered Pair
-3	-6	
-2	-4	
-1	-2	
0	0	
1	2	
2	4	
3	6	

Formula: $g(x)$ = _______________

In words:

Example 2: Use the **formula** for $H(t)$ to complete the **table**. **Graph** the results. Then use **words** to describe the relationship between the input and output variables.

Formula: $H(t) = -t - 2$

t	$H(t)$	Ordered Pair
–3		
–2		
–1		
0		
1		
2		
3		

In words:

Example 3: Use the **description** of the function $f(x)$ to complete the **table**. **Graph** the results and determine a **formula** for the function $f(x)$.

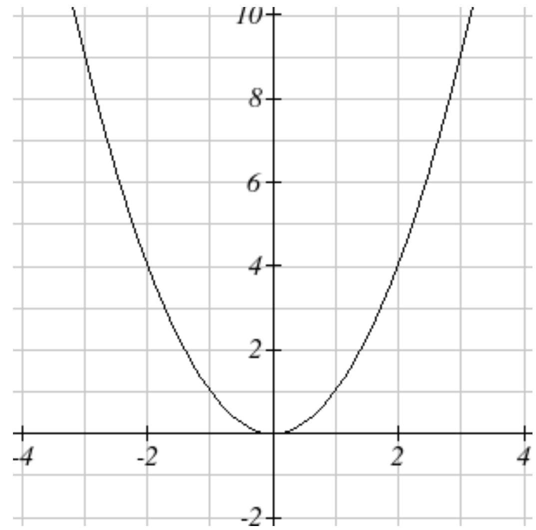
The function f(x) doubles the input value, then adds 5 to the result.

x	$f(x)$	Ordered Pair
-3		
-2		
-1		
0		
1		
2		
3		

Formulas: $f(x)$= _______________

Example 4: Refer to the **graph** of *k*(*n*) to complete the **table** of values. Determine the **formula** for the function *k*(*n*), then use **words** to describe the relationship between the input and output variables.

n	*k*(*n*)	Ordered Pair
-3		
-2		
-1		
0		
1		
2		
3		



Symbolic Rule: *k*(*n*) = _______________

In words:

Section 8.5 – You Try

Identify the pattern from the table, and use that information to construct the graph and determine the formula for the function $g(t)$. Then use words to describe the relationship between the input and output variables.

t	$g(t)$	Ordered Pair
-3	-1	
-2	0	
-1	1	
0	2	
1	3	
2	4	
3	5	

Symbolic Rule: $g(t)$ = _______________

In words:

Section 8.6: Applications

Example 1: A local towing company charges \$3.25 per mile driven plus a nonrefundable base fee of \$30.00. They tow a maximum of 25 miles.

a. Write a formula for the function $C(x)$ which represents total cost as a function of the number of miles driven.

b. Determine C(15). Write your answer as ordered pair then explain its meaning in a complete sentence.

c. Determine the value of x when $C(x) = 82$. Write your answer as ordered pair then explain its meaning in a complete sentence.

d. Identify the practical domain and practical range of this function by filling in the blanks below. Include units in your answers.

Practical Domain: _____________ $\leq x \leq$ ______________

Practical Range: _____________ $\leq C(x) \leq$ ______________

e. Construct a table of values and draw a good graph of $C(x)$

x	$C(x)$

Section 8.6 – You Try

The value, in dollars, of a washer/dryer set decreases as a function of time t in years. The function $V(t) = -125t + 1500$ models this situation. You own the washer/dryer set for 12 years.

a. Determine V(5). Show your work. Write your answer as ordered pair then explain its meaning in a complete sentence.

b. Determine the value of t when $V(t) = 500$. Show your work. Write your answer as ordered pair then explain its meaning in a complete sentence.

c. Identify the practical domain and practical range of this function by filling in the blanks below. Include units in your answers.

Practical Domain: _____________ $\leq t \leq$ ______________

Practical Range: _____________ $\leq V(t) \leq$ ______________

d. Construct a table of values and draw a good graph of $V(t)$

t	*V*(*t*)

Unit 8: Practice Problems

Skills Practice

1. Let $W(p) = 4p^2 - 9p + 1$. Show all steps. Write each answer in function notation *and* as an ordered pair.

a. Determine W(5) .

b. Determine W(0).

2. Let $k(m) = 8 - 3m$. Show all steps. Write each answer in function notation *and* as an ordered pair.

a. Determine $k(5)$.

b. For what value of m is $k(m) = 29$?

c. What is the domain of $k(m)$?

d. What is the range of $k(m)$?

3. Let $R(t) = 1500 + 40t$. Show all steps. Write each answer in function notation *and* as an ordered pair.

a. Determine $R(18)$.

b. For what value of t is $R(t) = 3000$?

4. Let $h(x) = 4$. Show all steps. Write each answer in function notation *and* as an ordered pair.

a. Determine $h(5)$.

b. Determine $h(81)$.

5. Let $p(x) = \frac{45}{2x}$. Show all steps. Write each answer in function notation *and* as an ordered pair.

a. Determine $p(5)$.

b. Determine $p(-6)$

c. What is the domain of $p(x)$?

d. What is the range of $p(x)$?

6. Identify the pattern from the table, and use that information to construct the graph and determine the formula for the function $g(x)$. Then use words to describe the relationship between the input and output variables.

x	$g(x)$	Ordered Pair
-3	3	
-2	2	
-1	1	
0	0	
1	-1	
2	-2	
3	-3	

Symbolic Rule: $g(x)$ = _______________

In words:

7. Use the formula for $H(t)$ to complete the table. Graph the results. Then use words to describe the relationship between the input and output variables.

Symbolic Rule: $H(t) = 5 - t^2$

t	$H(t)$	Ordered Pair
–3		
–2		
–1		
0		
1		
2		
3		

In words:

8. Use the description of the function $f(x)$ to complete the table. Graph the results and determine a symbolic rule for the function $f(x)$. Then use words to describe the relationship between the input and output variables.

The function f(x) subtracts 3 from the input.

x	$f(x)$	Ordered Pair
-3		
-2		
-1		
0		
1		
2		
3		

Symbolic Rule: $f(x)$ = _______________

Applications

9. A rock is dropped from the top of a building. The function $h(t) = 100 - 16t^2$ gives the height (measured in feet) of the rock after t seconds .

a. Complete the table below.

t	0	0.5	1	1.5	2	2.5
$h(t)$						

b. Is this function increasing or decreasing? ___________________________

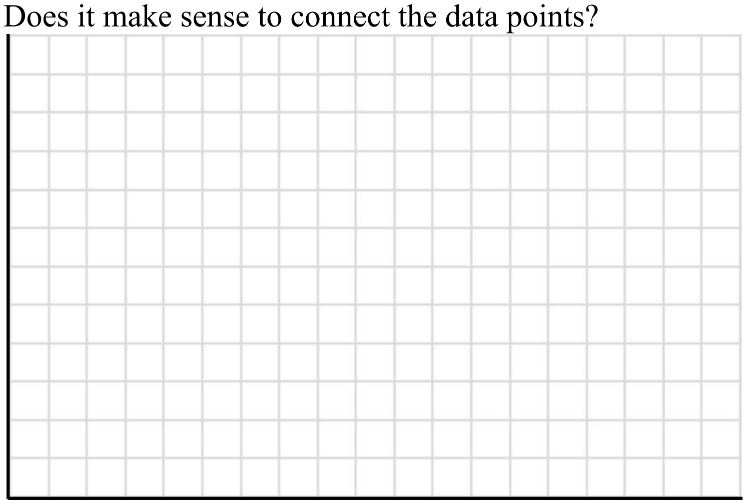
c. Determine $h(1)$. Write a sentence explaining the meaning of your answer.

d. For what value of t is $h(t) = 0$? Explain the meaning of your answer.

e. Determine the practical domain ____

f. Determine the practical range ____

g. Construct a good graph of $h(t)$. Does it make sense to connect the data points?



10. John is a door to door vacuum salesman. His weekly salary, in dollars, is given by the linear function $S(v) = 200 + 50v$, where v is the number of vacuums sold.

a. Determine $S(12)$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________

b. Determine $S(0)$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________

c. Determine v when $S(v) = 500$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________

11. The function $P(n) = 455n - 1820$ represents a computer manufacturer's profit, in dollars, when n computers are sold.

a. Write a complete sentence to explain the meaning of $P(5) = 455$ in words.

b. Determine $P(10)$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________

c. Determine $P(0)$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________

d. Determine n when $P(n) = 0$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________

12. The function $V(n) = 221.4 + 4.25n$ gives the value, in thousands of dollars, of an investment after n years. Determine $V(20)$, and write a sentence explaining the meaning of your answer.

13. The function $E(t) = 3861 - 77.2t$ gives the surface elevation (in feet above sea level) of Lake Powell t years after 1999.

a. Determine E(0), and write a sentence explaining the meaning of your answer.

b. Determine E(4), and write a sentence explaining the meaning of your answer.

c. This function accurately models the surface elevation of Lake Powell from 1999 to 2005. Determine the practical range of this linear function.

Extension

14. For a part-time student, the cost of tuition at a local community college is $85 per credit hour. The function C(n) gives the tuition cost for n credit hours. As a part-time student, Gabe can take a maximum of 11 credit hours.

a. Identify the input variable in this situation:______________________________________

b. Identify the output variable in this situation:______________________________________

c. Write a formula (symbolic rule) for the function C(n): C(n) = ____________________

d. Complete the table below and construct a properly scaled and labeled graph of C(n).

n	$C(n)$
0	
1	
2	
3	
5	
8	
11	

e. Does it make sense to connect the points on the graph? Why or why not?

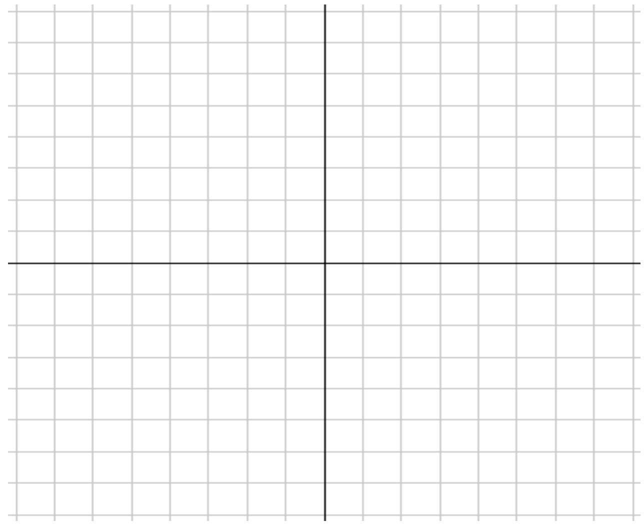
f. Determine the practical domain of C(n):______________________________________

g. Determine the practical range of C(n): ______________________________________

Unit 8: Review

1. Graph the function $p(r) = 3 - r$

r	$p(r)$	Ordered Pair



3. A local towing company charges $5.50 for each mile plus a reservation fee of $12. They tow a maximum of 30 miles.

   a. Write a formula for the function C(x), representing the total cost to tow the car x miles.

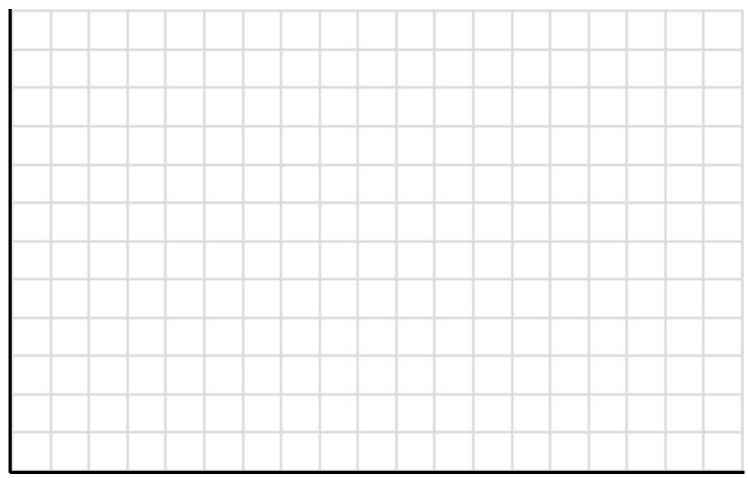
   b. Determine C(8). Show your work. Write your answer as an ordered pair and interpret its meaning in a complete sentence.

c. Determine x when $C(x) = 100$. Show your work. Write your answer as an ordered pair and interpret its meaning in a complete sentence.

d. Practical domain (include units): __________________ $\leq x \leq$ ____________________

e. Practical range (include units): ___________________ $\leq C(x) \leq$ __________________

f. Construct a good graph of $C(x)$.



4. The demand for Cupertino Skate Co. Skateboards, in weekly sales, is modelled by the function $q(p) = -4p + 64$, where p is the price per board in dollars. How many boards will be sold per week if they are priced at \$9 each? Determine the value of p that gives $q(p) = 20$, and interpret the meaning of this value of p.

Unit 9: Introduction to Linear Functions

Section 9.1: Intercepts

Section 9.2: Horizontal and Vertical Lines

Section 9.3: Linear Functions

Section 9.4: Graphing Linear Functions

Section 9.5: Interpreting the Slope of a Linear Function

Section 9.6: Using Rates of Change to Build Tables and Graphs

Section 9.7: Is the Function Linear?

KEY TERMS AND CONCEPTS Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms *in your own words*. Provide examples that are not identical to those in the Media Lesson.	
Horizontal Intercept	
Finding the Horizontal Intercept given an equation	
Vertical Intercept	

Finding the Vertical Intercept given an equation	
Horizontal Line	
Vertical Line	
Linear Functions	
Slope	
Using Slope to Graph a Linear Function	

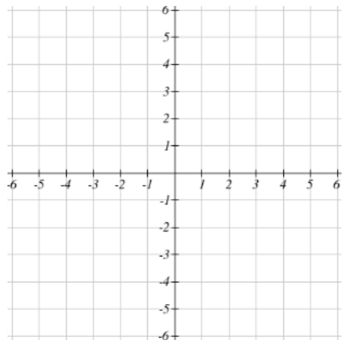
Units of Slope	
Rate of Change	
Constant Rate of Change	
Interpreting the Slope of a Linear Function	

Unit 9: Main Lesson

Section 9.1: Intercepts

Vertical and Horizontal Intercepts

The **vertical intercept** is the point at which the graph crosses the vertical axis.

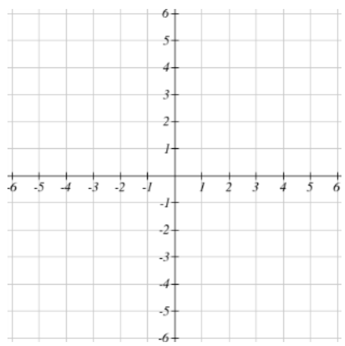


The input value of the vertical intercept is always_____________

The coordinates of the vertical intercept will be ______________

To determine the vertical intercept:

The **horizontal intercept** is the point at which the graph crosses the horizontal axis.



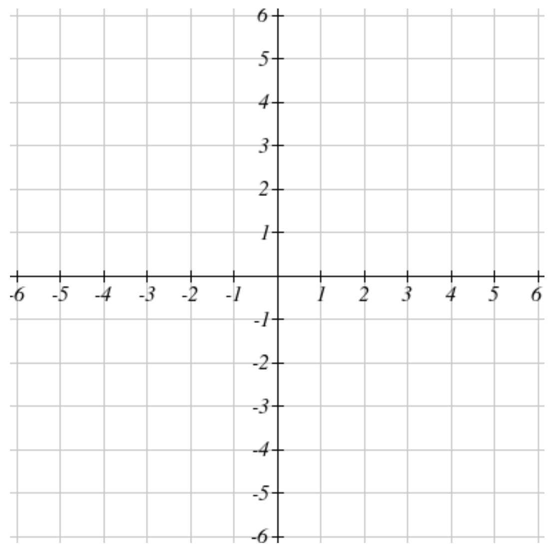
The output value of the horizontal intercept is always_________

The coordinates of the horizontal intercept will be ____________

To determine the horizontal intercept:

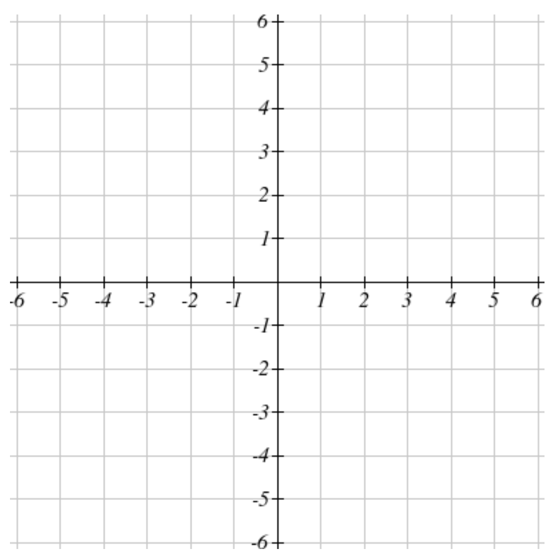
Example 1: Determine the vertical and horizontal intercepts for $y = 3x - 2$.

x	y	Ordered Pair



Example 2: Determine the vertical and horizontal intercepts for $4x - 2y = 10$.

x	y	Ordered Pair



Section 9.1 - You Try

Determine the vertical and horizontal intercepts for $y = 24 - 6x$. Show all steps as in the above examples.

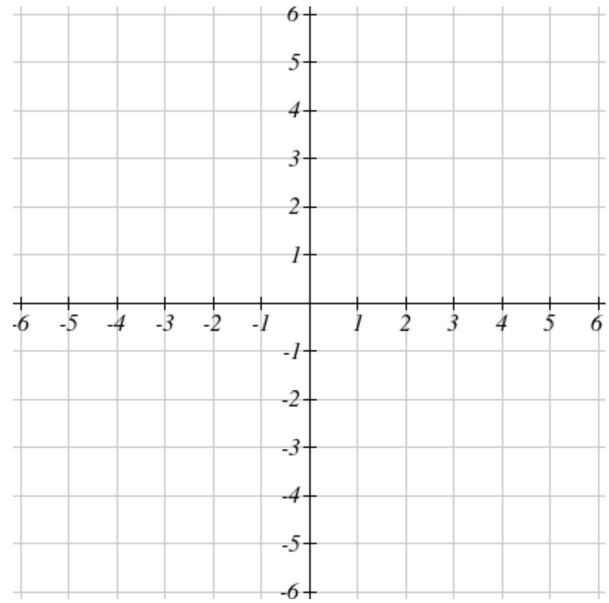
x	y	Ordered Pair

Section 9.2: Horizontal and Vertical Lines

Horizontal Lines $y = b$, where b is a real number

Example 1: Graph the equation $y = 2$

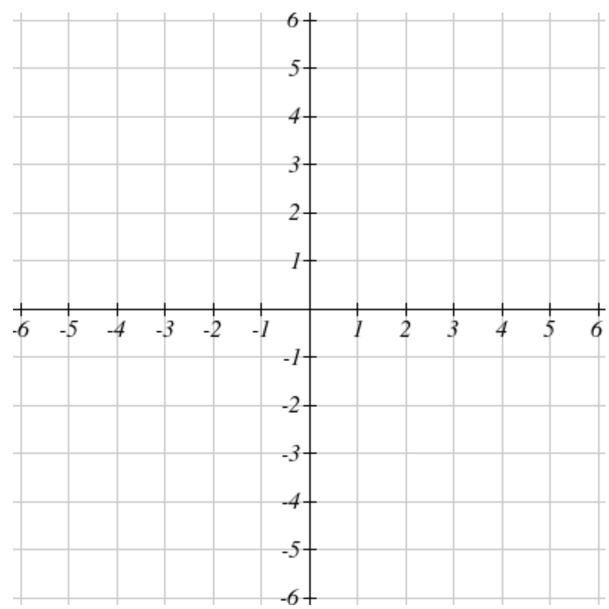
x	y	Ordered Pair



Vertical Lines $x = k$, where k is a real number

Example 2: Graph the equation $x = -3$

x	y	Ordered Pair

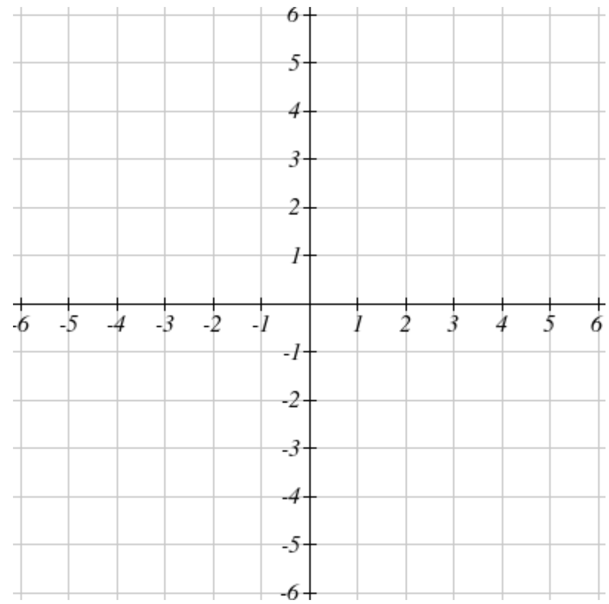


Section 9.2 - You Try

Complete the problems below.

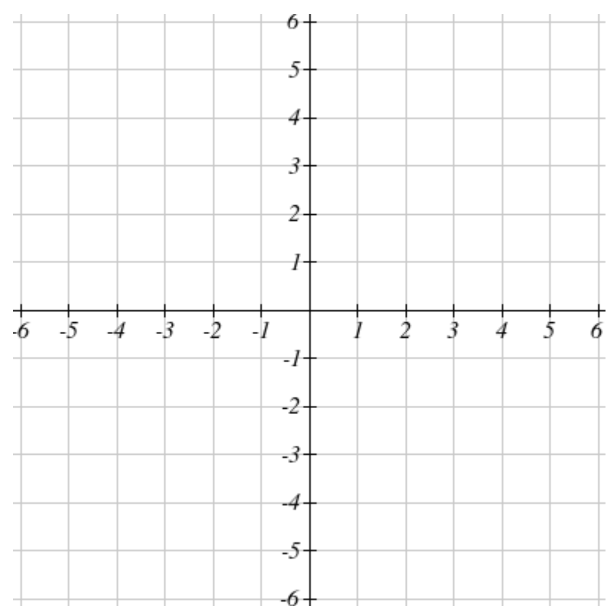
a. Graph the equation $y = -2$

x	y	Ordered Pair



b. Graph the equation $x = 4$

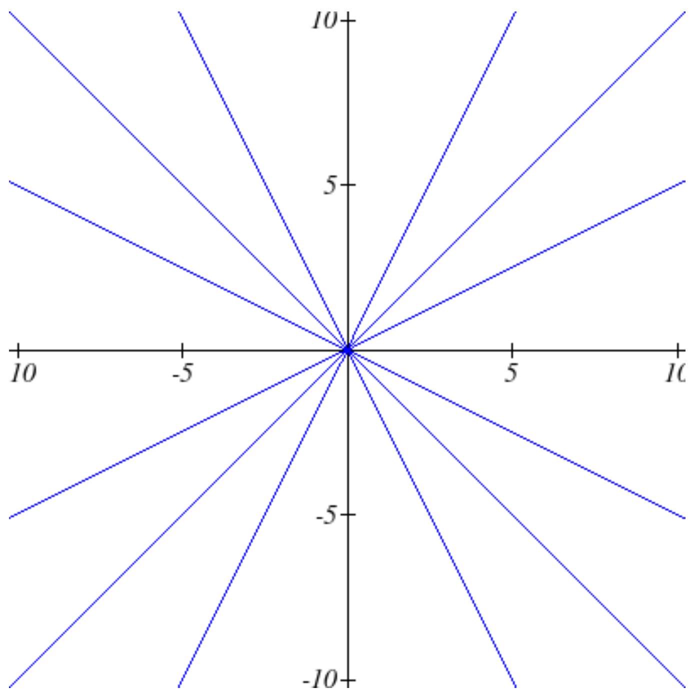
x	y	Ordered Pair

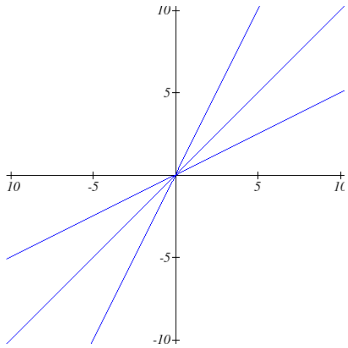


Section 9.3: Linear Functions

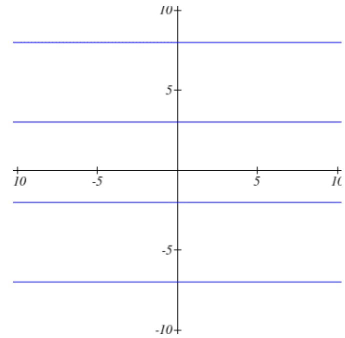
A linear function is a function that fits the form:

A linear function can be graphically represented by a ___________________________________

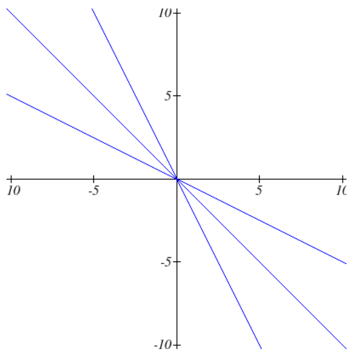




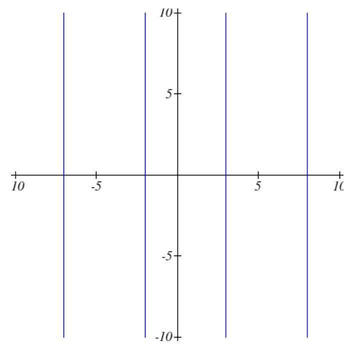
Increasing Linear Function
Slope > 0



Constant Function
Slope = 0

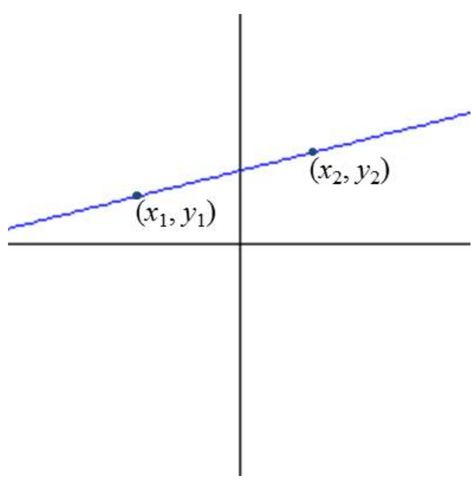


Decreasing Linear Function
Slope < 0



Not a Function
Slope is Undefined (No Slope)

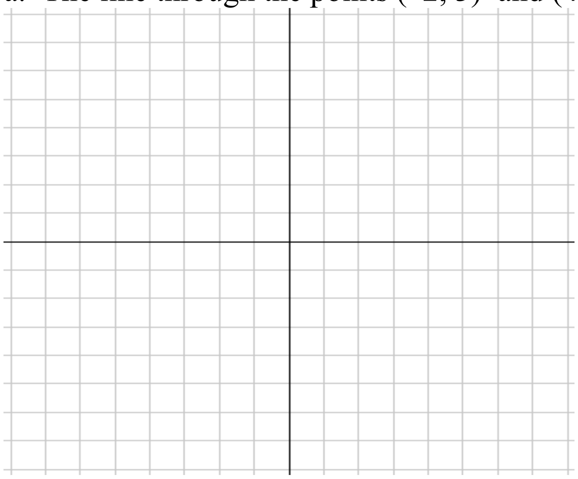
$$m = \text{Slope} = \frac{\text{Change in OUTPUT}}{\text{Change in INPUT}} = \frac{\Delta\ \text{OUTPUT}}{\Delta\ \text{INPUT}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



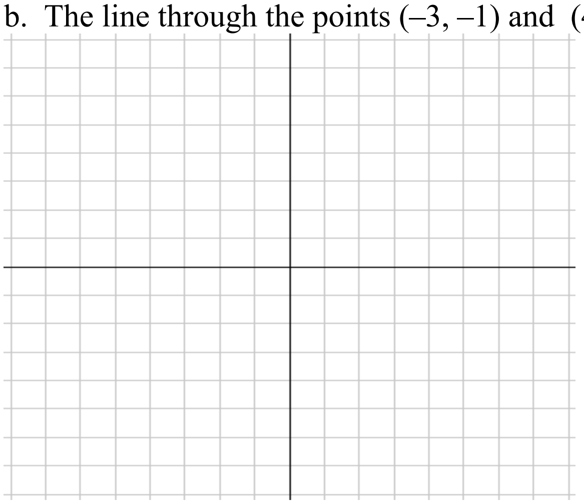
$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1: Determine the slope for each of the following:

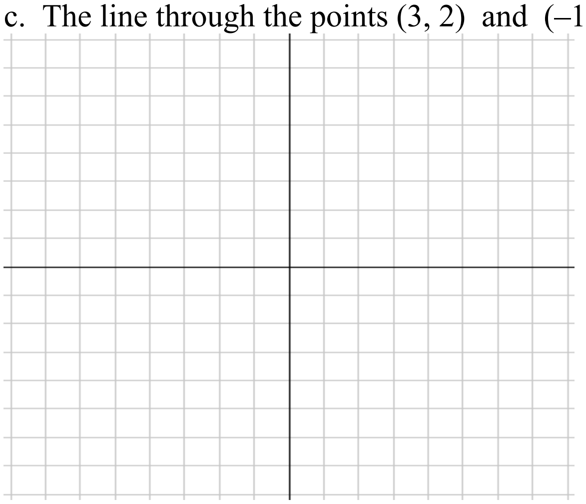
a. The line through the points (–2, 3) and (4, –1)



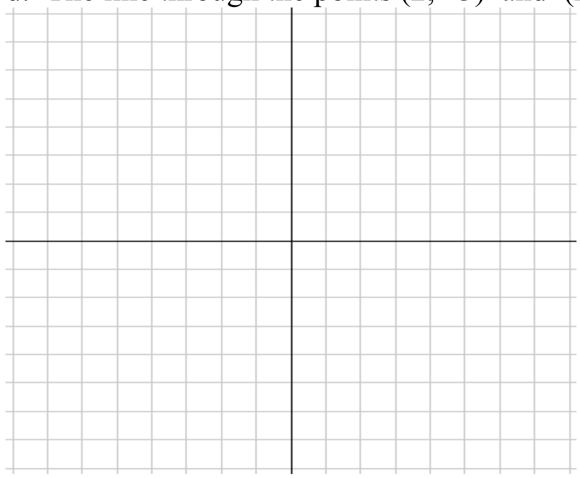
b. The line through the points (–3, –1) and (4, 2)



c. The line through the points (3, 2) and (–1, 2)

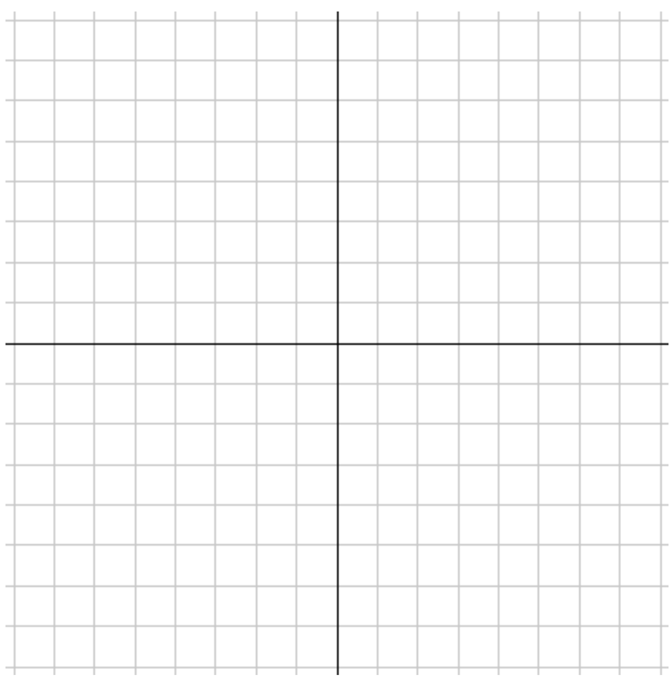


d. The line through the points (2, –3) and (2, 1)



Section 9.3 – You Try

Plot the points (–4, –1) and (5, –6) and draw a line connecting them. Determine the slope of this line. Show all steps, as in the above examples.



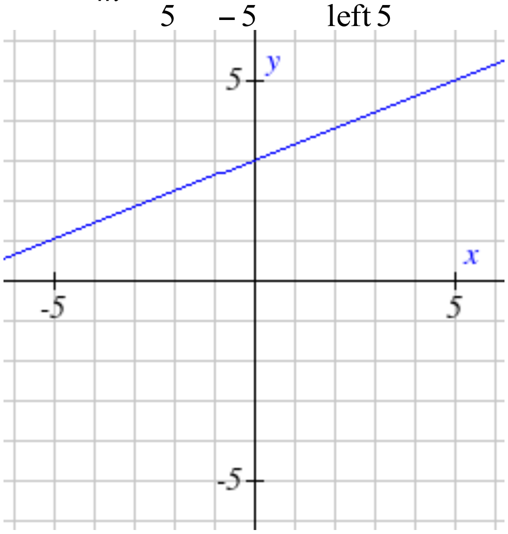
Section 9.4: Graphing Linear Functions

USING THE SLOPE TO GRAPH A LINEAR FUNCTION

$$m = \text{Slope} = \frac{\text{Change in OUTPUT}}{\text{Change in INPUT}} = \frac{\text{Vertical Change}}{\text{Horizontal Change}} \to \frac{\updownarrow}{\leftrightarrow}$$

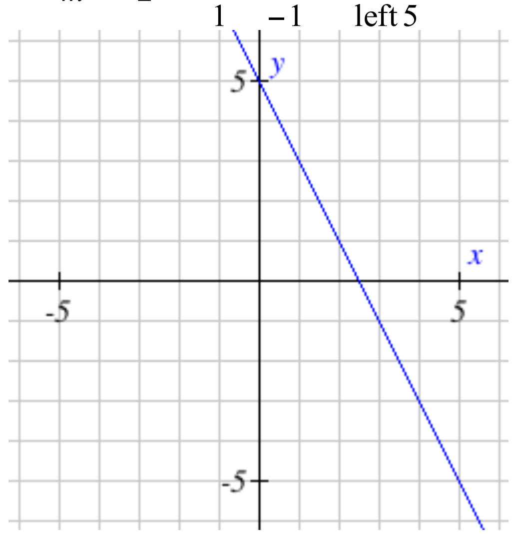
$$m = \frac{2}{5} \to \frac{\text{up } 2}{\text{right } 5}$$

$$m = \frac{2}{5} = \frac{-2}{-5} \to \frac{\text{down } 2}{\text{left } 5}$$



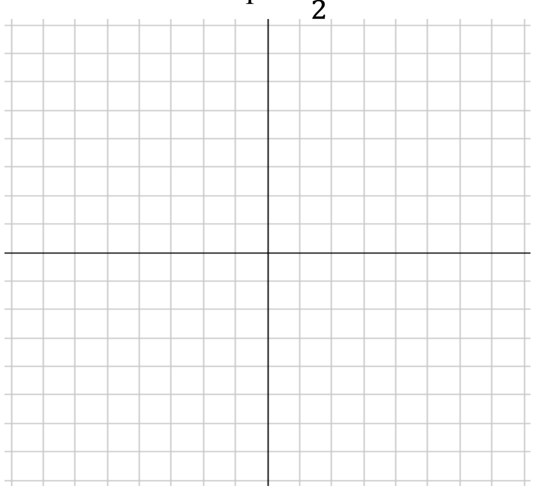
$$m = -2 = -\frac{2}{1} = \frac{-2}{1} \to \frac{\text{down } 2}{\text{right } 1}$$

$$m = -2 = -\frac{2}{1} = \frac{2}{-1} \to \frac{\text{up } 2}{\text{left } 5}$$

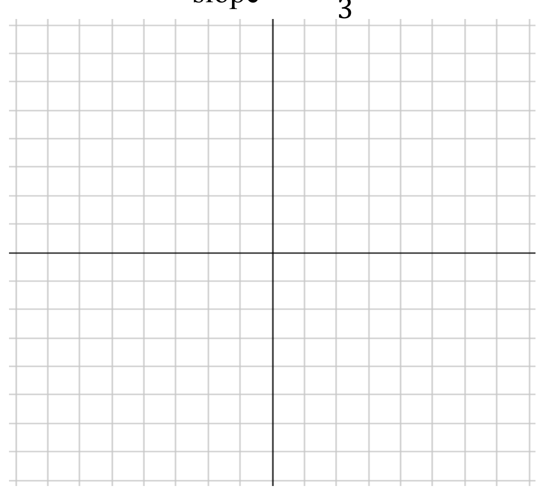


Example 1: Draw an accurate graph for the line that passes through the given point and has the given slope.

a. $(-2, -3)$
slope $= \frac{1}{2}$

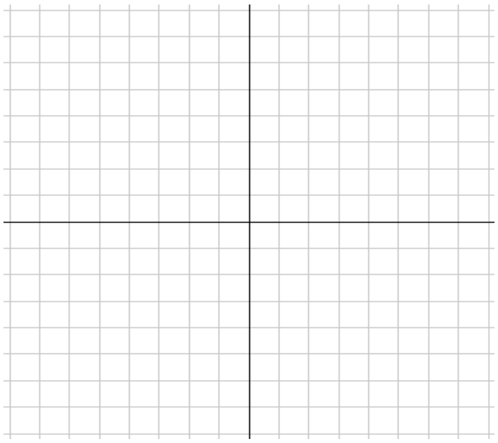


b. $(0, -1)$
slope $= -\frac{2}{3}$



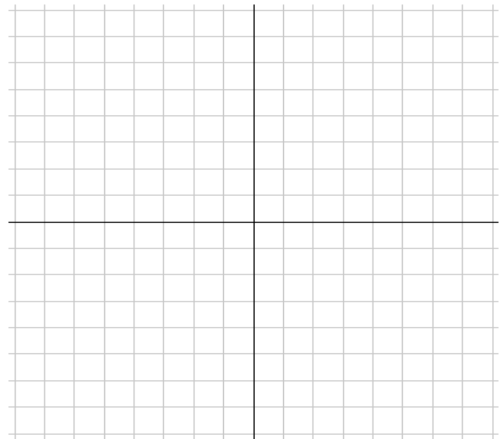
c. (2, 1)

slope = 3



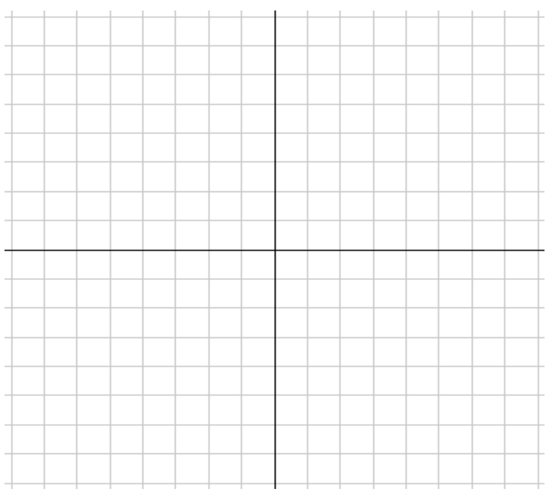
d. (1, –4)

slope = 0



Section 9.4 – You Try

Sketch the graph of a linear function that passes through the point (1, –2) with slope = $-\frac{3}{5}$.



Your line must extend accurately from edge to edge of the graph shown

Give the **coordinates** of at least two additional points on the line.

____

____

Section 9.5: Interpreting the Slope of a Linear Function

$$\text{Slope} = \frac{\text{Change in Output}}{\text{Change in Input}} \qquad \text{Units of Slope} = \frac{\text{Output Units}}{\text{Input Units}} \rightarrow \text{RATE OF CHANGE}$$

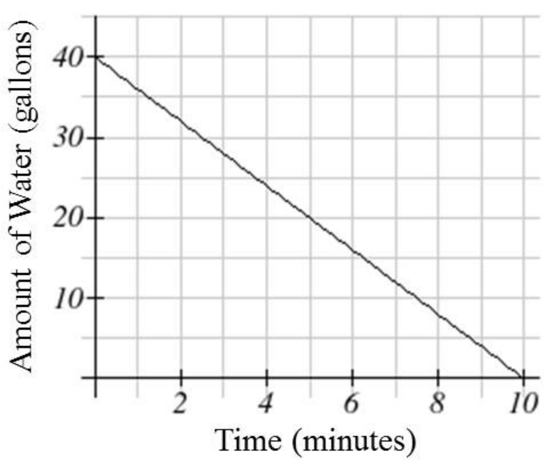
Example: Output = Height in Feet Input = Time in Seconds

$$\text{Slope} = \frac{\text{Change in Height}}{\text{Change in Time}} \qquad \text{Units of Slope} = \frac{\text{feet}}{\text{second}} = \text{feet/second}$$

What is the meaning of a slope of –5?

What is the meaning of a slope of 8?

Example 1: Consider the graph shown below.



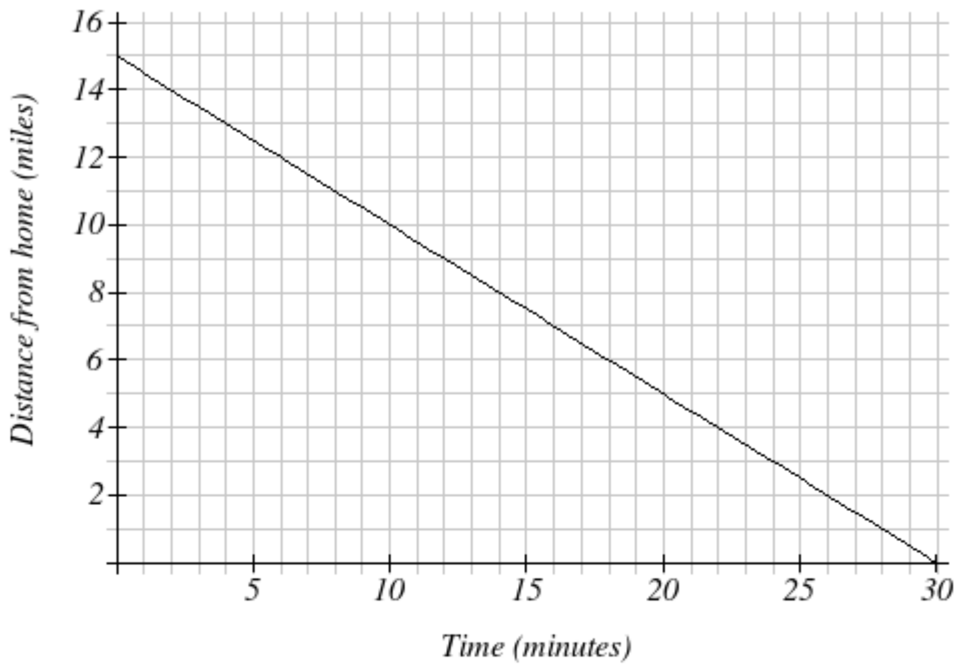
a. Identify the vertical intercept and interpret its meaning.

b. Identify the horizontal intercept and interpret its meaning.

c. Determine the slope, and interpret its meaning.

Section 9.5 – You Try

The graph below shows Sally's distance from home over a 30 minute time period.



a. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning.

b. Identify the horizontal intercept. Write it as an ordered pair and interpret its meaning.

c. Determine the slope, and interpret its meaning. Show your work and write your answer in a complete sentence.

Section 9.6: Using Rates of Change to Build Tables and Graphs

For each of the examples below, *circle* the rate of change in each situation and *underline* the starting value. Then use the given information to complete the table. Graph the results, and decide if it would make sense to connect the data points on the graph.

Example 1: A local carpet cleaning company charges $15 for each room plus a nonrefundable reservation fee of $25.

Number of Rooms	*Total Cost (dollars)*
0	
1	
2	
3	
4	
5	
6	

Example 2: Water is leaking out of a tank at a constant rate of 2 gallons per minute. The tank initially held 12 gallons of water.

Time (minutes)	*Amount of Water in Tank (gallons)*
0	
1	
2	
3	
4	
5	
6	

Section 9.6 – You Try

Sara is selling snow cones at the local carnival for $3 each.

Identify the rate of change in this situation. Be sure to include units in your answer.

Complete the table to show Sara's revenue from selling the snow cones. Graph the results, and decide if it would make sense to connect the data points on the graph.

Number of Snow Cones	Revenue (in dollars)
0	
1	
2	
3	
4	
5	
6	

Section 9.7: Is the Function Linear?

Rate of Change of a Linear Function
Given any two points (x_1, y_1) and (x_2, y_2), the **<u>rate of change</u>** between the points on the interval x_1 to x_2 is determined by computing the following ratio: $$\text{Rate of Change} = \frac{\text{Change in Output}}{\text{Change in Input}} = \frac{y_2 - y_1}{x_2 - x_1}$$ If the function is LINEAR, then the rate of change will be *the same* between any pair of points. This constant rate of change is the SLOPE of the linear function.

Example 1: Determine if the following function is linear by computing the rate of change between several pairs of points. If it is linear, give the slope.

x	*y*
-5	23
-2	14
0	8
3	-1
8	-16

Example 2: Determine if the following function is linear by computing the rate of change between several pairs of points. If it is linear, give the slope.

n	*T*(*n*)
-6	-3
-2	-1
0	1
1	2
4	6

Example 3: Determine if the following function is linear by computing the rate of change between several pairs of points. If it is linear, give the slope.

x	$g(x)$
-5	3
-2	3
0	3
4	3
6	3

Section 9.7 – You Try

Determine if the following function is linear by computing the rate of change between several pairs of points. If it is linear, give the slope. Show all of your work.

x	y
-8	-30
-3	-10
0	2
2	10
5	22

Unit 9: Practice Problems

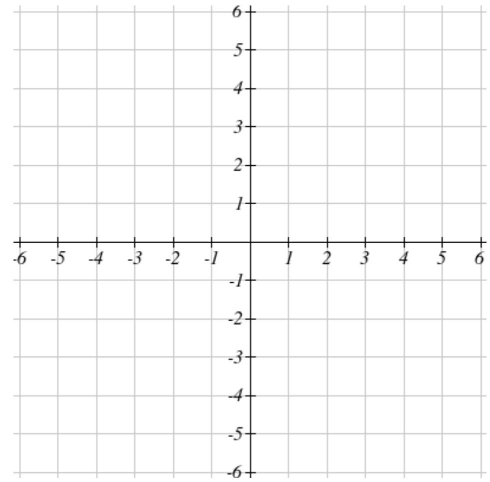
Skills Practice

1. Complete the table below. Write the intercepts as ordered pairs.

Equation	Vertical Intercept	Horizontal Intercept
$y = 5x - 3$		
$y = 4 - x$		
$y = 4x$		
$y = 3$		
$5x + 6y = 12$		
$3x - 4y = 24$		

2. Graph the equation $x = -2$

x	y	Ordered Pair



3. Determine the slope of the line between each of the following pairs of points. Show all steps, and reduce your answer to lowest terms.

a. (4, –5) and (–2, 3)

b. (–3, 2) and (1, 8)

c. (5, –9) and (5, 2)

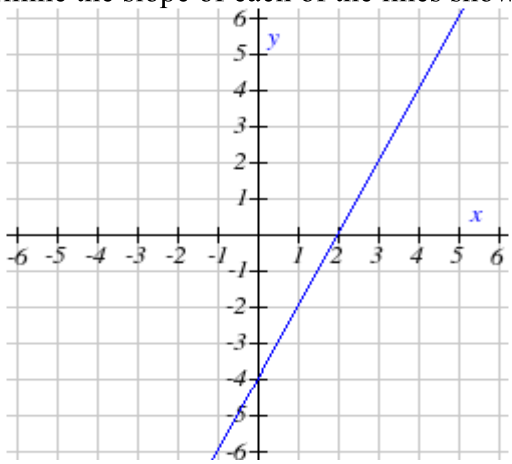
d. (2, –1) and (–2, 3)

e. (4, 3) and (12, –3)

f. (2, –4) and (7, –4)

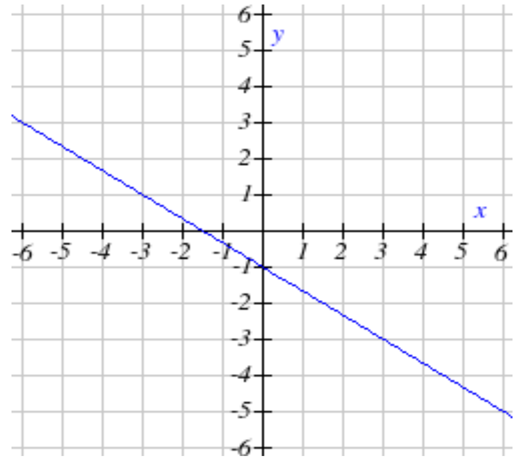
4. Determine the slope of each of the lines shown below.

a.



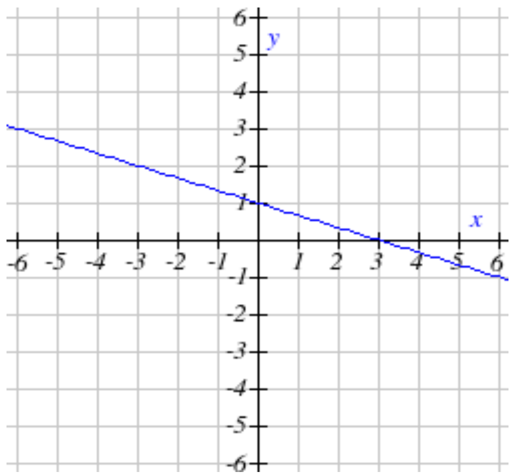
Slope = ____________

b.



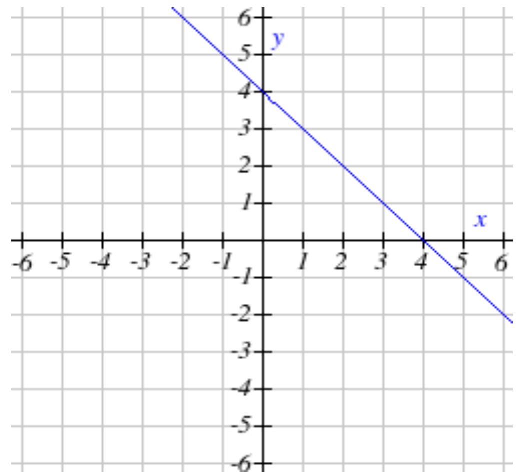
Slope = ____________

c.



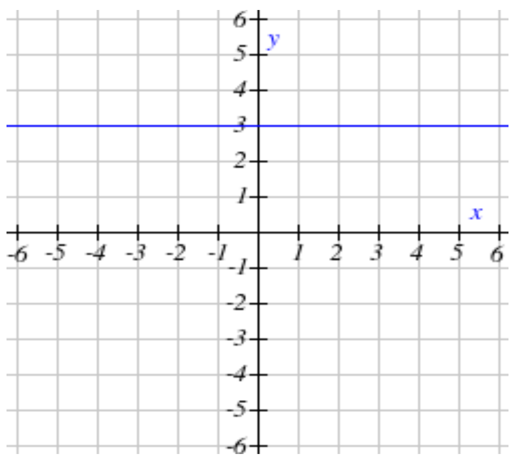
Slope = ____________

d.



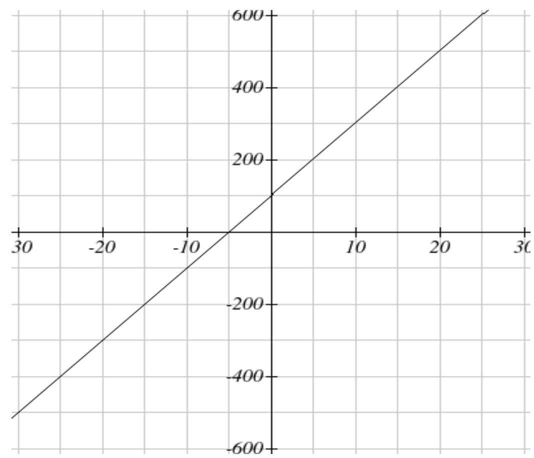
Slope = ____________

e.



Slope = ____________

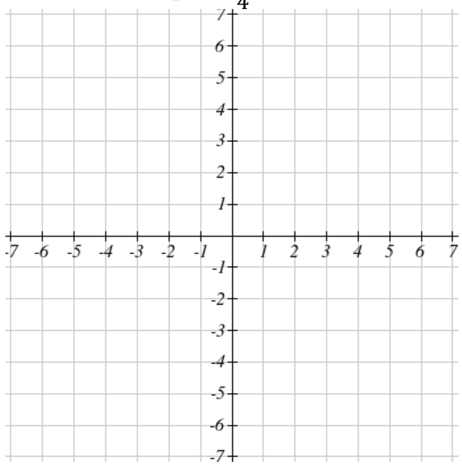
f.



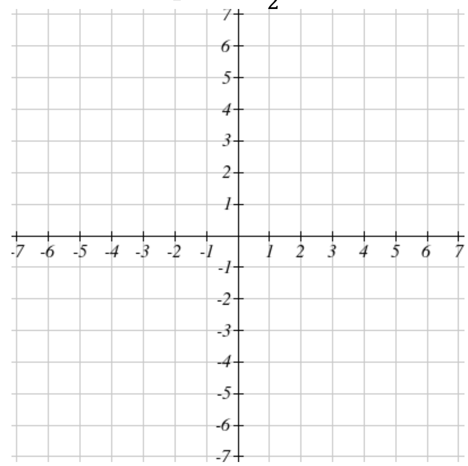
Slope = ____________

5. Draw an **accurate** graph for each of the following by
- Plotting the point
- Using the slope to find at least two additional points

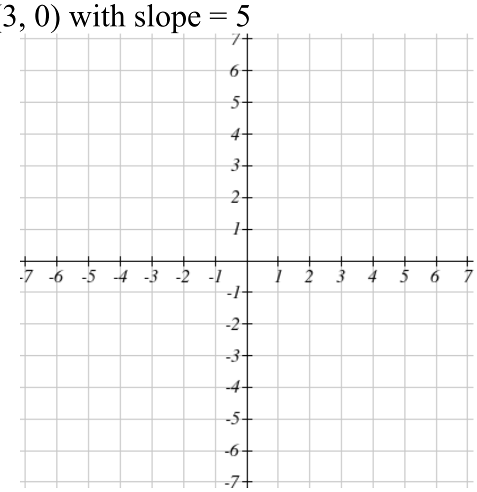
a. (1, –2) with slope = $\frac{1}{4}$



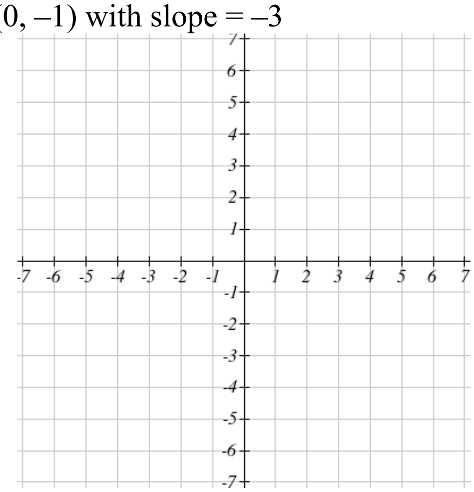
b. (–1,3) with slope = $-\frac{3}{2}$



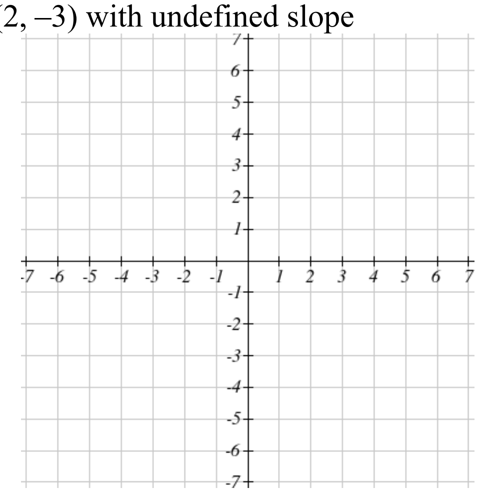
c. (3, 0) with slope = 5



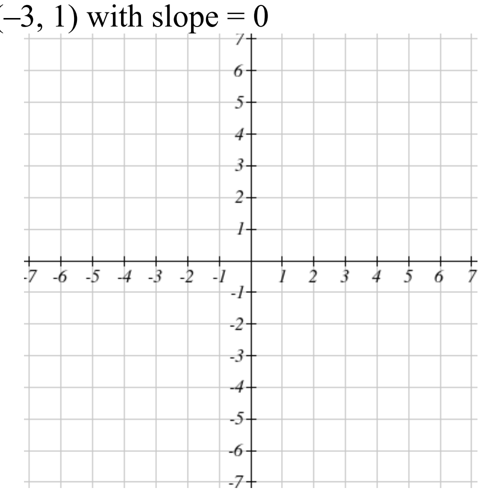
d. (0, –1) with slope = –3



e. (2, –3) with undefined slope



f. (–3, 1) with slope = 0



6. For each of the following, determine if the function is linear by computing the rate of change between several pairs of points. If it is linear, give the slope.

a.

x	y
-3	2
-1	8
0	16
2	64
3	128

b.

n	$A(n)$
-4	28
-1	19
5	1
11	-17
14	-26

c.

t	$r(t)$
-6	5
-3	6
4	7
11	8
18	9

Applications

7. A candy company has a machine that produces candy canes. The number of candy canes produced depends on the amount of time the machine has been operating. The machine produces 160 candy canes in five minutes. In twenty minutes, the machine can produce 640 candy canes. Determine the rate of change in this situation, and write a sentence explaining its meaning.

8. The enrollment at a local charter has been decreasing linearly. In 2006, there were 857 students enrolled. By 2015, there were only 785 students enrolled. Determine the rate of change of this school's enrollment during this time period.

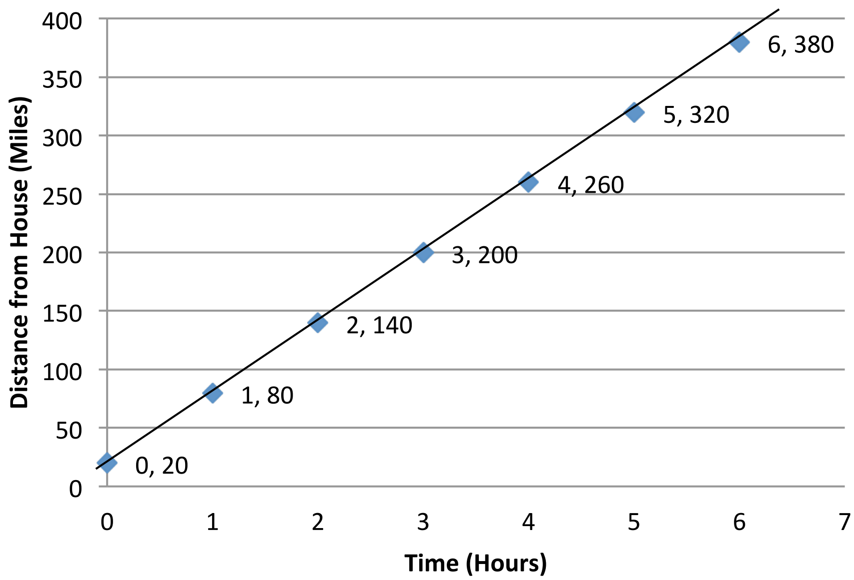
9. A tree grows 2 feet taller every 3 years. Determine the rate of change in this situation.

10. Oil is leaking from a tanker at a rate of 18 gallons every 30 minutes. Determine the rate of change in this situation.

11. In the year 1987, an investment was worth $30,200. In the year 1996, this investment was worth $43,700. Determine the rate of change in this situation, and write a sentence explaining its meaning. At this rate, how much was the investment worth in 2005?

12. In the year 1998, the surface elevation of Lake Powell was 3,843 feet above sea level. In the year 2001, the surface elevation of Lake Powell was 3,609 feet above sea level. Determine the rate of change in this situation, and write a sentence explaining its meaning.

13. The graph below shows the distance you are from your house if you leave work and drive in the opposite direction.



a. In a complete sentence, interpret the ordered pair (2, 140)

b. Identify the vertical intercept and interpret its meaning.

c. Determine the slope, and interpret its meaning.

d. At this rate, how far away from home will you be after 7 hours?

e. At this rate, how long will it take for you to be 680 miles from your home?

14. You need to hire a caterer for a banquet.

a. Caterer A charges a nonrefundable delivery fee of $45 plus $5 per guest.

b. Caterer B charges a fee of $150. This includes the delivery and food for up to 30 guests.

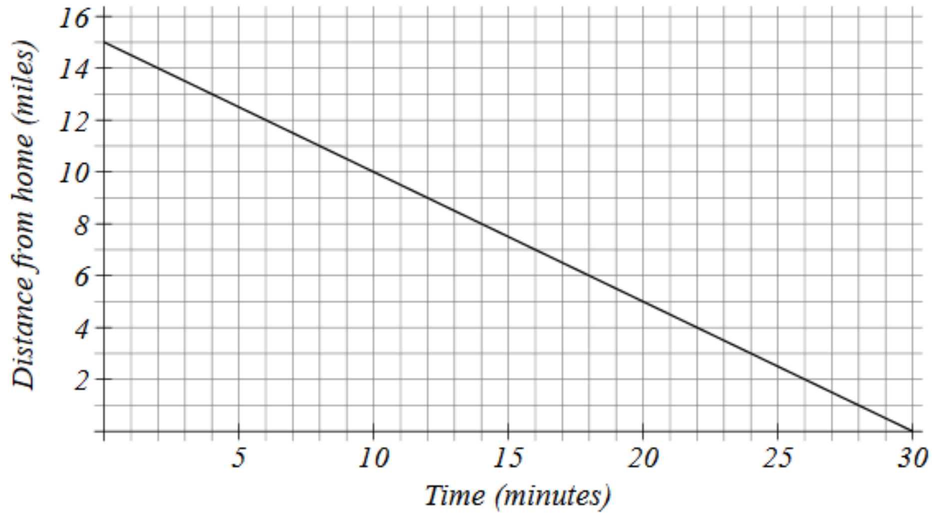
Use this information to complete the tables below. Draw good graphs of your results.

Number of Guests	*Cost (dollars) Caterer A*
0	
1	
2	
3	
4	
5	
6	

Number of Guests	*Cost (dollars) Caterer B*
0	
1	
2	
3	
4	
5	
6	

Which caterer should you choose? What considerations should be made before making this decision?

15. The function D(t) below shows Sally's distance from home over a 30 minute time period.



a. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning.

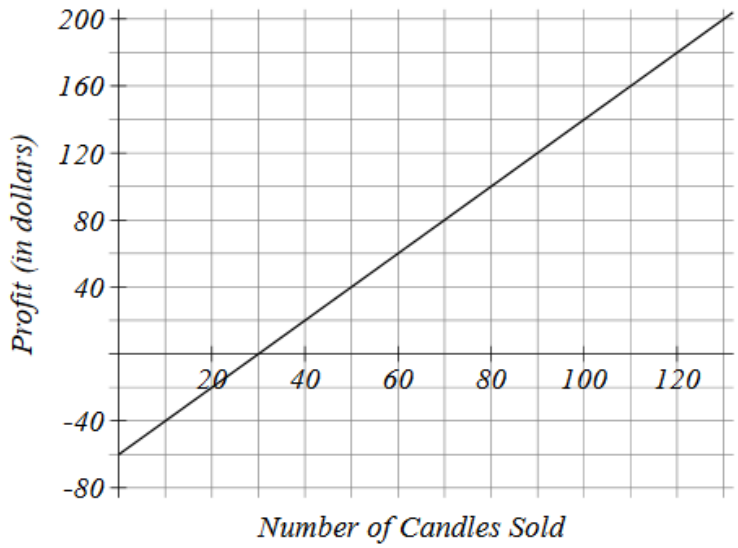
b. Identify the horizontal intercept. Write it as an ordered pair and interpret its meaning.

c. Determine the slope of D(t), and interpret its meaning.

d. Determine the practical domain of this linear function. Use inequality notation and include units.

e. Determine the practical range of this linear function. Use inequality notation and include units.

16. Janey is selling homemade scented candles. The graph below shows her profit from selling the candles.



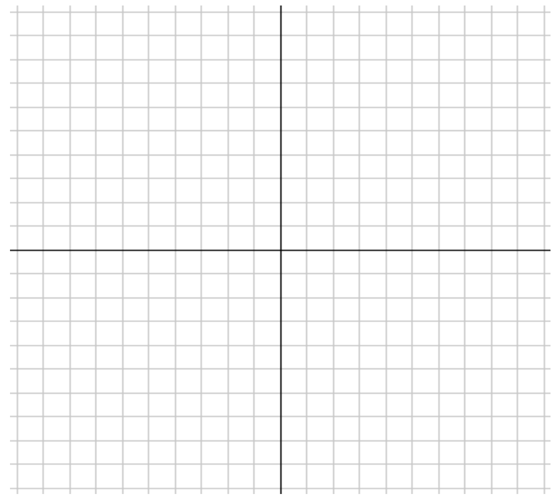
a. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning.

b. Identify the horizontal intercept. Write it as an ordered pair and interpret its meaning.

c. Determine the slope, and interpret its meaning.

Extension

17. Graph the lines A, B, C, and D on the grid below.



A: Passes through the point (0, -5) with slope $\frac{2}{3}$

B: Passes through the point (0, -1) with slope $\frac{2}{3}$

C: Passes through the point (0, 3) with slope $\frac{2}{3}$

D: Passes through the point (0, 7) with slope $\frac{2}{3}$

How are these lines geometrically related?

18. Amber starts off with $1000 in her savings account. Determine the balance in the account after 1 year in each of the following situations:

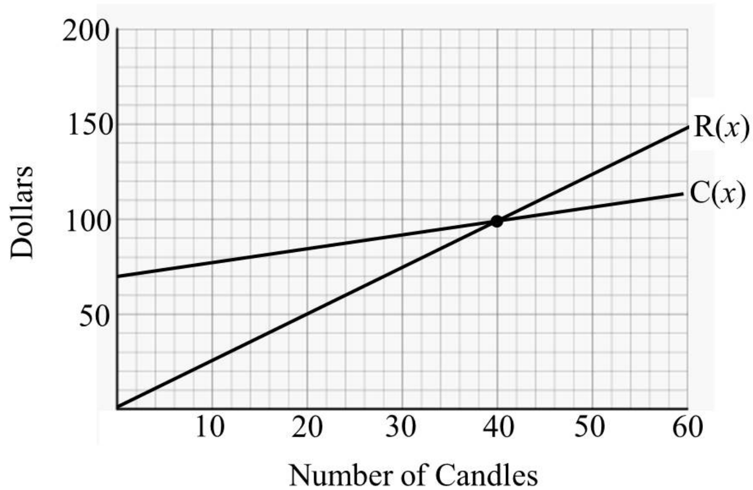
a. Amber deposits $50 every month.

b. Amber withdraws $50 from her account every month.

c. Amber deposits $500 into the account every six months.

d. Amber makes no withdrawals or deposits.

19. The graph below shows the cost and revenue for a company that produces and sells scented candles. The function R(x) gives the revenue earned when x candles are sold. The function C(x) gives the total cost to produce x candles.



a. Identify the vertical intercept of C(x). Write it as an ordered pair, and interpret its meaning.

b. Determine the slope of C(x). Interpret its meaning.

c. Identify the vertical intercept of R(x). Write it as an ordered pair, and interpret its meaning.

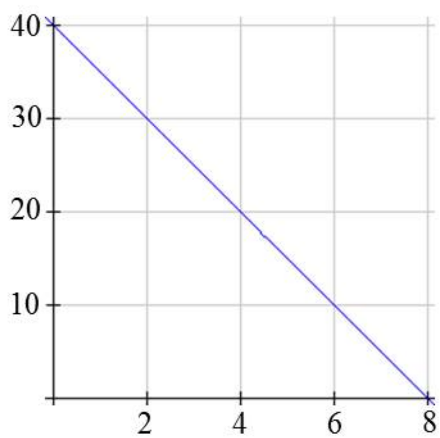
d. Determine the slope of R(x). Interpret its meaning.

e. Discuss the significance of the point (40, 100) in terms of the cost, revenue, and *profit* for this company.

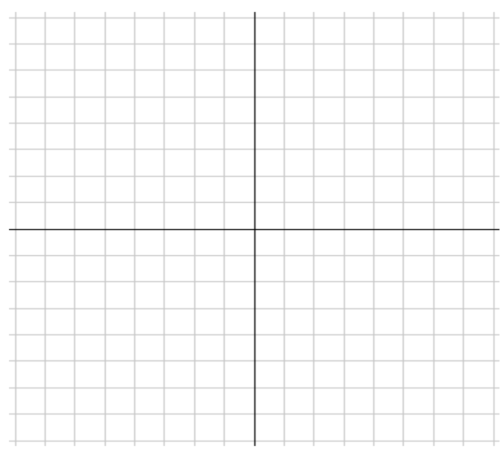
Unit 9: Review

1. Determine the slope of the line between the points (2, –1) and (–2, 3). Show all steps, and reduce your answer to lowest terms.

2. Determine the slope of the line shown below.



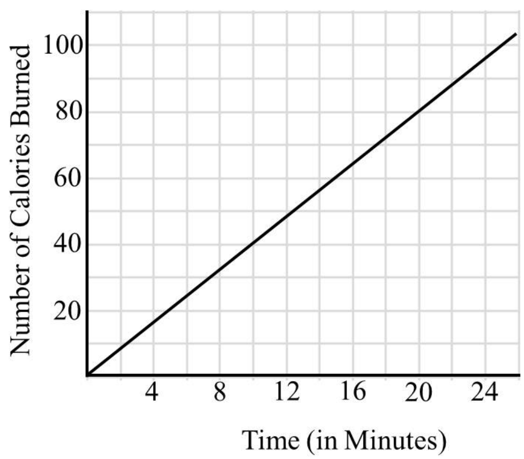
3. Draw an **accurate** graph of the line passing through the point (–2,4) with slope $-\frac{2}{5}$.



4. Determine if the function $g(x)$ is linear by computing the rate of change between several pairs of points. If it is linear, give the slope.

x	$g(x)$
-8	39
-2	18
0	11
4	-3
12	-31

5. The graph of the function $C(n)$ below shows the number of calories burned after riding a stationary bike for n minutes.



a. Interpret the meaning of the statement $C(8) = 32$

b. Determine $C(10)$ and interpret its meaning in a complete sentence.

c. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.

d. Determine the slope of $C(n)$ and interpret its meaning in a complete sentence.

Unit 10: The Equation of a Linear Function

KEY TERMS AND CONCEPTS	
Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms *in your own words*. Provide examples that are not identical to those in the Media Lesson.	
Slope-Intercept Form	
How to graph a Linear Equation given in Slope-Intercept Form	
How to write the Equation of a line in Slope-Intercept Form given two points.	
Slopes of Parallel Lines	

Slopes of Perpendicular Lines	
The Slope of a Vertical Line	
The Equation of a Vertical Line	

Unit 10: Main Lesson

Section 10.1: The Equation of a Linear Function

Slope – Intercept Form

SLOPE-INTERCEPT FORM:

$y = mx + b$

$y = b + mx$

$f(x) = mx + b$

Slope	Behavior
$m > 0$	Increasing
$m < 0$	Decreasing
$m = 0$	Horizontal
m is undefined	Vertical

Example 1: Fill in the table below.

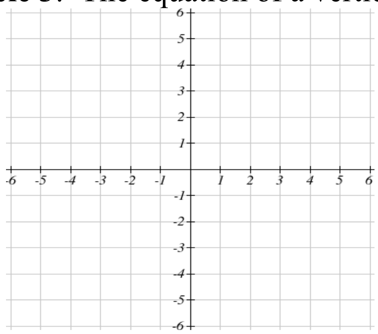
Equation	Slope	I, D, H, V	Vertical Intercept
$y = 3x + 5$			
$y = 8 - x$			
$y = 2x$			
$y = -8$			

Example 2: Determine the *horizontal* intercepts of each of the following.

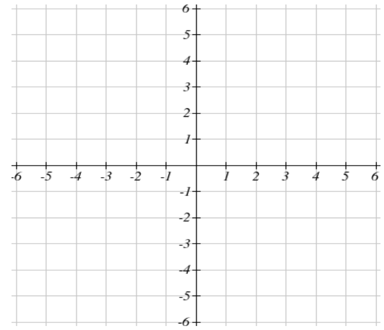
$y = 3x + 5$ $\qquad$ $y = 8 - x$ $\qquad$ $y = 2x$ $\qquad$ $y = -8$

To find a horizontal intercept: ______________________________

Example 3: The equation of a vertical line



x = 4



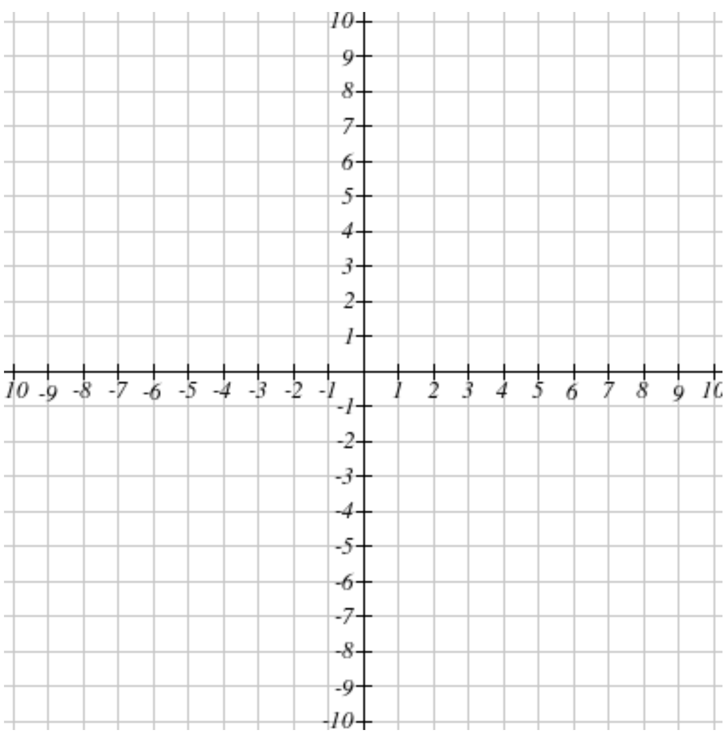
x = -1

Example 4: Draw an **accurate** graph of the function $f(x) = 4 - 3x$.

Slope: ___________

Vertical Intercept: ___________

Horizontal Intercept: __________



To find the Horizontal Intercept:

____

____

Two additional points on the line:

___________ ____________

Slope-Intercept Form
$f(x) = mx + b$
$f(x) = b + mx$

Section 10.1 – You Try

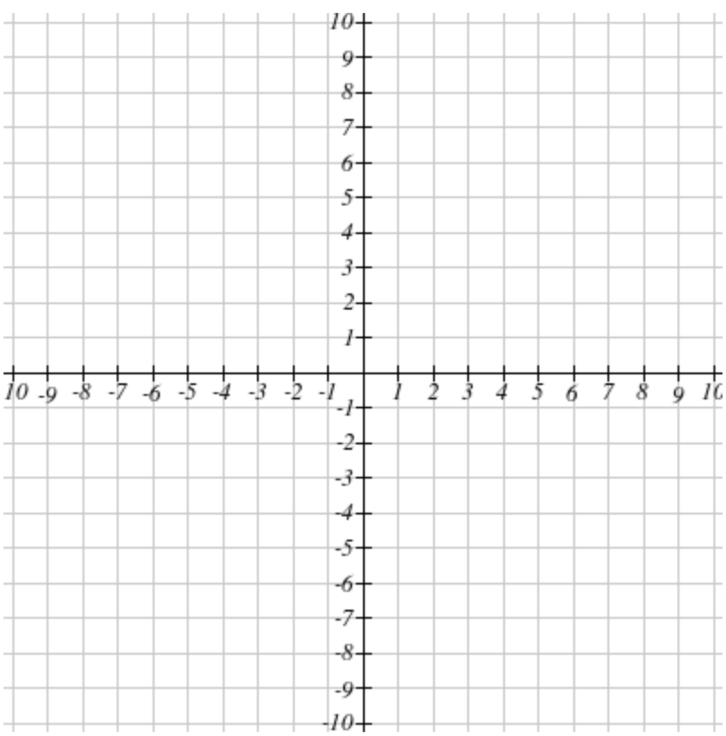
Complete the problems below.

a. Fill in the table below. Write intercepts as ordered pairs. Write "DNE" if the answer does not exist.

Equation	Slope	I, D, H, V	Vertical Intercept
$y = x - 11$			
$G(x) = -2x$			
$x = 5$			

I = Increasing, D = Decreasing, H = Horizontal (Constant), V = Vertical

b. Draw an **accurate** graph of the function $y = \frac{3}{4}x - 5$. Identify the slope, intercepts, and two additional points on the line.



Slope: ___________

Vertical Intercept: _____________

Horizontal Intercept: _____________

Two additional points on the line:

____________ ____________

Section 10.2: Writing the Equation of a Line in Slope-Intercept Form

Slope-Intercept Form $y = mx + b$

Example 1: Give the equation of the line in slope-intercept form

a. With vertical intercept (0, 2) and slope –9

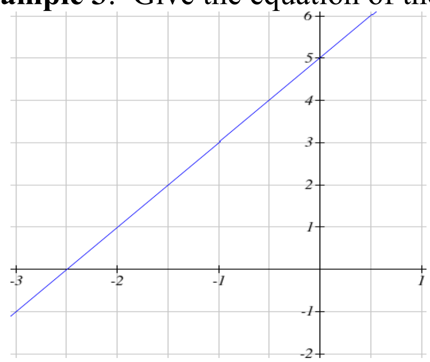
b. Passing through (2, 3) with slope –5

c. Passing through (2, 6) and (4, 16)

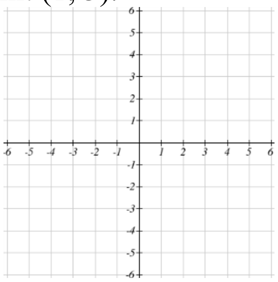
Example 2: Give the equation of the linear function that would generate the following table of values. Use your calculator to check.

x	$f(x)$
–5	238
–3	174
–1	110
1	46
7	–146
12	–306

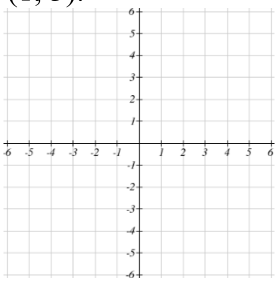
Example 3: Give the equation of the linear function shown below.



Example 4: Give the equation of the horizontal line passing through the point (1, 3).



Example 5: Give the equation of the vertical line passing through the point (1, 3).



Section 10.2 – You Try

Complete the problems below. Show as much work as possible, as demonstrated in the above examples.

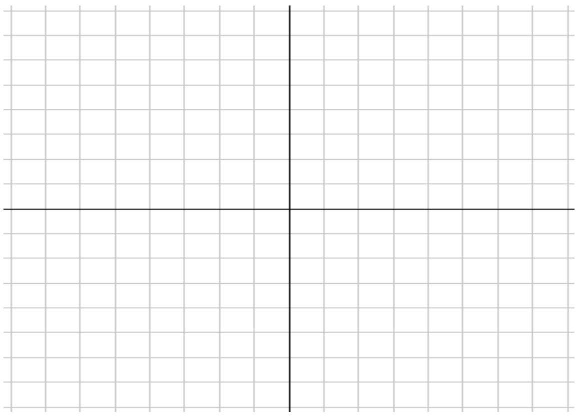
a. Give the equation of the line passing through the points (1, 7) and (3, –9).

b. Give the equation of the horizontal line passing through the point (5, 11).

Section 10.3: Parallel and Perpendicular Lines

Parallel Lines

The slopes of Parallel Lines are ____



Slope-Intercept Form

$y = mx + b$ $\qquad f(x) = mx + b$

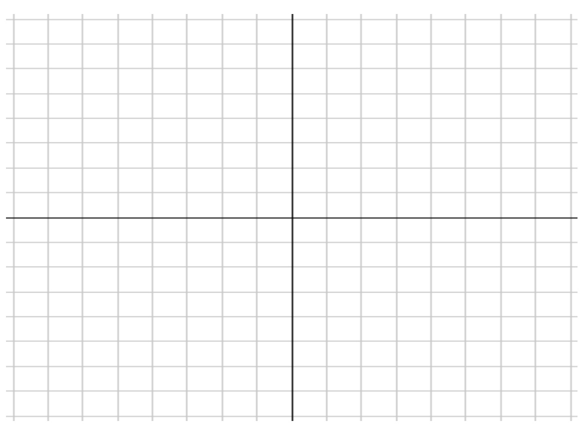
m = slope

b = vertical intercept $(0,b)$

Example 1: Give the equation of the line passing through the point (8, 3) that is **parallel** to the line $y = -2x + 3$.

Perpendicular Lines

The slopes of perpendicular lines are ____



If Line 1 and Line 2 are perpendicular to each other, then

Slope of Line 1	Slope of Line 2
$\frac{2}{3}$	
5	
–8	
$-\frac{4}{5}$	

Negative (Opposite) Reciprocals

$\frac{a}{b}$ and $-\frac{b}{a}$

Example 2: Give the equation of the line passing through the point (8, 3) that is **perpendicular** to the line $y = -2x + 3$.

Section 10.3 – You Try

Give the equation of the line passing through the point (–3, 1) that is:

a. **Parallel** to the line $y = 8x - 5$.

b. **Perpendicular** to the line $y = 8x - 5$.

Section 10.4: Applications – Slope-Intercept Form

Slope-Intercept Form	**If we are not given the slope and vertical intercept, we need:**
$y = mx + b$ $f(x) = mx + b$ m = slope = rate of change b = vertical intercept (initial value)	• One point and the slope • Two points

Example 1: You have just bought a new Sony 55" 3D television set for $2300. The TV's value decreases at a rate of $250 per year. Construct a linear function to represent this situation.

Example 2: In 1998, the cost of tuition at a large Midwestern university was $144 per credit hour. In 2008, tuition had risen to $238 per credit hour. Determine a linear equation to represent the cost, C, of tuition as a function of *x,* the number of years since 1990.

Section 10.4 – You Try

For each of the following, determine a linear equation to represent the given situation. Use the indicated variables and proper function notation. Show all possible steps.

a. A tree is 3 feet tall when it is planted, and it grows by approximately half a foot each year. Let $H(t)$ represent the height of the tree (in feet) after t years.

b. The enrollment at a local charter has been decreasing linearly. In 2006, there were 857 students enrolled. By 2015, there were only 785 students enrolled. Let $S(n)$ represent the number of students enrolled in this school n years after the year 2000.

Section 10.5
Interpreting a Linear Function in Slope-Intercept Form

Example 1: The function $A(m) = 200 - 1.25m$ represents the balance in a bank account (in thousands of dollars) after m months.

a. Identify the slope of this linear function and interpret its meaning in a complete sentence.

b. What are the **practical domain** and **practical range** of this function?

c. Identify the vertical intercept. Write it as an ordered pair and interpret its practical meaning in a complete sentence.

Ordered Pair:_________________

d. Determine the horizontal intercept of this linear function. Write it as an ordered pair and interpret its practical meaning in a complete sentence.

Ordered Pair:_________________

e. Determine $A(12)$. Write your answer as an ordered pair and interpret its practical meaning in a complete sentence.

Ordered Pair:_________________

f. How long will it take for the balance in this account to reach $80,000? Write the corresponding ordered pair.

Ordered Pair:_________________

Section 10.5 – You Try

The function $E(t) = 3860 - 77.2t$ gives the surface elevation (in feet above sea level) of Lake Powell t years after 1999. Your answers must include all appropriate units.

a. Identify the slope of this linear function and interpret its meaning in a complete sentence.

b. What are the **practical domain** and **practical range** of this function?

c. Identify the vertical intercept. Write it as an ordered pair and interpret its practical meaning in a complete sentence.

Ordered Pair:__________________

d. Determine $E(5)$. Write your answer as an ordered pair and interpret its practical meaning in a complete sentence. Show your work.

Ordered Pair:__________________

Unit 10: Practice Problems

Skills Practice

1. Determine the slope, behavior (increasing, decreasing, constant, or vertical), and vertical intercept (as an ordered pair) of each of the following. Write "DNE" if an answer does not exist.

Equation	Slope	Behavior	Vertical Intercept
$y = x - 2$			
$f(a) = 6 - 4a$			
$\mathrm{P}(n) = 3n$			
$y = 4$			
$x = 7$			
$y = \frac{3}{5}x - 4$			
$y = x$			
$B(x) = 8 - x$			
$V(t) = -70$			

2. Determine the horizontal intercepts for each of the following. Write "DNE" if there is no horizontal intercept.

a. $y = x - 2$

b. $f(a) = 6 - 4a$

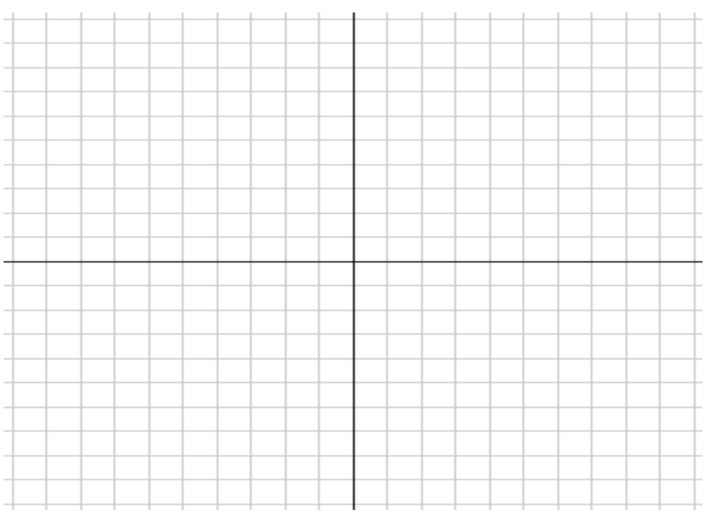
c. $P(n) = 3n$

d. $y = 4$

e. $x = 7$

f. $y = \frac{3}{5}x - 4$

3. Draw an **accurate** graph of the function $f(x) = 4x + 5$.

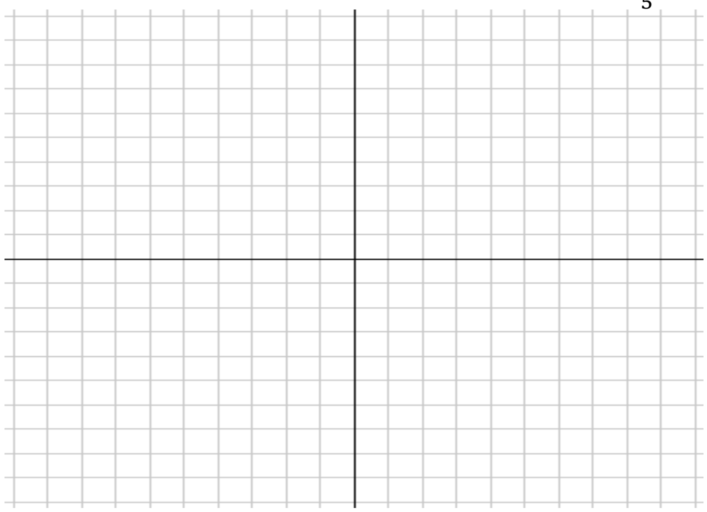


Slope: ___________

Vertical Intercept: ____________

Horizontal Intercept: ____________

4. Draw an **accurate** graph of the function $y = \frac{2}{5}x - 3$

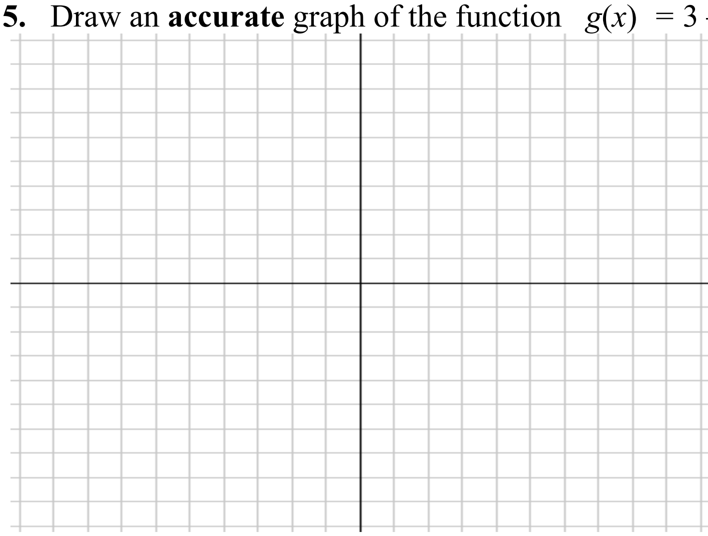


Slope: ___________

Vertical Intercept: ____________

Horizontal Intercept: ____________

5. Draw an **accurate** graph of the function $g(x) = 3 - x$.

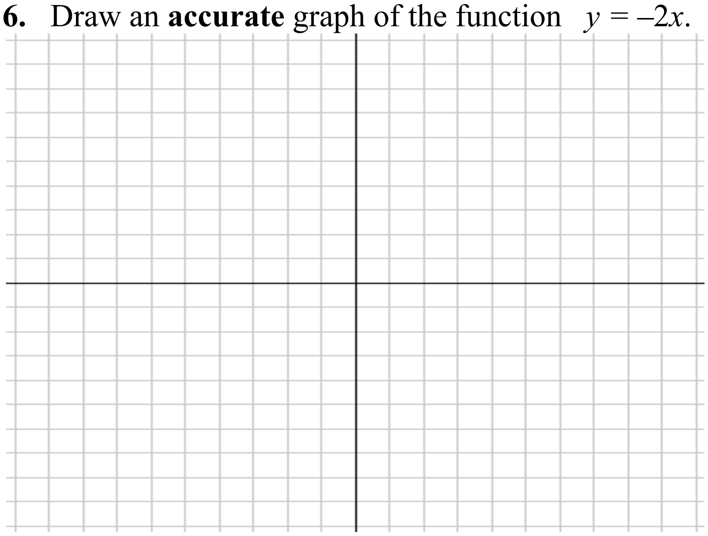


Slope: ___________

Vertical Intercept: ____________

Horizontal Intercept: ____________

6. Draw an **accurate** graph of the function $y = -2x$.

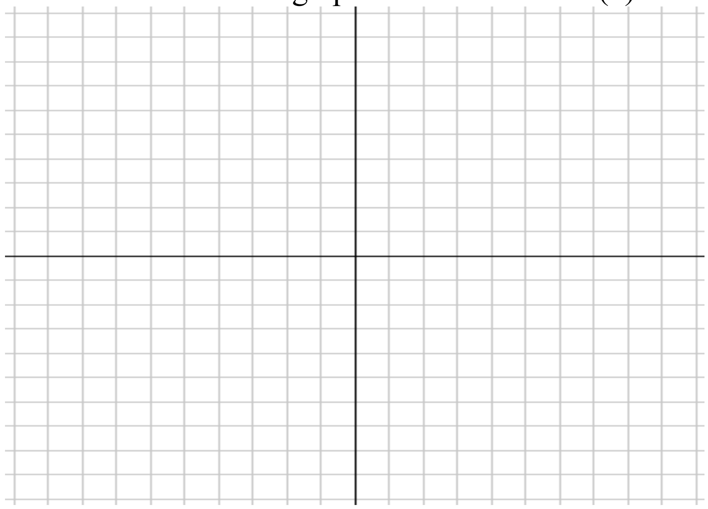


Slope: ___________

Vertical Intercept: ____________

Horizontal Intercept: ____________

7. Draw an **accurate** graph of the function $r(a) = 5$.

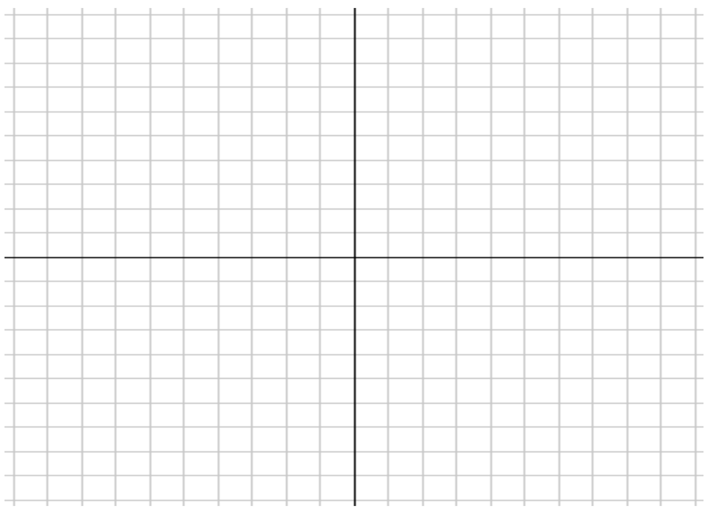


Slope: ____________

Vertical Intercept: _____________

Horizontal Intercept: _____________

8. Draw an **accurate** graph of the function $C(x) = \frac{x}{5}$

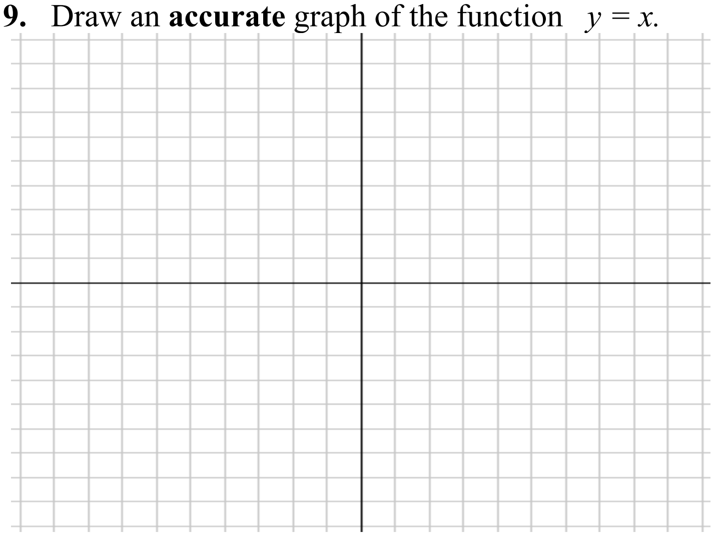


Slope: ____________

Vertical Intercept: _____________

Horizontal Intercept: _____________

9. Draw an **accurate** graph of the function $y = x$.



Slope: ____________

Vertical Intercept: _____________

Horizontal Intercept: _____________

10. Determine the equation of the line between each of the following pairs of points.

a. (4, –5) and (2, 3)

b. (–3, 2) and (1, 8)

c. (5, –9) and (5, 2)

d. (2, –1) and (–2, 3)

e. (4, 3) and (12, –3)

f. (2, –4) and (7, –4)

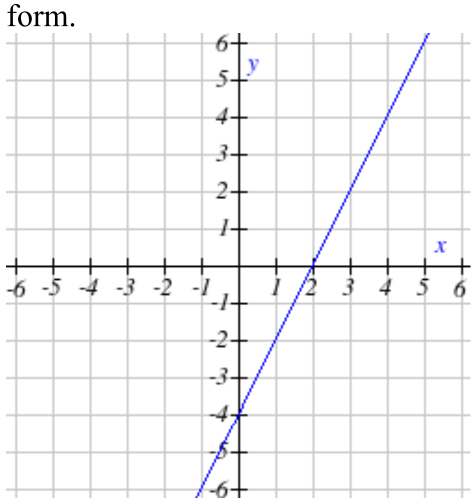
11. Give the equation of the linear function that generates the following table of values. Write your answer in slope-intercept form.

x	$f(x)$
–5	91
–2	67
1	43
4	19
9	–21

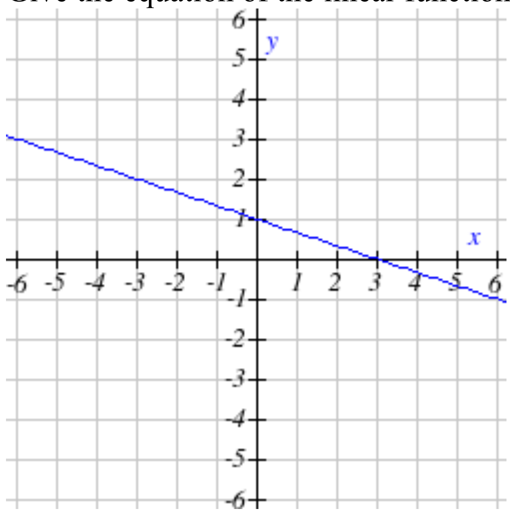
12. Give the equation of the linear function that generates the following table of values. Write your answer in slope-intercept form.

t	$C(t)$
5	–1250
15	–900
20	–725
35	–200
45	150

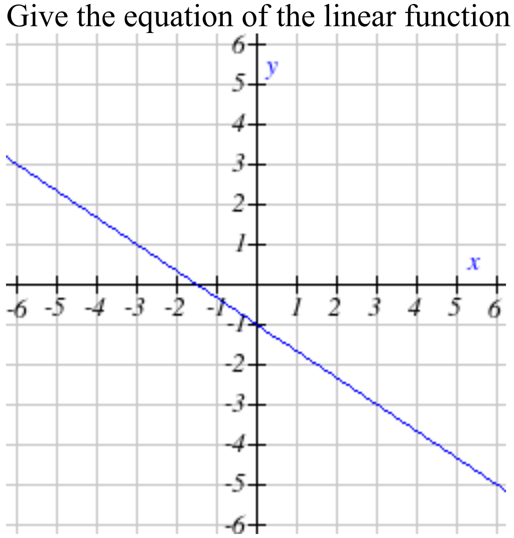
13. Give the equation of the linear function shown below. Write your answer in slope-intercept form.



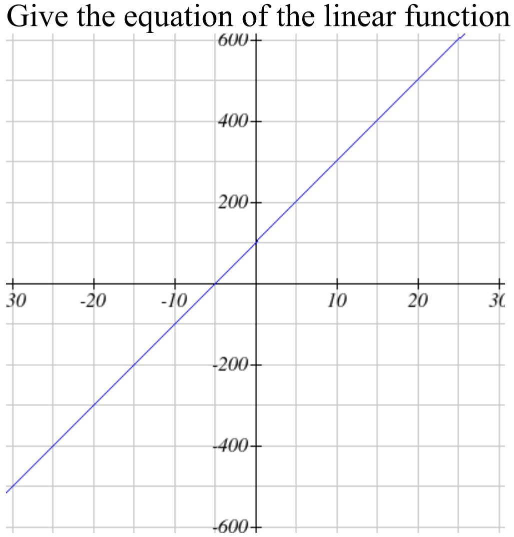
14. Give the equation of the linear function shown below.



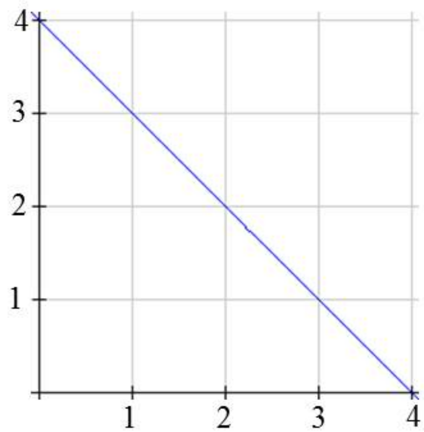
15. Give the equation of the linear function shown below.



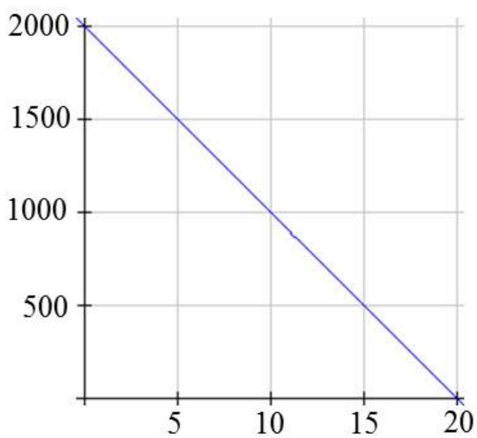
16. Give the equation of the linear function shown below.



17. Give the equation of the linear function shown below. Write your answer in slope-intercept form.



18. Give the equation of the linear function shown below. Write your answer in slope-intercept form.



19. Give the equation of the horizontal line passing through the point (–6, 11). _______________

20. Give the equation of the vertical line passing through the point (4, 7). ________________

21. Give the equation of the *x*-axis. ________________

22. Give the equation of the *y*-axis. ________________

23. Give the equation of the line passing through the point (1, –5) that is parallel to $y = 12 - 8x$.

24. Give the equation of the line passing through the point (4, 0) that is parallel to $y = 9 - \frac{3}{2}x$.

25. Give the equation of the line passing through the point (10, 3) that is perpendicular to $y = \frac{2}{5}x + 1$.

26. Give the equation of the line passing through the point (–12, –1) that is perpendicular to $y = 3 - 4x$.

Applications

27. A candy company has a machine that produces candy canes. The number of candy canes produced depends on the amount of time the machine has been operating. The machine produces 160 candy canes in five minutes. In twenty minutes, the machine can produce 640 candy canes.
   a. Determine the equation of the linear function that represents this situation. Let C(x) represent the number of candy canes produced in x minutes. Write your answer in function notation.

   b. What are the **practical domain** and **practical range** of this function?

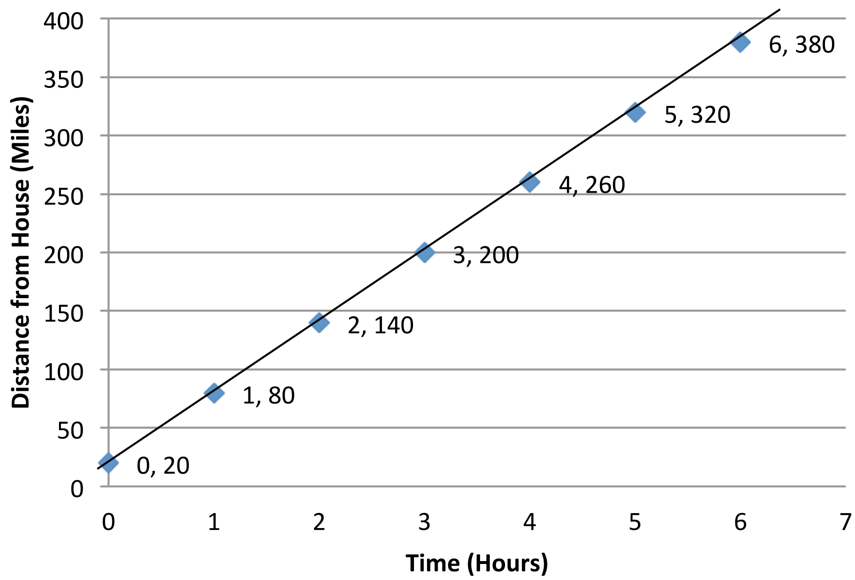
   c. Determine C(10). Write a sentence explaining the meaning of your answer.

   d. What is the practical meaning of the slope of this linear function? Include units.

   e. Determine horizontal intercept of this linear function. Write it as an ordered pair and interpret its meaning.

   f. How many candy canes will this machine produce in 1 hour?

28. Your workplace is 20 miles from your house. The graph below shows the distance you are from your house if you leave work and drive in the opposite direction.



a. Determine the equation of the linear function that represents this situation. Let D(t) represent your distance from home after t hours. Write your answer in function notation.

b. Use the equation from part a to determine how long it would take for you to be 500 miles from your house. Express your answer in hours and minutes.

c. How far from your house would you be after 12 hours?

d. Interpret the meaning of the slope of this linear function.

29. A local carpet cleaning company charges $10 for each room plus a reservation fee of $25. They clean a maximum of 12 rooms. Also, they have the policy that once a reservation is made, if you cancel, the reservation fee is non-refundable.

a. Determine the equation of the linear function C(*n*) that represents the total cost for cleaning *n* rooms.

b. Complete the table below. Graph the results, and decide if it would make sense to connect the data points on the graph.

n	C(*n*)
0	
1	
2	
3	
6	
12	

30. Water is leaking out of a tank at a constant rate of 1 gallon every 2 minutes. The tank initially held 30 gallons of water.

a. Determine the equation of the linear function A(*t*) that represents the amount of water (in gallons) remaining in the tank after *t* minutes.

b. Complete the table below. Graph the results, and decide if it would make sense to connect the data points on the graph.

t	A(*t*)
0	
1	
2	
3	
5	
10	
60	

c. Determine the practical domain of A(*t*):______________________________________

d. Determine the practical range of A(*t*): _______________________________________

31. With good credit, and a $5000 down payment, you can finance a new 2012 Chevrolet Camaro convertible for 60 months for $615.17 per month.
    a. Determine the equation of the linear function, $T(n)$, that represents the total amount paid for this car after n months.

    b. Determine the **practical domain** and **practical range** of this function.

    c. Use the equation from part a to determine the total payment over the 60-month time period.

    d. A new 2012 Chevrolet Camaro convertible has a base MSRP of $35,080. Why is this value lower than your answer in part b?

32. The function $P(n) = 455n - 1820$ represents a computer manufacturer's profit when n computers are sold.

a. Identify the slope, and interpret its meaning in a complete sentence.

b. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.

c. Determine the horizontal intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.

33. John is a door to door vacuum salesman. His weekly salary is given by the linear function $S(v) = 200 + 50v$, where v is the number of vacuums sold.

a. Identify the slope, and interpret its meaning in a complete sentence.

b. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.

34. The function $V(n) = 221.4 + 4.25n$ gives the value, in thousands of dollars, of an investment after n years.

a. Identify the slope, and interpret its meaning in a complete sentence.

b. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.

c. Determine the **practical domain** and **practical range** of this function

35. The function $V(t) = 86.4 - 1.2t$ gives the value, in thousands of dollars, of an investment after t years.

a. Identify the slope, and interpret its meaning in a complete sentence.

b. Determine the **practical domain** and **practical range** of this function

c. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.

d. Determine the horizontal intercept. Write it as an ordered pair and discuss its meaning.

36. When a new charter school opened in 2005, there were 300 students enrolled. Write a formula for the function N(t) representing the number of students attending this charter school t years after 2005, assuming that the student population

a. Increases by 20 students per year.

b. Decreases by 40 students per year.

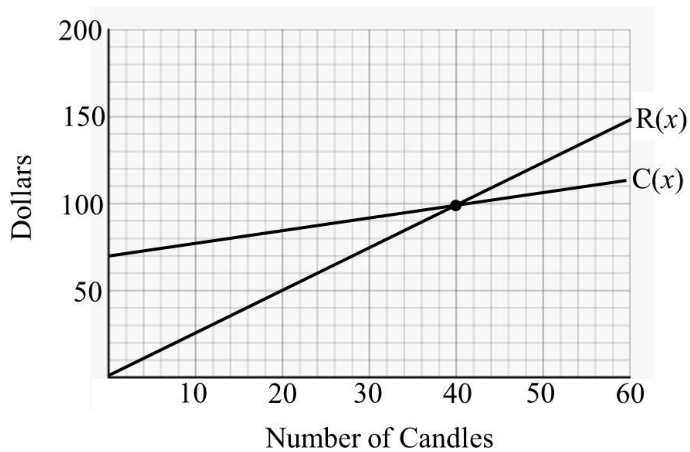
c. Increases by 100 students every 4 years.

d. Decreases by 60 students every two years.

e. Remains constant (does not change).

Extension

37. The graph below shows the cost and revenue for a candle company. The function R(x) gives the revenue earned when x candles are sold. The function C(x) gives the total cost to produce x candles.



a. Determine the formula for C(x): C(x) = ______________________________

b. Determine the formula for R(x): R(x) = ______________________________

c. Profit is found by subtracting the costs from the revenue. Determine the formula for the profit, P(x), earned from selling x candles.

d. Identify the vertical intercept of P(x). Write it as an ordered pair, and interpret its meaning.

e. Identify the slope of P(x). Interpret its meaning.

f. Discuss the cost, revenue, and *profit* for this company when 40 candles are sold.

38.World famous mathematician Cedric Villani is starting to produce his signature bow tie in bulk. It costs him \$5 to produce each tie and he has monthly fixed costs of \$600. Compute the monthly cost function C(x) and the average cost function A(x) = C(x)/x. Also compute A(10) and A(40) and interpret your answer in a complete sentence.

39.Sysc O'Systems produces computer chips. Their costs per month is modelled by the function C(x) = 30x + 90 thousands of dollars, where x is the number of computer chips in hundreds of chips. Sysc O'Systems' monthly revenue is R(x) = 60x thousands of dollars.

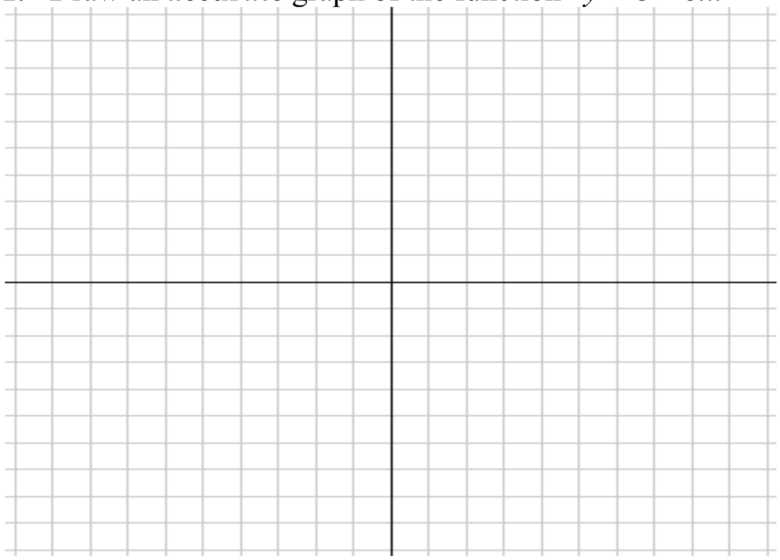
a. Monthly profit is given by the function P(x) = R(x) - C(x). Find the function P(x) that models the monthly profit for Sysc O'Systems.

b. Compute P(2) and P(7) and interpret your answer in a complete sentence.

c. Determine what value of x gives P(x) = 0 and interpret your answer.

Unit 10: Review

1. Draw an **accurate** graph of the function $y = 3 - 5x$.



Slope: ___________

Vertical Intercept: ___________

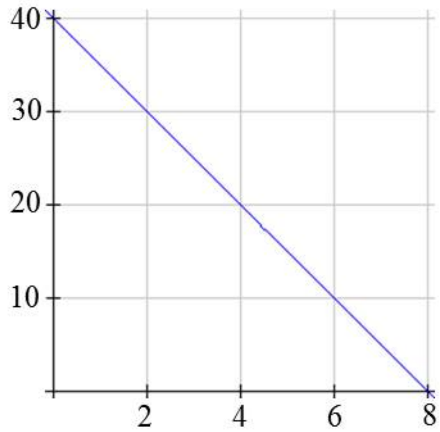
Horizontal Intercept: _________

2. Determine the equation of the line between the points (-6, 14) and (18, –2). Your answer must be written in slope-intercept form.

3. Give the equation of the vertical line passing through the point (1, 8). _______________

4. Give the equation of the horizontal line passing through the point (1, 8). _______________

5. Give the equation of the linear function shown below. Write your answer in slope-intercept form.



6. In the year 2000, the median cost for in-state tuition and fees at a public 4-year college was \$3412. In the year 2010, the median cost for tuition had risen to \$7231.

   a. Determine a linear function, $C(t)$ to represent the cost for tuition and fees t years since 2000. Show all of your work. Write your answer in function notation, $C(t) = mt + b$.

   c. Determine the **practical domain** and **practical range** of this function.

   d. Determine $C(13)$. Show all of your work. Write your answer in a complete sentence.

   c. Identify the slope of this linear function and write a sentence explaining its meaning in this situation.

Unit 11: Linear Equations

Section 11.1: General Form: $ax + by = c$

Section 11.2: Applications – General Form

Section 11.3: Point-Slope Form: $y - y_1 = m(x - x_1)$

KEY TERMS AND CONCEPTS	
Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms *in your own words*. Provide examples that are not identical to those in the Media Lesson.	
General (Standard) Form	
Finding the intercepts of a linear equation given in general form.	
Finding the slope of a linear equation given in general form.	
Converting from general form to slope-intercept form.	
Write a linear equation in general form given an application problem.	

How to Graph a Linear Equation given in General Form	
Point-Slope Form of a linear equation	
Converting from point-slope form to slope-intercept form.	
How to Graph a Linear Equation given in Point-Slope Form	
Write a linear equation in Point-Slope form.	

Unit 11: Main Lesson

Section 11.1: General Form: $ax + by = c$

Slope-Intercept Form of a Linear Equation	**General (Standard) Form of a Linear Equation**
$y = mx + b$	$ax + by = c$
x = input, y = output m = slope b = vertical intercept $(0, b)$	x = input, y = output a, b, and c are constants

Example 1: Consider the linear equation $3x - 5y = 30$

a. Write this equation in slope-intercept form.

b. Identify the slope.

Determining Intercepts:
To find the **vertical intercept**, set $x = 0$ and solve for y.
To find the **horizontal intercept**, set $y = 0$ and solve for x.

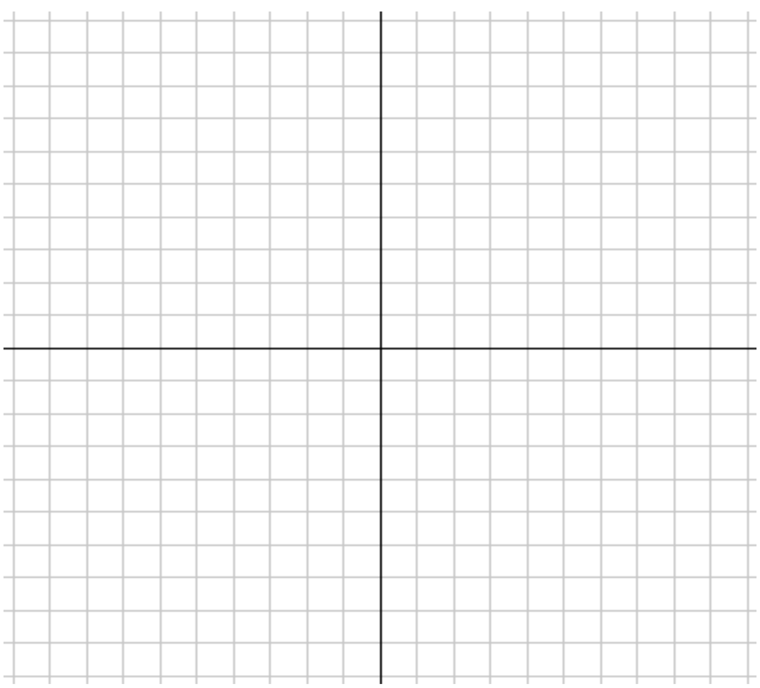
c. Determine the vertical intercept.

d. Determine the horizontal intercept.

Example 2: Draw an **accurate** graph of the linear equation $3x + 2y = 16$.

Slope-Intercept Form:

____



Slope: ____________

Vertical Intercept: ____________

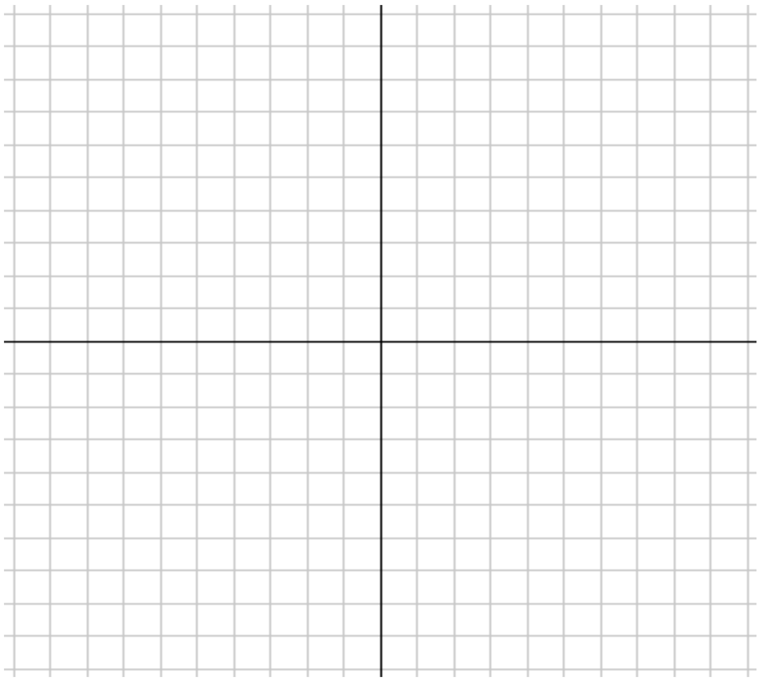
Horizontal Intercept: __________

Additional points on the line:

____

Section 11.1 – You Try

Draw an **accurate** graph of the linear equation $4x - y = 7$



Slope-Intercept Form:

____

Slope: ____________

Vertical Intercept: ___________

Horizontal Intercept: _________

Additional points on the line:

____

Section 11.2: Applications – General Form

Example 1: Movie tickets cost $7 for adults (matinee), $5.50 for children. A total of $668 was collected in ticket sales for the Saturday matinee.

a. Write an equation representing the total amount of money collected.

b. If 42 adult tickets were purchased for this matinee, how many children were there?

Example 2: Juan has a pocket full of dimes and quarters. The total value of his change is $6.25.

a. Write a linear equation in general form to represent this situation. Clearly indicate what each variable represents.

b. If Juan has 7 quarters in his pocket, how many dimes are there?

Example 3: Ivan invested money into two mutual funds. Fund A earned 6% interest during the first year, while Fund B earned 8% interest. At the end of the year, he receives a total of $774 in interest.

a. Write a linear equation in general form to represent this situation. Clearly indicate what each variable represents.

b. If Ivan invested $8500 in Fund A, how much did he invest in Fund B?

Example 4: Kim invested money into two mutual funds. Fund A earned 6% profit during the first year, while Fund B suffered a 3.5% loss. At the end of the year, she receives a total of $177 in profit.

a. Write a linear equation in general form to represent this situation. Clearly indicate what each variable represents.

b. If Kim invested $3650 in Fund A, how much did she invest in Fund B?

Section 11.2 – You Try

At a concession stand, two hot dogs and three sodas cost $12.

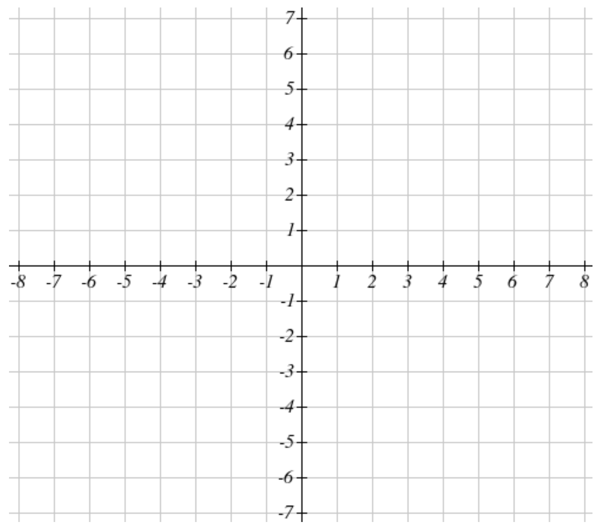
a. Let h represent the price of each hot dog, and s represent the price of each soda. Write a linear equation in general form to represent this situation.

b. If sodas cost $1.50 each, how much is each hot dog?

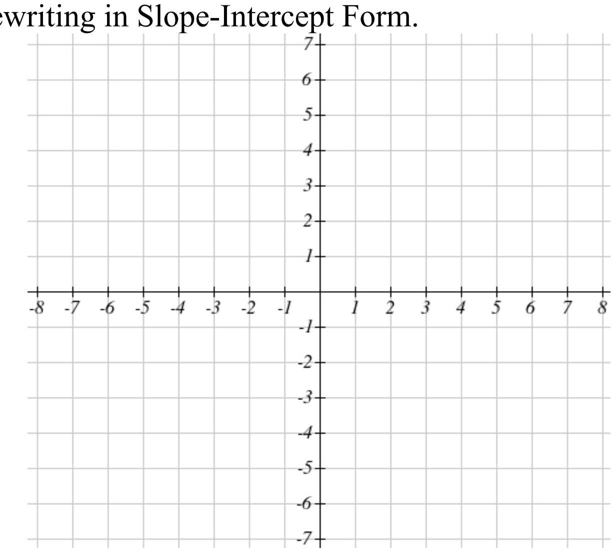
Section 11.3: Point-Slope Form of a Line

Slope-Intercept Form of a Linear Equation	**Point-Slope Form of a Linear Equation**
$y = mx + b$ x = input, y = output m = slope b = vertical intercept $(0, b)$	$y - y_1 = m(x - x_1)$ x = input, y = output m = slope (x_1, y_1) is a point on the line.

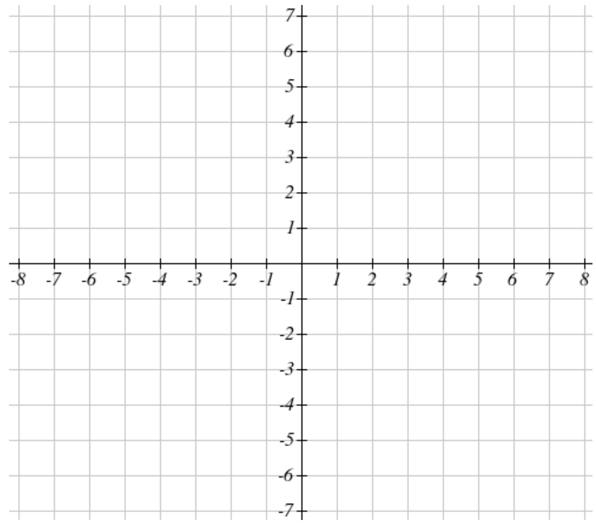
Example 1: Graph the line $y - 3 = -2(x + 4)$.



Example 2: Graph the line $y - 3 = -2(x + 4)$ by rewriting in Slope-Intercept Form.



Example 3: Graph the line $y + 3 = \frac{1}{2}(x - 1)$.



Example 4: Find the equation of the line with slope = ½ and passing through the point (6, -3).

Point-Slope Form	**Slope-Intercept Form**
$y - y_1 = m(x - x_1)$	$y = mx + b$

Example 5: Find the equation of the line containing the points (-1, 4) and (3, 5).

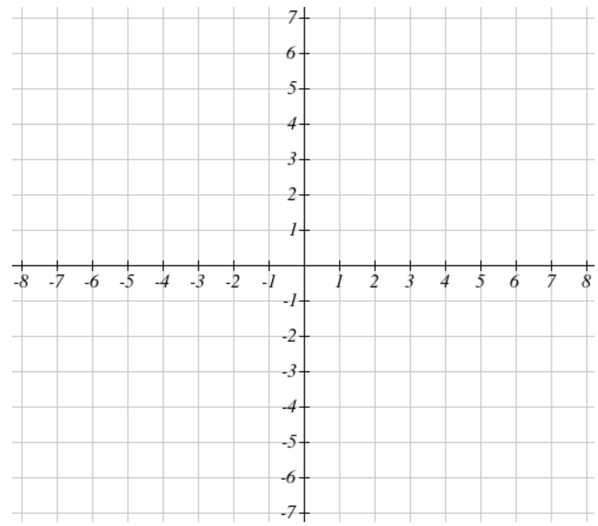
Point-Slope Form	**Slope-Intercept Form**
$y - y_1 = m(x - x_1)$	$y = mx + b$

Section 11.3 – You Try

Complete the problems below. Show as much work as possible, as shown in the examples.

a. Draw an *accurate* graph of the line.

$$y - 5 = \frac{2}{3}(x - 3)$$



b. Rewrite the linear equation $y + 7 = -3(x - 5)$ in Slope-Intercept Form, $y = mx + b$.

c. Find the equation of the line containing the points (-2, 4) and (8, -1).

Point-Slope Form	**Slope-Intercept Form**
$y - y_1 = m(x - x_1)$	$y = mx + b$

Unit 11: Practice Problems

Skills Practice

1. Which of the ordered pairs below satisfy the equation $x - y = 5$?
   (-2, 3) (6, 1) (0, -5) (-3, -8)

2. Which of the ordered pairs below satisfy the equation $2x + 3y = 6$?
   (0, 3) (6, -2) (3, 0) (-3, 4)

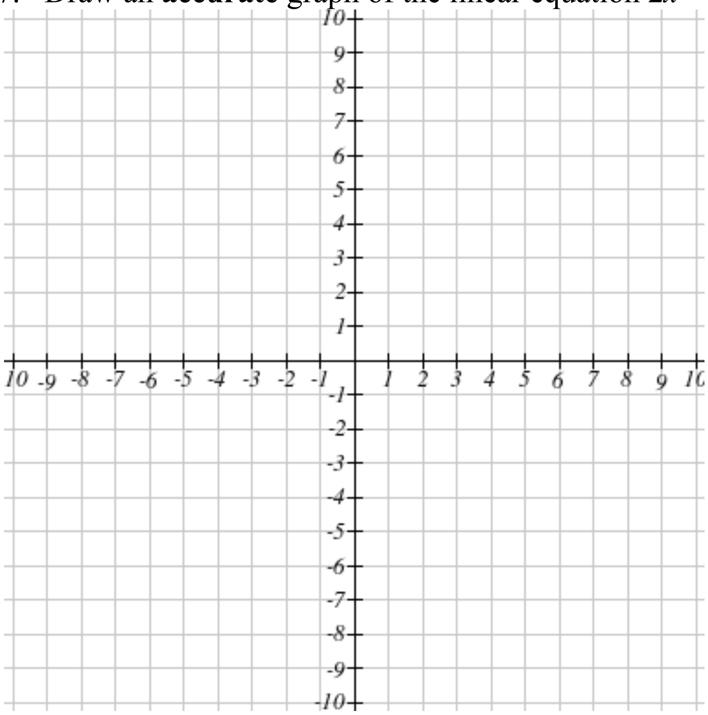
3. Which of the ordered pairs below satisfy the equation $y - 5 = 2(x + 1)$?
   (5, -1) (1, -5) (-5, 1) (-1, 5)

4. Which of the ordered pairs below satisfy the equation $y + 8 = -\frac{3}{2}(x - 4)$?
   (8, -4) (0, -2) (-8, 4) (0, 8)

5. Write the equation $x - y = 5$ in Slope-Intercept Form.

6. Write the equation $2x + 3y = 6$ in Slope-Intercept Form.

7. Draw an **accurate** graph of the linear equation $2x + 4y = 12$.



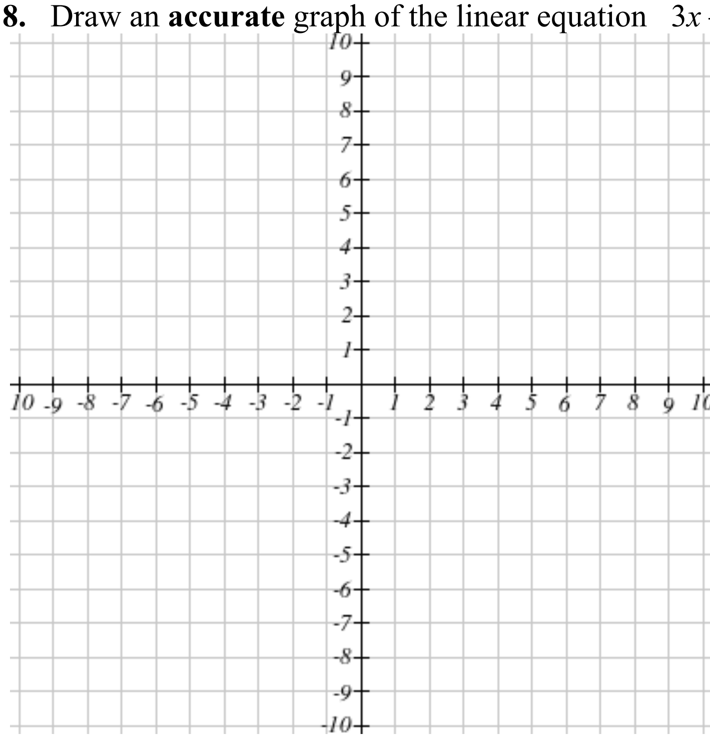
Slope-Intercept Form:

Slope: ____________

Vertical Intercept: _____________

Horizontal Intercept: _____________

8. Draw an **accurate** graph of the linear equation $3x - 2y = 10$.



Slope-Intercept Form:

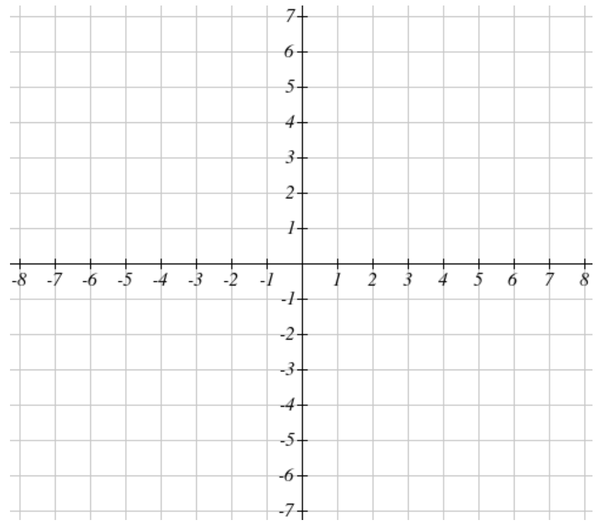
Slope: ____________

Vertical Intercept: _____________

Horizontal Intercept: _____________

9. Draw an **accurate** graph for each of the following by identifying the slope and one point on the line.

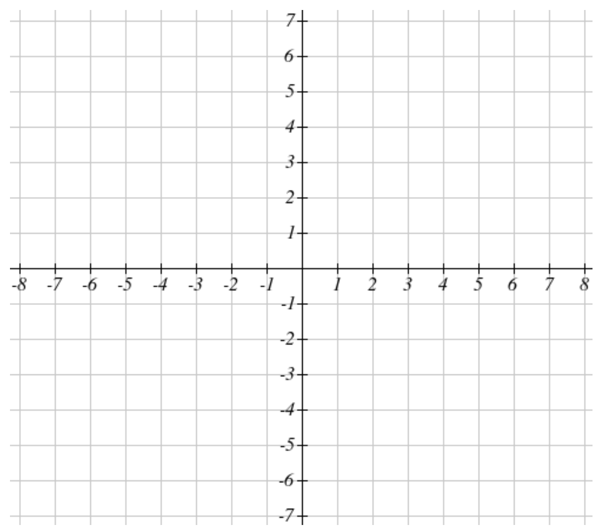
   a. $y + 1 = 2(x - 5)$

   Slope: ________________

   Point: ________________

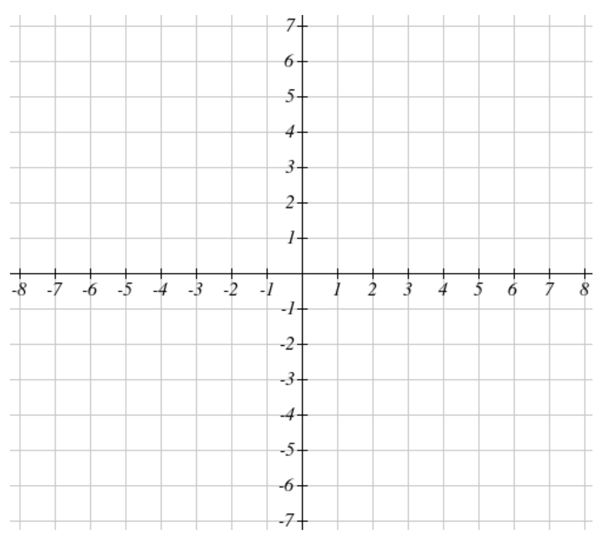
   b. $y - 2 = -3(x + 1)$

   Slope: ________________

   Point: ________________

   c. $y + 4 = -\frac{2}{5}(x - 2)$

   Slope: ________________

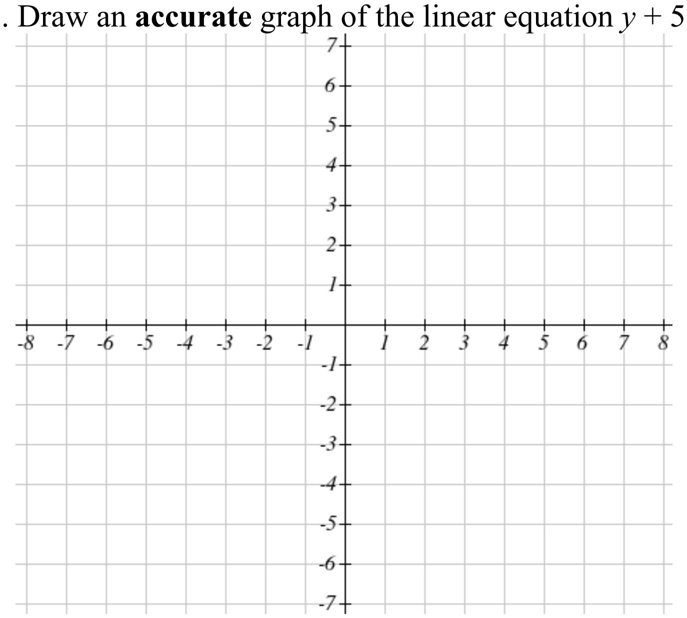
   Point: ________________

10. Rewrite the linear equation $y + 1 = 2(x - 5)$ in Slope-Intercept Form, *y = mx + b*.

11. Rewrite the linear equation $y - 2 = -3(x + 1)$ in Slope-Intercept Form, *y = mx + b*.

12. Rewrite the linear equation $y + 4 = -\frac{2}{5}(x - 2)$ in Slope-Intercept Form, *y = mx + b*.

13. Draw an **accurate** graph of the linear equation $y + 5 = 2(x + 1)$.



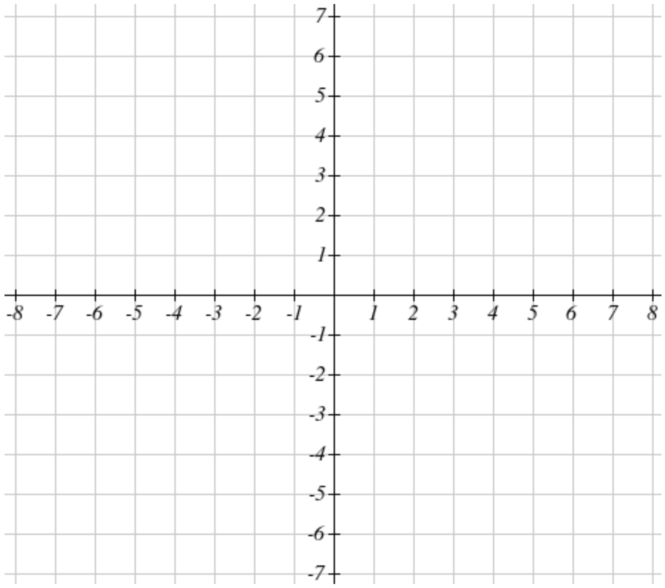
Slope-Intercept Form:

Slope: ____________

Vertical Intercept: _____________

Horizontal Intercept: _____________

14. Draw an **accurate** graph of the linear equation $y + 2 = -\frac{1}{2}(x - 3)$.



Slope-Intercept Form:

Slope: ___________

Vertical Intercept: ____________

Horizontal Intercept: ____________

15. Find the equation of the line with slope = -4 and passing through the point (8, -10).

Point-Slope Form	**Slope-Intercept Form**
$y - y_1 = m(x - x_1)$	$y = mx + b$

16. Find the equation of the line with slope = $\frac{4}{5}$ and passing through the point (-2, 5).

Point-Slope Form	**Slope-Intercept Form**
$y - y_1 = m(x - x_1)$	$y = mx + b$

17. Find the equation of the line containing the points (2, 3) and (9, -4).

Point-Slope Form	**Slope-Intercept Form**
$y - y_1 = m(x - x_1)$	$y = mx + b$

18. Find the equation of the line containing the points (-5, 2) and (4, -1).

Point-Slope Form	**Slope-Intercept Form**
$y - y_1 = m(x - x_1)$	$y = mx + b$

Applications

19. At a concession stand, three hot dogs and five sodas cost $18.50.
    a. Let *h* represent the price of each hot dog, and *s* represent the price of each soda. Write a linear equation in general form to represent this situation.

    b. If hot dogs cost $3.25 each, how much is each soda?

20. The Science Museum charges $14 for adult admission and $11 for each child. The museum bill for a school field trip was $896.
    a. Write a linear equation in general form to represent this situation. Clearly indicate what each variable represents.

    b. Nine adults attended the field trip. How many children were there?

21. Jamaal invested money into two mutual funds. Fund A earned 6% interest during the first year, while Fund B earned 2.5% interest. At the end of the year, he receives a total of $390 in interest. Write a linear equation in general form to represent this situation. Clearly indicate what each variable represents.

22. Marisol invested money into two mutual funds. Fund A earned 4% profit during the first year, while Fund B suffered a 2% loss. At the end of the year, she receives a total of $710 in profit. Write a linear equation in general form to represent this situation. Clearly indicate what each variable represents.

23. Jake has a pocket full of dimes and quarters. The total value of his change is $4.00. Write a linear equation in general form to represent this situation. Clearly indicate what each variable represents.

24. Bill begins a 50 mile bicycle ride. Unfortunately, his bicycle chain breaks, and he is forced to walk the rest of the way. Bill walks at a rate of 4 miles per hour, and rides his bike at a rate of 18 miles per hour.
    a. Let *b* represent the amount of time Bill spent bicycling before the chain broke, and *w* represent the amount of time Bill spent walking. Write a linear equation in general form to represent this situation. (Hint: Distance = rate · time)

    b. Bill had been riding his bike for two hours when the chain broke. Use the equation in part a to determine the amount of time he spent walking.

Extension

25. **Refer to your course syllabus**

    a. The Final Exam for this class is worth ____________ % of your course grade.

    b. Let *x* represent the score you make on the Final Exam (as a percent), and *y* represent your grade in the class (as a percent) just prior to taking the Final Exam. Write a linear *inequality* in general form to represent this situation, assuming that you want your final course grade to be:

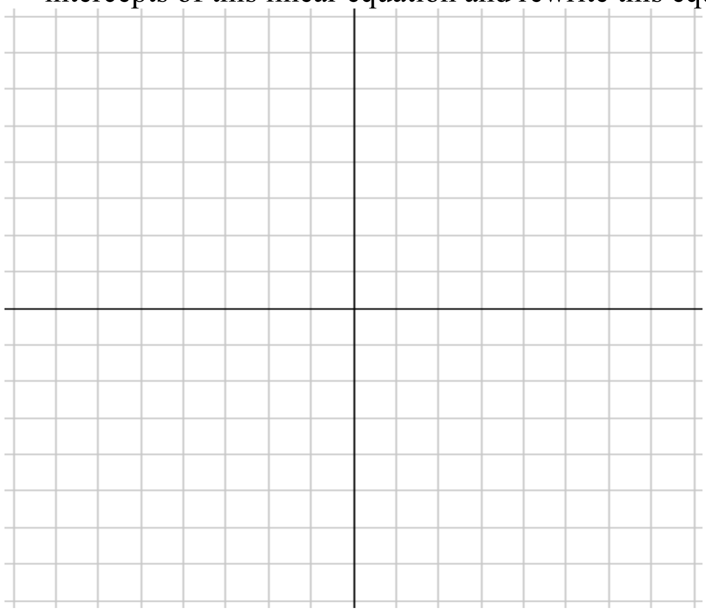
    A: At least 90% B: At least 80% C: At least 70%

    Hint: If your Final Exam is worth 30% of your course grade, then everything else would be worth 100% – 30% = 70% of your course grade.

    c. Suppose you have a 77% in the class just before taking the final exam. What score do you need to make on the Final Exam to earn an A, B, or C in the class? Assume that your instructor *does not* round up!

Unit 11: Review

1. Draw an **accurate** graph of the linear equation $2x + 3y = 6$. Determine the slope and intercepts of this linear equation and rewrite this equation in Slope-Intercept Form.



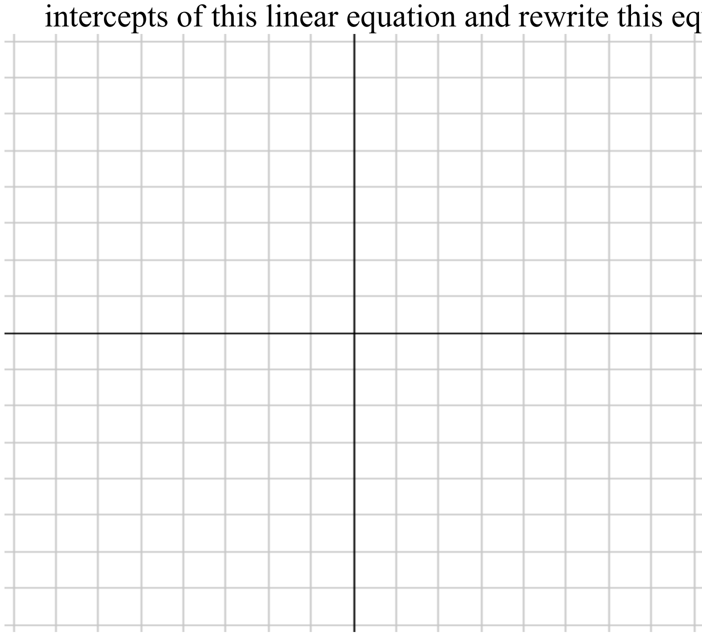
Slope-Intercept Form:

Slope: ____________

Vertical Intercept: _____________

Horizontal Intercept: _____________

2. Draw and **accurate** graph of the linear equation $y + 1 = \frac{1}{2}(x - 6)$. Determine the slope and intercepts of this linear equation and rewrite this equation in Slope-Intercept Form.



Slope-Intercept Form:

Slope: ____________

Vertical Intercept: _____________

Horizontal Intercept: _____________

3. Find the equation of the line containing the points (2, -5) and (-1, 4).

Point-Slope Form	**Slope-Intercept Form**
$y - y_1 = m(x - x_1)$	$y = mx + b$

4. Rashid invested money into two mutual funds. Fund A earned 4% interest during the first year, while Fund B earned 1.5% interest. At the end of the year, he receives a total of $117 in interest.

   a. Write a linear equation in general form to represent this situation. Clearly indicate what each variable represents.

   b. If Rashid invested $3,000 in Fund B, how much did he invest in Fund A?

5. Water boils at 212°F and 100°C, and water freezes at 32°F and 0°C.

   a. Using the above information, and the fact that the relationship between Fahrenheit and Celsius is linear, write a linear equation in slope intercept form that converts temperature in Celsius to Fahrenheit.

b. You are planning a trip to São Paulo, Brazil, in January. You notice that the average temperature for São Paulo is about 30^{o}C. Should you pack for cold or warm weather on your trip and why?

6. Your company produces t-shirts. You make an initial investment of $50,000 to get your company up and running. It costs you $6 to produce each t-shirt, and you currently sell your t t-shirts to various retailers at $15 each.
How many t-shirts do you need to sell in order to break even?

Unit 12: Systems of Equations

KEY TERMS AND CONCEPTS	
Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms *in your own words*. Provide examples that are not identical to those in the Media Lesson.	
System of Linear Equations	
Solution to a System of Linear Equations	

Types of Solutions to a System of Linear Equations	
Substitution Method	
Addition (Elimination) Method	

Unit 12: Main Lesson

Section 12.1: Systems of Linear Equations

Definitions
Two linear equations that relate the same two variables are called a **system of linear equations.**
A **solution** to a system of linear equations is an **ordered pair** that satisfies both equations.

Example 1: Verify that the point (5, 4) is a solution to the system of equations

$$y = 2x - 6$$
$$y = x - 1$$

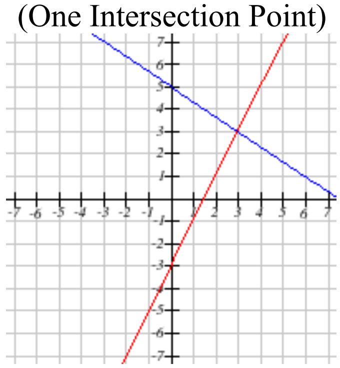
Types of Solutions to a Linear System of Equations

Graphically, the solution to a system of linear equations it a point at which the graphs intersect.

Types of Solutions to a Linear System of Equations:

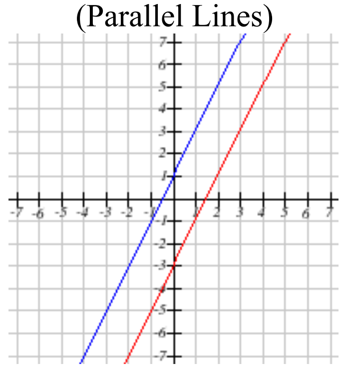
- **One unique solution**: The lines intersect at exactly one point
- **No solution**: The two lines are parallel and will never intersect
- **Infinitely many solutions**: This occurs when both lines graph as the same line

One Unique Solution
(One Intersection Point)



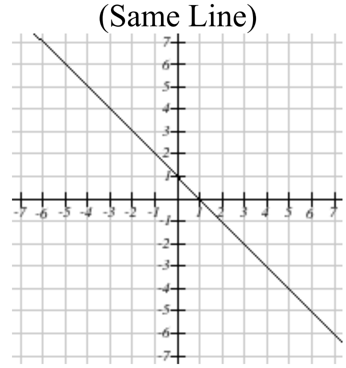
Consistent and Independent

No Solution
(Parallel Lines)



Inconsistent

Infinitely Many Solutions
(Same Line)



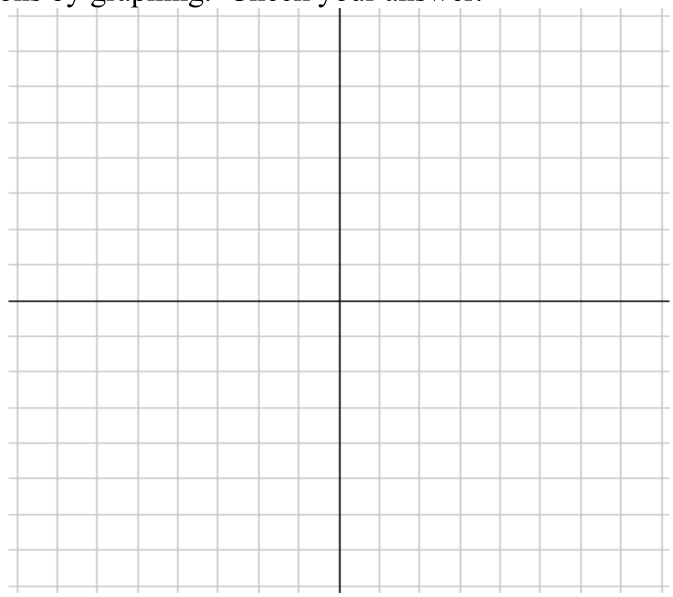
Consistent and Dependent

Solving a System of Linear Equations by Graphing

Example 2: Solve the system of equations by graphing. Check your answer.

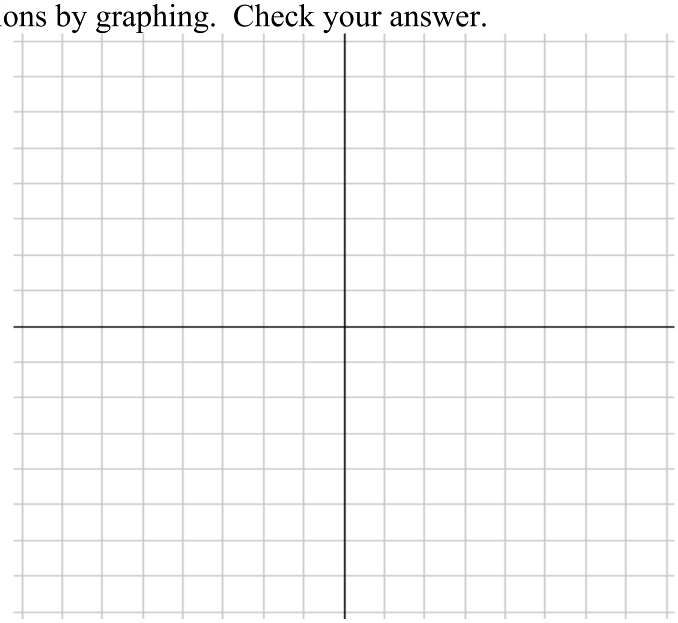
$$y = 6 - \frac{2}{3}x$$

$$y = x + 1$$



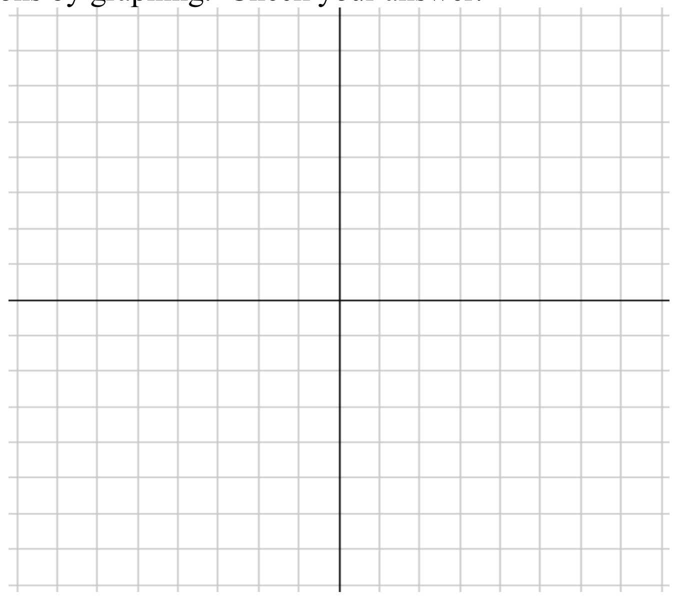
Example 3: Solve the system of equations by graphing. Check your answer.

$$4x - 3y = -18$$
$$2x + y = -4$$



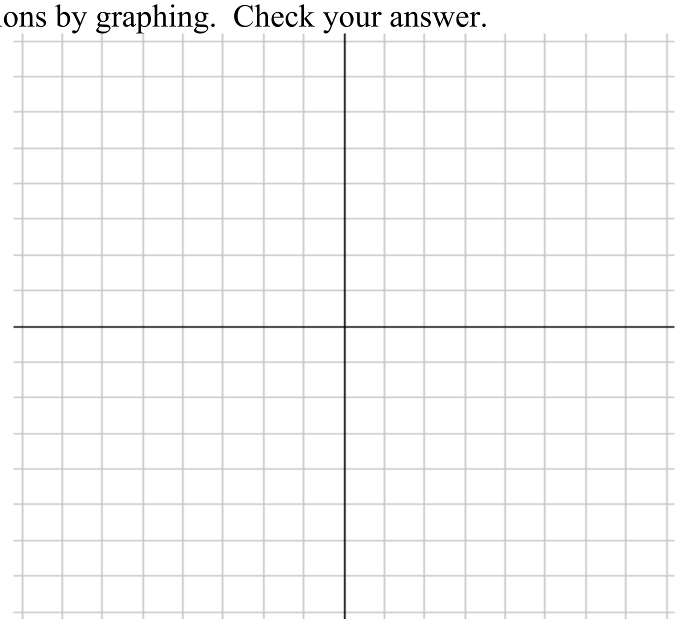
Example 4: Solve the system of equations by graphing. Check your answer.

$x - 3y = 3$
$3x - 9y = -18$



Example 5: Solve the system of equations by graphing. Check your answer.

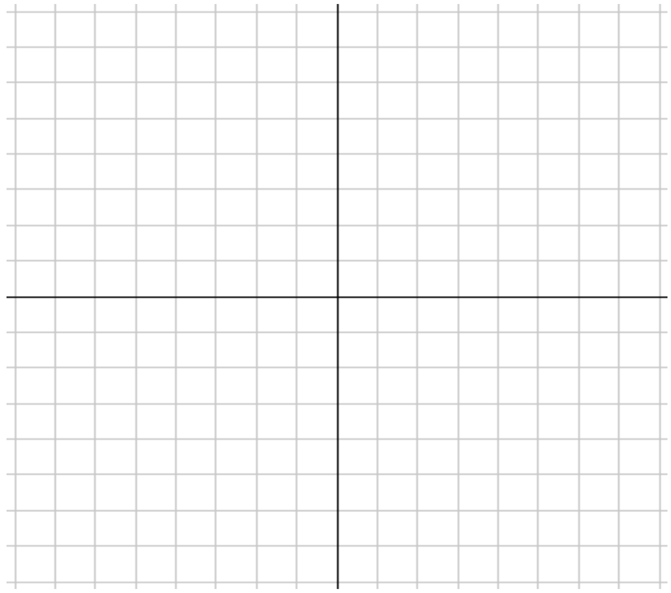
$2x + y = 3$
$6x + 3y = 9$



Section 12.1 – You Try

Solve the system of equations by graphing. Write your answer as an ordered pair and verify that it is correct.

$$x - y = 2$$
$$x + y = 6$$



Verify that your solution is correct:

Section 12.2: The Substitution Method

Consider the following equations: $y = 2x$
$x + y = 3$

Using Substitution to Solve a Linear System of Equations
Step 1: Solve one of the equations of the system for one of the variables.
Step 2: Substitute the expression for the variable obtained in step 1 into the other equation.
Step 3: Solve the equation.
Step 4: Substitute the result back into one of the original equations to find the ordered pair solution.
Step 5: Check your result by substituting your result into either one of the original equations.

Example 1: Solve the system of equations using the Substitution Method.

$3x - 2y = 16$
$2x + y = 20$

Example 2: Solve the system of equations using the Substitution Method.

$5x - 4y = 9$
$x - 2y = -3$

Example 3: Solve the system of equations using the Substitution Method.

$3x + y = 5$

$6x + 2y = 11$

Example 4: Solve the system of equations using the Substitution Method.

$x - y = -1$

$y = x + 1$

Section 12.2 – You Try

Solve the system of equations using the Substitution Method. Show all steps. Check your answer.

$x - 2y = -11$

$5x + 2y = 5$

Section 12.3: The Addition (Elimination) Method

Consider the following systems of equations:

$x - 2y = -11$
$5x + 2y = 5$

Using the Addition (Elimination) Method to Solve a Linear System of Equations
Step 1: "Line up" the variables.
Step 2: Determine which variable you want to eliminate. Make those coefficients opposites.
Step 3: Add straight down (one variable should "drop out")
Step 4: Solve resulting equation
Step 5: Substitute this result into either of the ORIGINAL equations
Step 6: Solve for the variable
Step 7: CHECK!!!!!!! Plug solution into BOTH equations!

Example 1: Solve the system of equations using the Addition (Elimination) Method.

$4x - 3y = -15$
$x + 5y = 2$

Example 2: Solve the system of equations using the Addition (Elimination) Method.

$3x - 2y = -12$
$5x - 8y = 8$

Example 3: Solve the system of equations using the Addition (Elimination) Method.

$7x - 2y = 41$
$3x - 5y = 1$

Section 12.3 – You Try

Solve the system of equations using the Addition (Elimination) Method. Show all steps. Check your answer.

$2x + 3y = 18$
$x - y = 4$

Section 12.4: Applications

Example 1: Movie tickets cost \$7 for adults (matinee), \$5.50 for children. There are 218 seats in the theater. A total of \$1,463 was collected in ticket sales for the sold-out Saturday matinee. How many adults and how many children were in the theater?

a. Write an equation representing the total number of tickets sold.

b. Write an equation representing the total amount of money collected from the sale of all tickets.

c. Solve this system of linear equations.

Section 12.4 – You Try

Tickets to a 3D movie cost $12.50 for adults and $8.50 for children. The theater can seat up to 180 people. A total of $1,826 was collected in ticket sales for the sold-out 7:15PM show. Determine the number of adult tickets and the number of children's tickets that were sold.

a. Write an equation representing the total number of tickets sold. Clearly indicate what each variable represents.

b. Write an equation representing the total amount of money collected from the sale of all tickets.

c. Solve this system of linear equations. Show all steps.

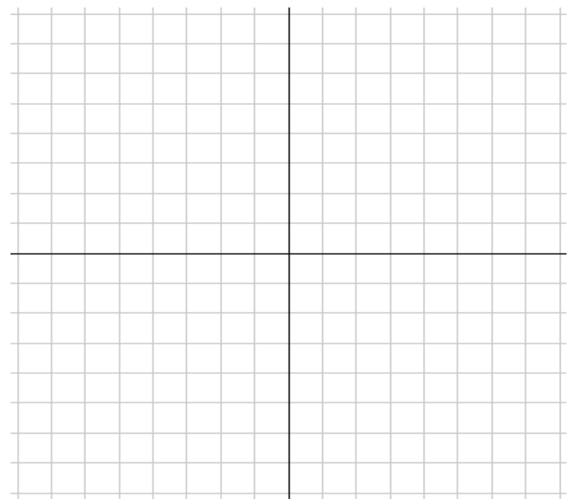
Number of adult tickets sold: ____________

Number of children's tickets sold: ____________

Section 12.5: Linear Inequalities in Two Variables

The Solution Set

Example 1: Graph the linear equation $y = 2x - 3$.



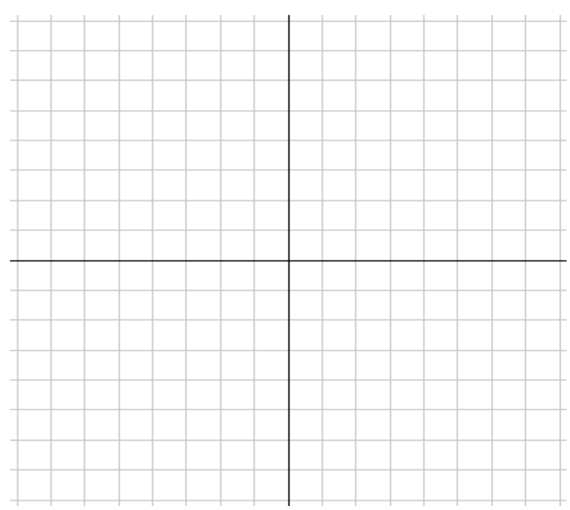
Example 2: Which of the ordered pairs below satisfy the **equation** $y = 2x - 3$?

(5,3)   (2,1)   (0,0)

Example 3: Which of the ordered pairs below satisfy the **inequality** $y \leq 2x - 3$?

(5,3)   (2,1)   (0,0)

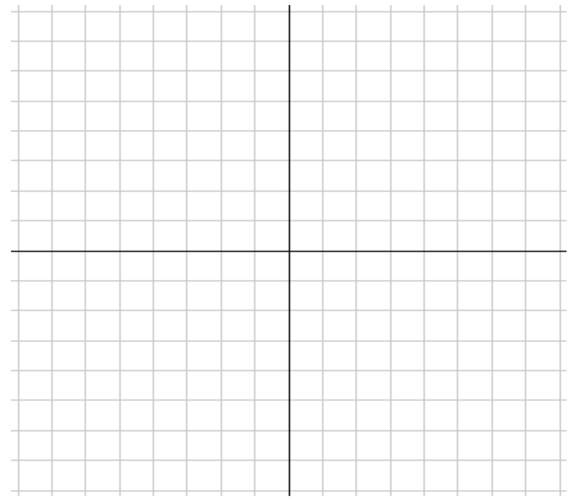
Example 4: Graph the linear equation $y \leq 2x - 3$.



Example 5: Which of the ordered pairs below satisfy the **inequality** $y < 2x - 3$?

(5,3) (2,1) (0,0)

Example 6: Graph the linear equation $y < 2x - 3$.



You Try

Which of the ordered pairs below satisfy the linear inequality $y \geq 4 - 2x$?

(1,2) (0,0) (5,0)

Which of the ordered pairs below satisfy the linear inequality $y < 4 - 2x$?

(1,2) (0,0) (5,0)

Section 12.6: Graphing Linear Inequalities in Two Variables

Graphing The Solution Set of a Linear Inequality in Two Variables

Step 1: Rewrite the inequality as an equality statement.

Step 2: Graph the linear equation. This is the boundary of the solution region.

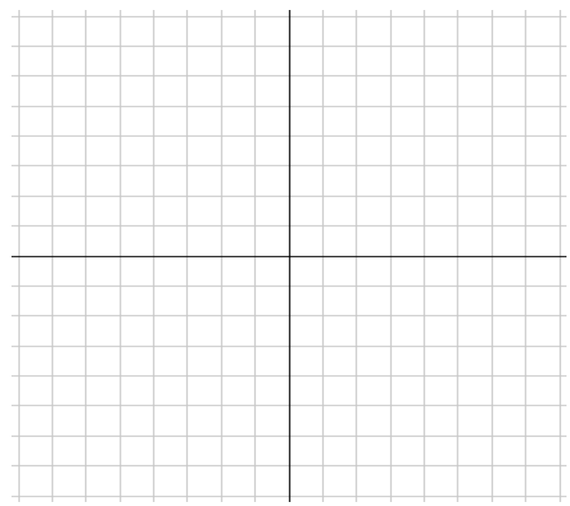
Step 3: Determine if the line should be solid or dotted.

- If the original inequality statement is either $<$ or $>$, draw a dotted line.
- If the original inequality statement is either $\leq$ or $\geq$, draw a solid line.

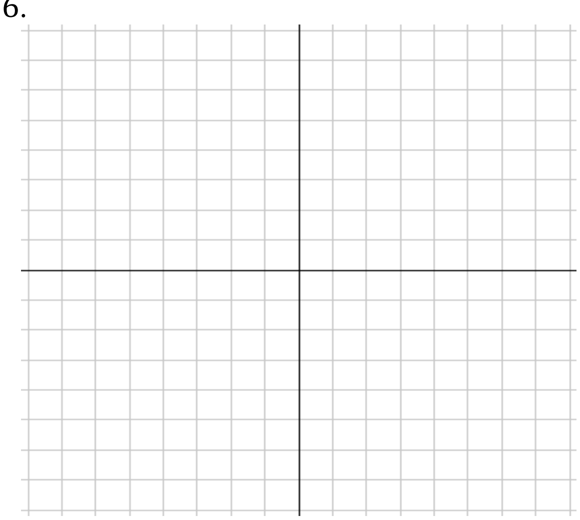
Step 4: Choose a test point and plug it into the original inequality.

- If the test point satisfies the inequality, shade in the direction of the test point.
- If the test point does not satisfy the inequality, shade in the opposite direction of the test point.

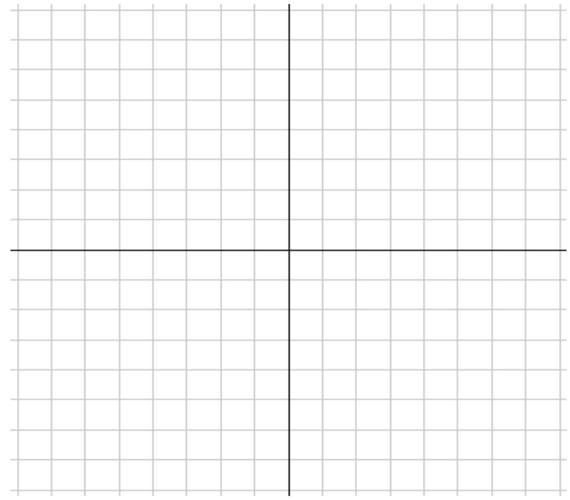
Example 1: Graph the linear inequality $y < 5 - 3x$.



Example 2: Graph the linear inequality $3x - 2y \geq 6$.

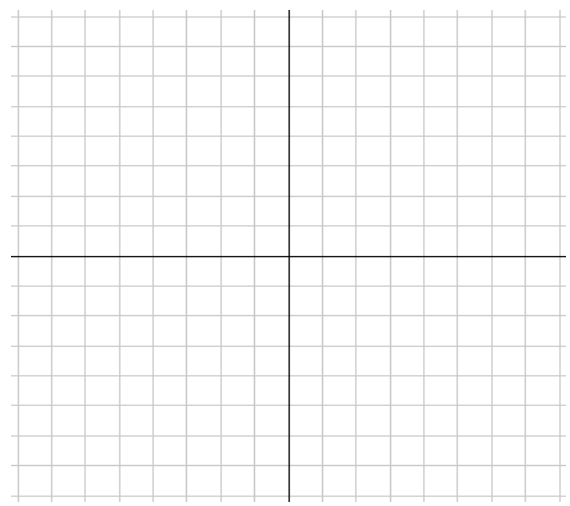


Example 3: Graph the linear inequality $y \geq 2x$.



You Try

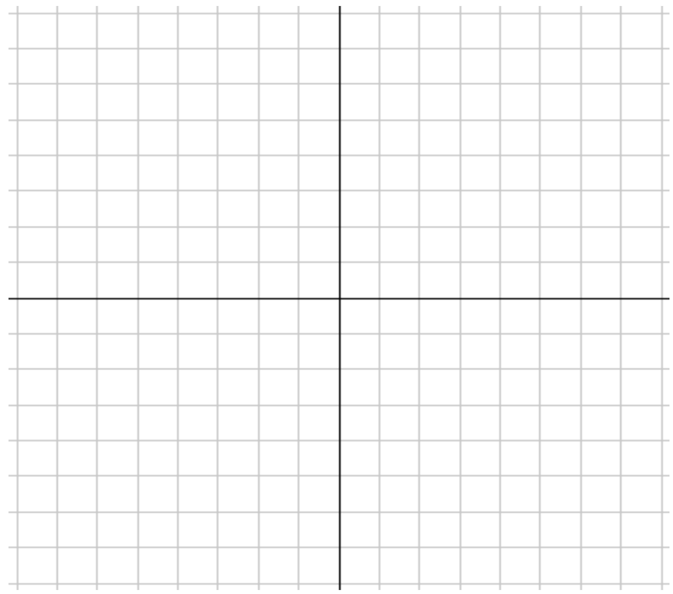
Graph the linear inequality $y > 2x - 1$.



Section 12.7: Graphing Systems of Linear Inequalities

Example 1: Solve the **system** of linear **equations** by graphing.

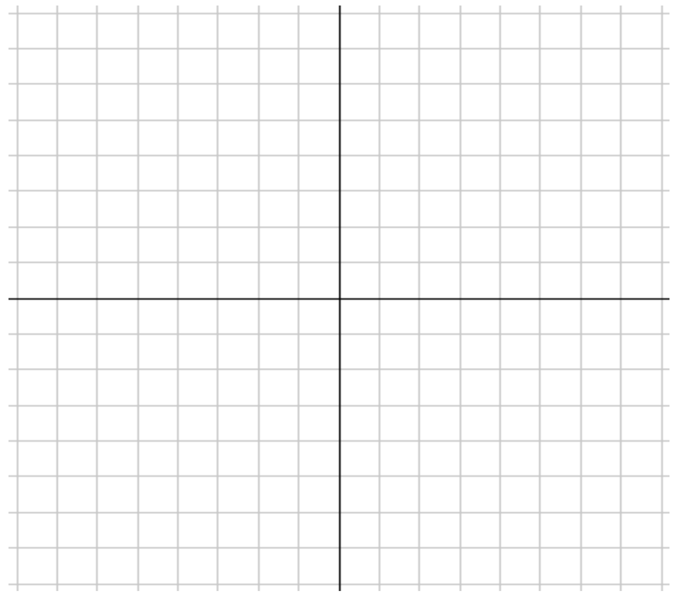
$y = x + 2$
$y = -2x + 4$



Solutions to Systems of Linear Equations and Inequalities
The solution to a system of two linear **equations** is a **point** (if the solution exists).
The solution to a system of two or more linear **inequalities** is a **region** of the plane. We must **graph** the solution by shading in the appropriate region.

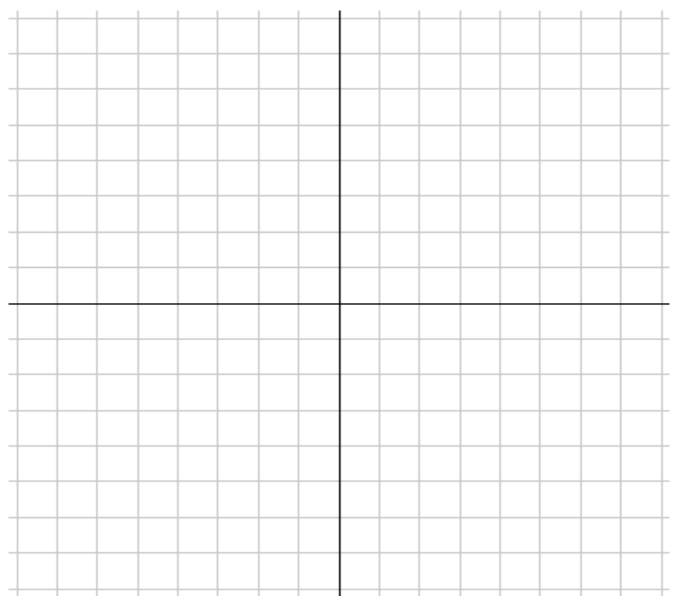
Example 2: Solve the **system** of linear **inequalities**.

$y > x + 2$
$y < -2x + 4$



Example 2: Solve the system of linear inequalities.

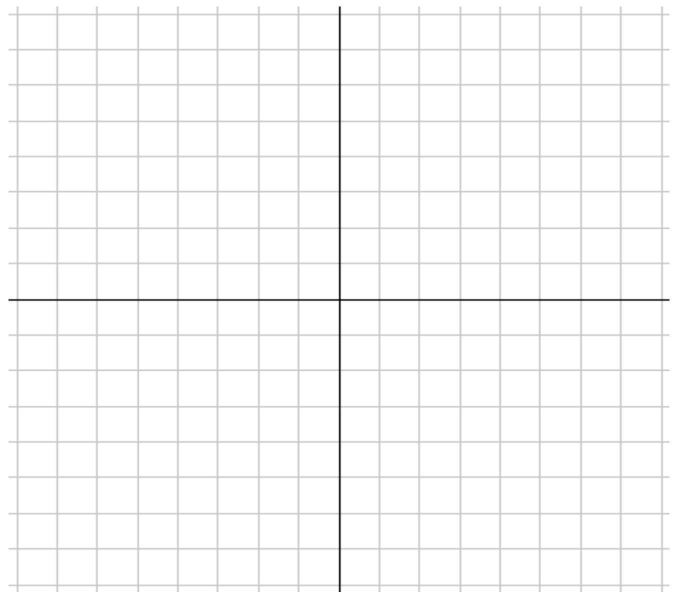
$2y + x \leq 2$
$y - x < 4$



You Try

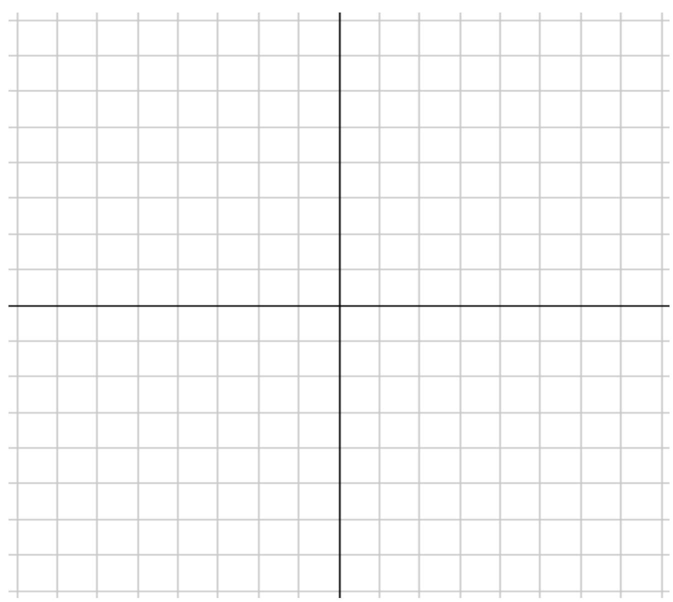
Solve the system of linear inequalities.

$$2y - x \geq 4$$
$$y < 3$$



Solve the system of linear inequalities.

$$y \geq -x - 3$$
$$y \leq 2$$
$$x < 4$$



Unit 12: Practice Problems

Skills Practice

1. Is the point (6, 1) a solution to the system of equations below? You must show correct work to justify your answer.

$$y = x - 5$$
$$y = 2x + 4$$

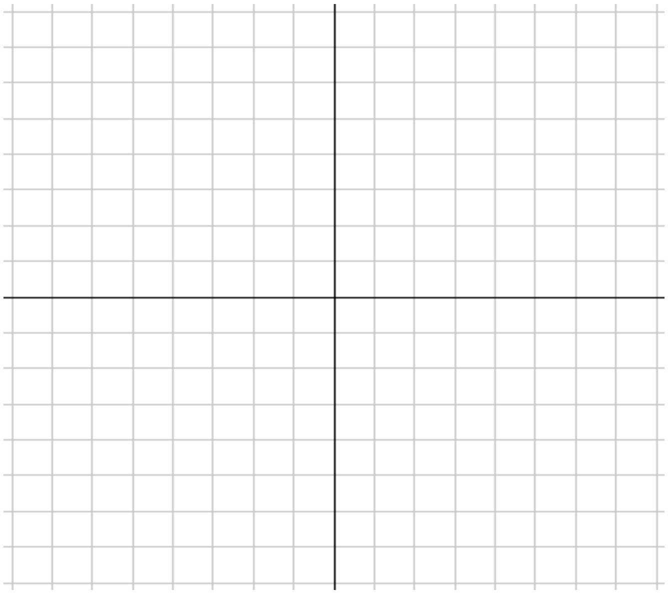
2. Is the point (–2, 5) a solution to the system of equations below? You must show correct work to justify your answer.

$$2x + y = 1$$
$$3x - 2y = -16$$

3. Is the point (5, 3) a solution to the system of equations below? You must show correct work to justify your answer.

$$3x - 2y = 9$$
$$2x + 5y = 4$$

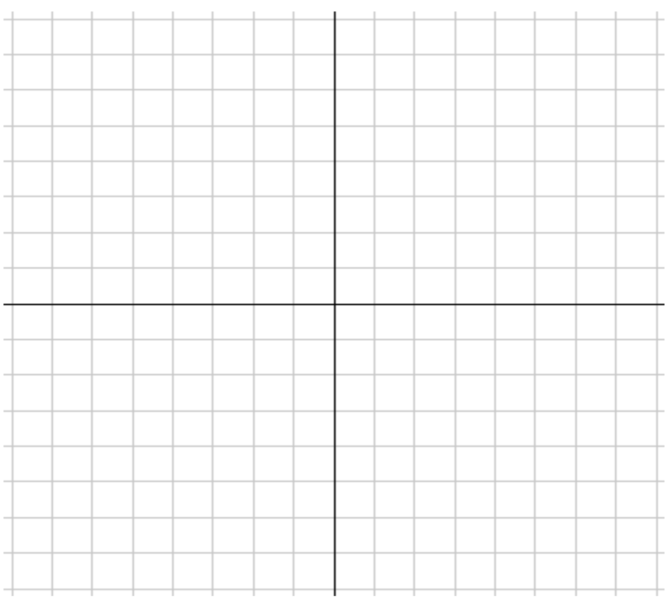
4. Solve the system of equations by **graphing**. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.



$y = 7 - x$
$y = 3x - 5$

Solution: ______________

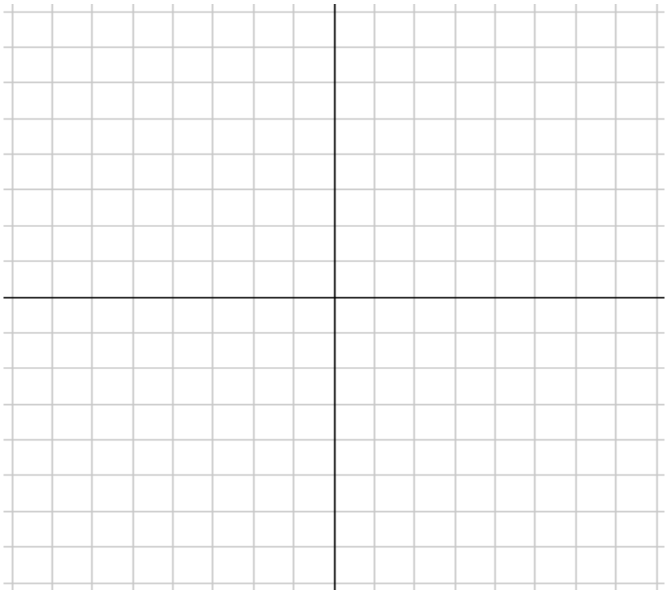
5. Solve the system of equations by **graphing**. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.



$x - y = -2$
$x + y = 4$

Solution: ______________

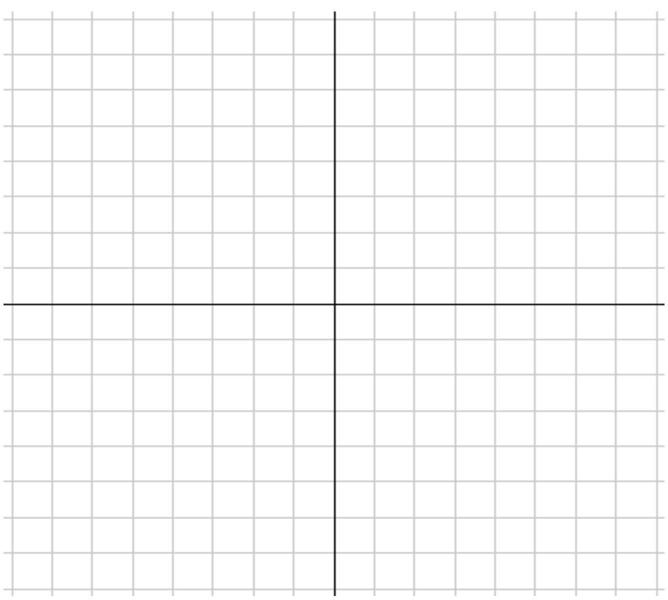
6. Solve the system of equations by **graphing**. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.



$x - 2y = 10$
$5x - y = -4$

Solution: ______________

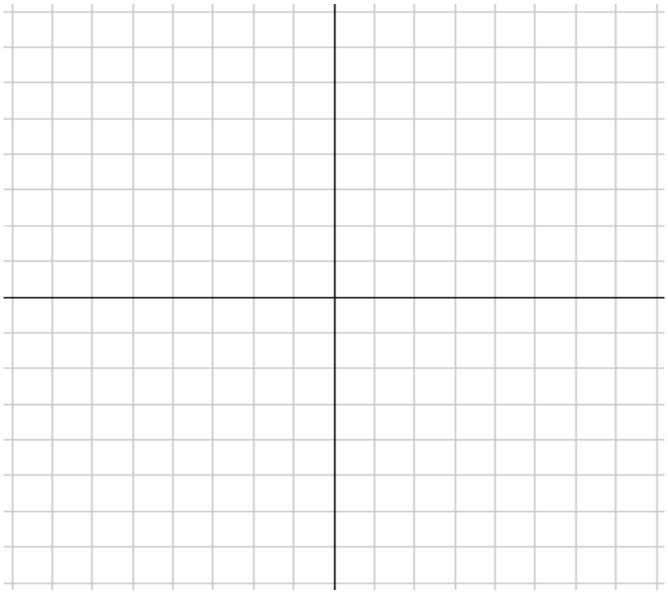
7. Solve the system of equations by **graphing**. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.



$3x - y = 8$
$-3x + y = 1$

Solution: ______________

8. Solve the system of equations by **graphing**. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.



$x + 2y = -4$
$2x + 4y = -8$

Solution: _____________

9. Solve the system of equations using the **substitution** method. Show all steps.
$5x + y = 2$
$3x - 4y = 15$

Solution: ______________

10. Solve the system of equations using the **substitution** method. Show all steps.

$2x + y = 8$

$6x + 3y = 24$

Solution: _______________

11. Solve the system of equations using the **substitution** method. Show all steps.

$x - y = 9$

$5x + 3y = 21$

Solution: _______________

12. Solve the system of equations using the **addition (elimination) method**. Show all steps.

$-3x + 2y = 12$

$x + y = 16$ Solution: _______________

13. Solve the system of equations using the **addition (elimination) method**. Show all steps.

$3x - 2y = -12$

$12x - 8y = 22$ Solution: _______________

14. Solve the system of equations using the **addition (elimination) method**. Show all steps.

$3x + 2y = -18$

$4x - 3y = -24$ Solution: _______________

15. Solve the system of equations using the **addition (elimination) method**. Show all steps.

$5x + 2y = -10$

$3x + 4y = 8$

Solution: ______________

16. The functions $f(x)$ and $g(x)$ are defined by the following tables. At what point is $f(x) = g(x)$?

x	–2	–1	0	1	2	3	4
$f(x)$	11	8	5	2	–1	–4	–7

x	–2	–1	0	1	2	3	4
$g(x)$	7	6	5	4	3	2	1

Solution (write the ordered pair):__________________

17. The functions $f(x)$ and $g(x)$ are defined by the following tables. At what point is $f(x) = g(x)$?

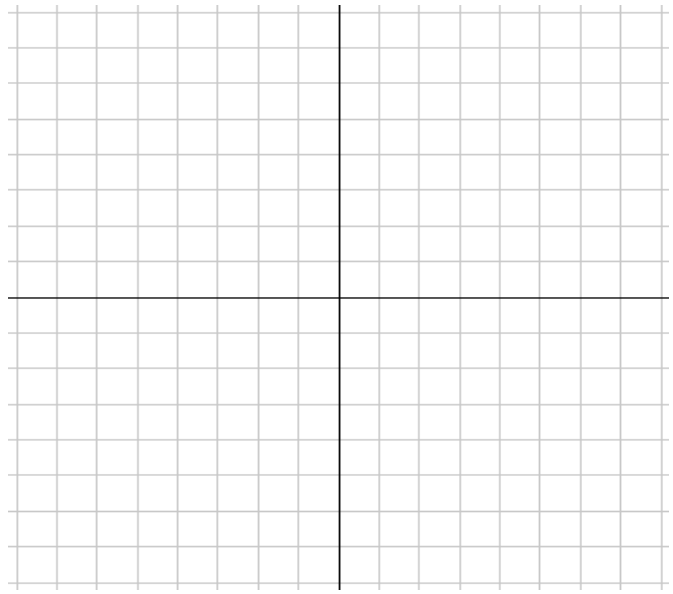
x	–2	–1	0	1	2	3	4
$f(x)$	8	1	0	–1	–8	–27	–64

x	–2	–1	0	1	2	3	4
$g(x)$	8	10	12	14	16	18	20

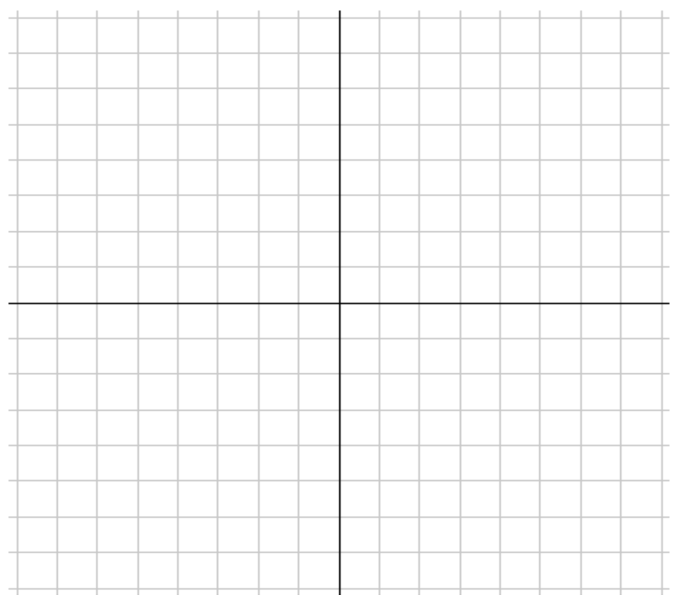
Solution (write the ordered pair):__________________

18. Graph the solution sets to the following linear inequalities.

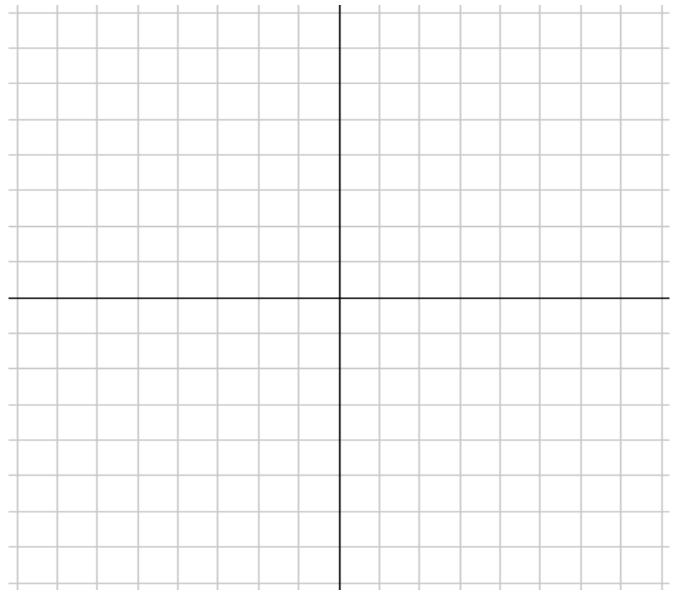
a. $y > 3 - x$



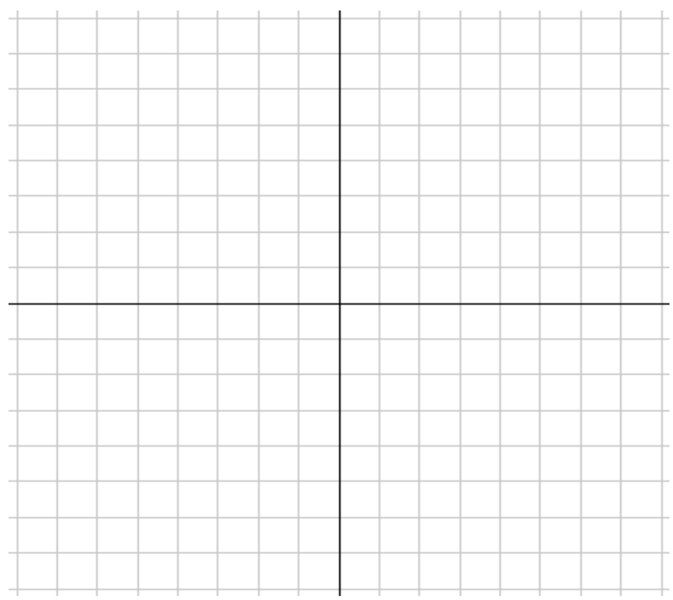
b. $y \geq \frac{3}{5}x - 1$



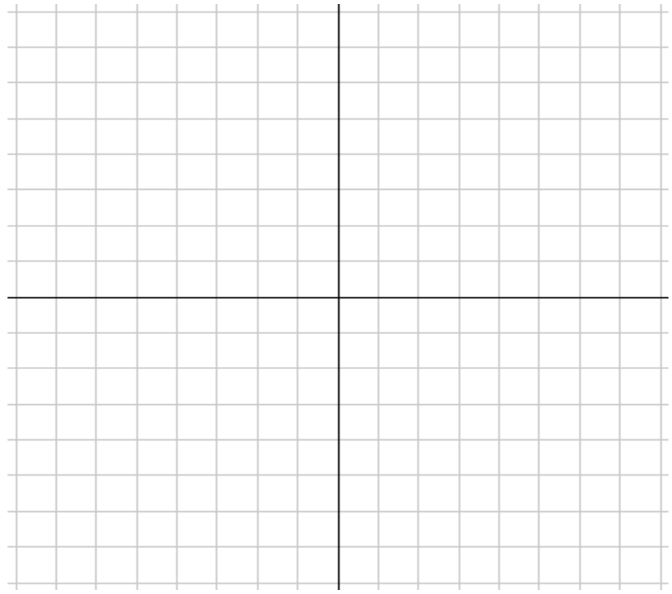
c. $4x - y < 3$



d. $y > \frac{1}{2}x$

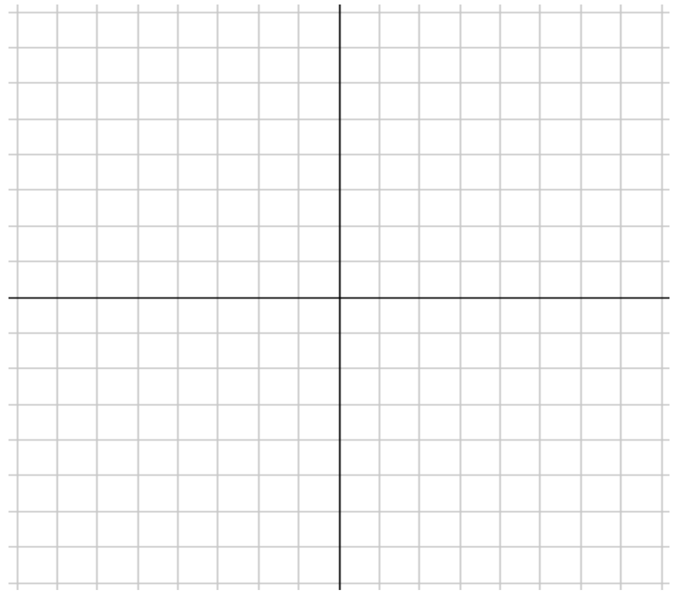


e. $x \geq 2$

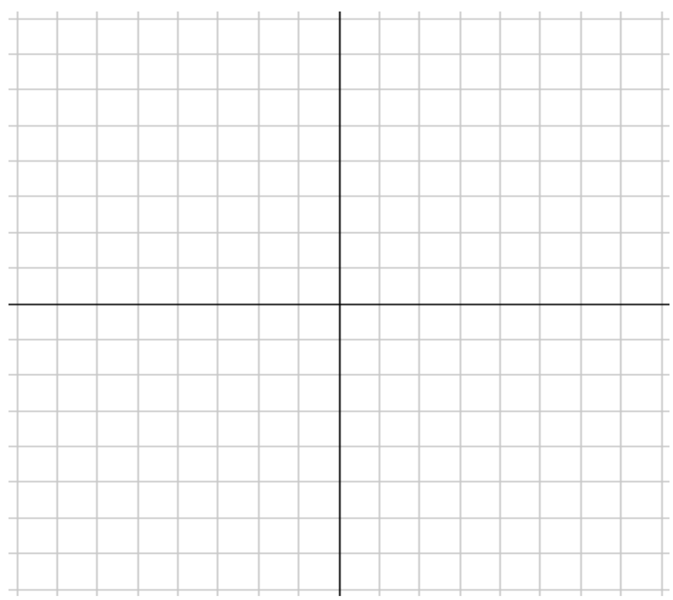


19. Graph the solutions to the following systems of inequalities.

a. $y - x \geq -2$
$3y - 2x < 6$



b. $y \leq -3x + 5$
$y > 2$
$y \leq x + 5$



Applications

20. Your yard is a mess, and you decide to hire a landscaper. The Greenhouse charges a $80 consultation fee plus $14 per hour for the actual work. Garden Pros does not charge a consulting fee, but charges $30 per hour for the actual work.

   a. Write an equation that describes the cost, C, if you hire The Greenhouse for *h* hours of work.

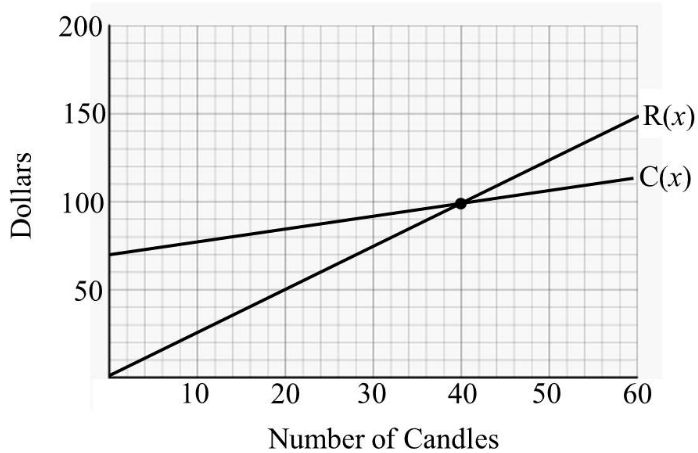
   b. Write a second equation that describes Garden Pros' charge, C, for *h* hours of work.

   c. Solve this system of linear equations. Write your answer as an ordered pair.

   d. Interpret the solution in a complete sentence.

   e. Your yard needs a lot of work, and you anticipate that the job will take at least 6 hours. Which service do you choose? Why?

21. The graph below shows the cost and revenue for a company that produces and sells scented candles. The function R(x) gives the revenue earned when x candles are sold. The function C(x) gives the total cost to produce x candles.



a. Discuss the significance of the point (40, 100) in terms of the cost, revenue, and *profit* for this company.

b. What happens if *fewer than* 40 candles are sold?

c. What happens if *more than* 40 candles are sold?

22. At a concession stand, five hot dogs and five sodas cost $30. Two hot dogs and four sodas cost $15. Determine the price of each hot dog and each soda.

Price for each soda: ____________

Price for each hot dog:___________

23. The Science Museum charges $14 for adult admission and $11 for each child. The total bill for 68 people from a school field trip was $784. How many adults and how many children went to the museum?

Number of children _____________

Number of adults _____________

24. Tickets to a 3D movie cost $12.50 for adults and $8.50 for children. The theater can seat up to 260 people. A total of $1,734 was collected in ticket sales for the 7:15PM show, in which only 60% of the tickets were sold. How many adults and how many children were in the theater?

Number of children _____________

Number of adults _____________

25. Jake has 20 coins in his pocket, all of which are dimes and quarters. If the total value of his change is $4.10, how many dimes and how many quarters does he have?

Number of dimes _____________

Number of quarters _____________

26. Juan had \$17400 and chose to split the money into two different mutual funds. During the first year, Fund A earned 3% interest and Fund B earned 6% interest. If he received a total of \$774 in interest, how much had he invested into each account?

Amount invested in Fund A: ____________

Amount invested in Fund B: ____________

27. Emery invested \$10,000 in two mutual funds. Fund A earned 4% profit during the first year, while Fund B suffered a 2% loss. If she received a total of \$130 profit, how much had she invested in each mutual fund?

Amount invested in Fund A: ____________

Amount invested in Fund B: ____________

28. Bill begins a 100 mile bicycle ride. Unfortunately, his bicycle chain breaks, and he is forced to walk the rest of the way. The whole trip takes 6 hours. If Bill walks at a rate of 4 miles per hour, and rides his bike at a rate of 20 miles per hour, find the amount of time he spent walking. Write your answer in a complete sentence. (Hint: Distance = rate · time)

Extension

29. The functions $f(x)$ and $g(x)$ are defined by the following tables.
    At what point(s) is $f(x) = g(x)$?

x	**–2**	**–1**	**0**	**1**	**2**	**3**	**4**
f(x)	4	1	0	1	4	9	16

x	**–2**	**–1**	**0**	**1**	**2**	**3**	**4**
g(x)	-1	1	3	5	7	9	11

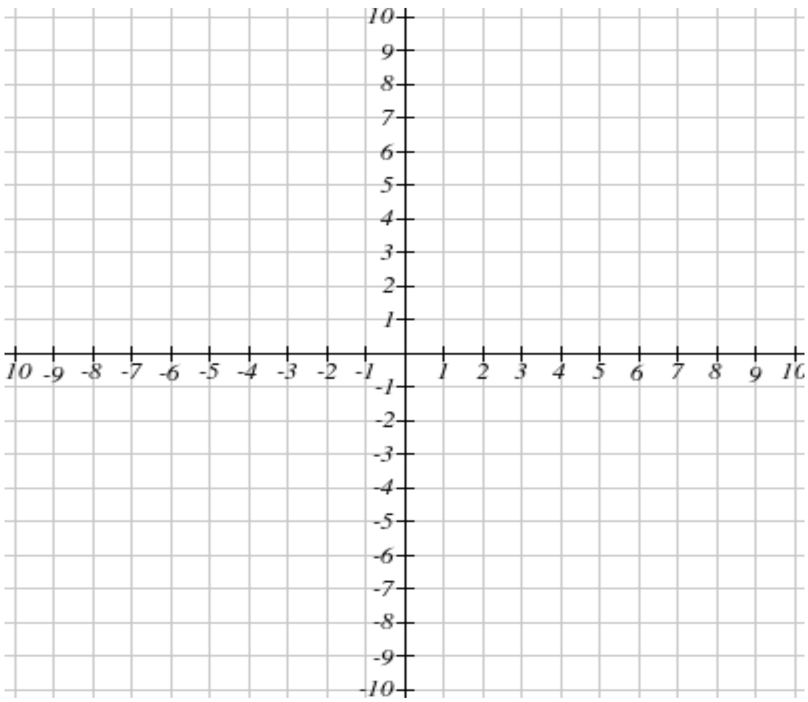
Solutions (write the ordered pairs):_________________

30. Construct a system of linear equations (in slope-intercept form) that has the ordered pair (3,5) as a solution.

31. Construct a system of linear equations (in general form) that has the ordered pair (2,4) as a solution.

Unit 12: Review

1. Solve the system of equations by **graphing**. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.



$4x - 3y = -18$
$3x + y = -7$

Solution: ______________

2. Solve the system of equations using the **substitution** method. Show all steps. Verify that your solution is correct.

$2x - 3y = -19$
$x + 2y = 8$

Solution: _______________

3. Solve the system of equations using the **addition (elimination)** method. Show all steps. Verify that your solution is correct.

$5x - 2y = -3$
$7x - y = 12$

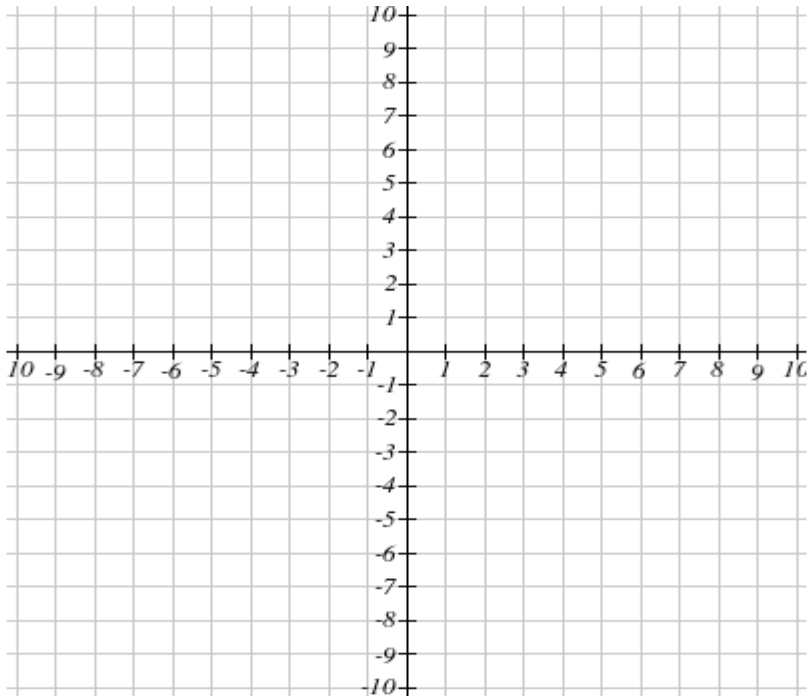
Solution: ______________

4. Jamaal invested $10,000 in two mutual funds. Fund A earned 6% profit during the first year, and Fund B earned 2% profit. If he received a total of $374 profit, how much had he invested in each mutual fund? Show all steps. Verify that your solution is correct.

Amount invested in Fund A: ____________

Amount invested in Fund B: ____________

5. Solve the system of inequalities by graphing. Your lines must extend accurately to the edge of the graph.



$$y - 2x < 6$$
$$2y - x \leq -6$$

6. You Scream for Ice Cream parlor is starting to sell their signature ice cream sandwiches to local supermarkets. The supply curve for these ice cream sandwiches is given by $p(q) = -.45q + 17$, where p is price in dollars per box of ice cream sandwiches and q is number of boxes of ice cream sandwiches (quantity). The demand curve for these ice cream sandwiches is $p(q) = .05q + 2$. The market equilibrium is point of intersection of the two curves.
What is the market equilibrium for this situation?

Unit 13: Exponents

Section 13.1: Properties of Exponents

Section 13.2: Division Properties of Exponents

Section 13.3: Multiplication of Linear Expressions

KEY TERMS AND CONCEPTS Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms *in your own words*. Provide examples that are not identical to those in the Media Lesson.	
The Multiplication Property of Exponents	
Raising a Power to a Power	
Raising a Product to a Power	
The Division Property	

Raising a Quotient to a Power	
Multiplying Linear Expressions	
Squaring Linear Expressions	

Unit 13: Main Lesson

Section 13.1: Properties of Exponents

Given any real numbers *a, b, c, m,* and *n*

$n^1 =$ ______ $1^n =$ ______ $n^0 =$ ______ ($n \neq 0$) $0^n =$ ______ ($n \neq 0$)

$3^4 =$ ______

$3^3 =$ ______

$3^2 =$ ______

$3^1 =$ ______

$3^0 =$ ______

$3^{-1} =$ ______

$3^{-2} =$ ______

$3^{-3} =$ ______

$3^{-4} =$ ______

Multiplication Properties of Exponents	
$a^m \cdot a^n = a^{m+n}$	$(a^m)^n = a^{mn}$
Why?	Why?

Example 1: Evaluate and simplify the following expressions.
Assume $x \neq 0$, $x \neq -1/2$, $a \neq 0$, $b \neq 0$, and $c \neq 0$.

$5x^0$ $(2x + 1)^0$ $a^0 + b^0 + c^0$

The Multiplication Property: $a^m \cdot a^n = a^{m+n}$

Example 2: Simplify the following expressions

n^3n^9 $b^5 \cdot b^4 \cdot b$ $5x^2y^5(7xy^9)$

Raising a Power to a Power: $(a^m)^n = a^{mn}$

Example 3: Simplify the following expressions

$(x^3)^9$ $5b^2(b^5)^8$

Raising a Product to a Power: $(ab)^n = a^n b^n$

Example 4: Simplify the following expressions

$(5x)^2$ $(x^3y^2)^9$ $(-8ab^5)^2$

$5(-2w^7)^3$ $5n^4(-3n^3)^2$

Section 13.1 – You Try

Simplify the following expressions. Show all steps.

a. $(2x^4)^2$

b. $2(x^2)^3$

c. $8g^3 \cdot 5g^4$

d. $2n^0$

Section 13.2: Division Properties of Exponents

The Division Property: $\frac{a^m}{a^n} = a^{m-n}$ $a \neq 0$

$\frac{x^5}{x^2} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} = \frac{x \cdot x \cdot x}{1} = \frac{x^3}{1} = x^3$ $\frac{x^5}{x^2} = x^{5-2} = x^3$

Example 1: Simplify the following expressions. Variables represent nonzero quantities.

$\frac{x^{50}}{x^4}$ $\frac{4a^{10}b^5}{6ab^2}$

Raising a Quotient to a Power: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $b \neq 0$

Example 2: Simplify the following expressions. Variables represent nonzero quantities.

$\left(\frac{5}{7}\right)^2$ $\left(\frac{x^5}{y^3}\right)^4$ $\left(\frac{-4t^{10}}{u^6}\right)^2$

Section 13.2 – You Try

Simplify the following expressions. Variables represent nonzero quantities. Show all steps.

a. $\left(\frac{3a^{10}}{7}\right)^2$

b. $\frac{6x^3y^8}{9xy^5}$

Section 13.3: Multiplication of Linear Expressions

Multiplication of Linear Expressions

Example 1: Multiply and simplify.

a. $(x + 3)(x + 4)$

b. $(m - 5)(m - 6)$

c. $(2d - 4)(3d + 5)$

Squaring a Linear Expression

Example 2: Multiply and simplify

a. $(n + 5)^2$

b. $(3 - 2a)^2$

Section 13.3 – You Try

Multiply and simplify. Show all steps.

a. $-3x^2(x^5 + 6x^3 - 5x)$

b. $(3x - 4)(5x + 2)$

c. $(2p - 5)^2$

Name: ________________________________ Date: _____________

Unit 13: Practice Problems

Skills Practice

1. Simplify completely. Show all steps, and box your answers.

a. $(2x)^3$

b. $5(3n)^2$

c. $y^3 \cdot y^7 \cdot y$

d. $(-2x)^3$

e. $5w(8w^3)$

f. $(-2x^5)^2$

g. $(-5w^8)^2$

h. $3x^0 + 2x^0$

i. $(-4x)^2 + 4x^2$

j. $(5x - 7)^0$

2. Multiply and simplify completely. Show all steps, and box your answers.

a. $(p + 5)(p + 7)$

b. $(x + 2)(x - 2)$

c. $(2x - 4)(3x - 5)$

d. $(5w - 8)(3w + 11)$

e. $(x + 2)^2$

f. $(2x - 4)^2$

g. $3(x + 2)(x + 4)$

h. $4(x + 2)^2$

3. Simplify completely. Show all steps, and box your answers.

a. $\frac{x^8}{x^3}$

b. $\left(\frac{2}{5}\right)^4$

c. $\frac{8n^8p^5}{12np^4}$

d. $\left(\frac{3a^5}{7b}\right)^2$

4. Evaluate the algebraic expression x^2 given $x = -7$. Show your work.

5. Evaluate the algebraic expression $5x^3$ given $x = -2$. Show your work.

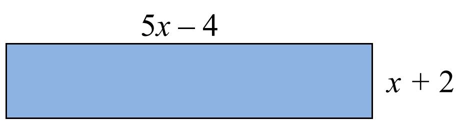
6. Evaluate the algebraic expression $(5x)^2$ given $x = -2$. Show your work.

7. Evaluate the algebraic expression $5(2x)^2$ given $x = -3$. Show your work.

8. Evaluate the algebraic expression $\frac{1}{4x^2}$ given $x = -5$. Show your work.

Application

9. Write an algebraic expression that represents the total area of the figure shown below. Simplify completely. Show your work.



Extension

10. If possible, simplify each of the following by combining like terms or using properties of exponents.

a. $2n^5 + 3n^5 =$ ________________

b. $2n^5 \cdot 3n^5 =$ ________________

c. $3n^3 + 3n^5 =$ ________________

d. $3n^3 \cdot 3n^5 =$ ________________

11. Simplify completely. Show all steps, and box your answers.

a. $4p(-5p^3)^2$

b. $3(-2x)^3 - 3x(-2)^3$

c. $4w^5(3w^8)^2$

d. $10p^3(-5p^7)^2$

e. $2a^3b(3ab^5)^2$

f. $(3x^4)^3 - (5x^6)^2$

Name: ________________________________ Date: _____________

Unit 13: Review

1. If possible, simplify each of the following by combining like terms or using properties of exponents.

   a. $8n^3 + 5n^3 =$ _______________ b. $8n^3 \cdot 5n^3 =$ _______________

   c. $8n^3 + 8n^5 =$ _______________ d. $8n^3 \cdot 8n^5 =$ _______________

2. Simplify completely. Show all steps, and box your answers.

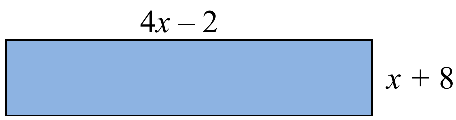
   a. $(-5x^5)^3$ b. $(3 - 5x)^2$

   c. $\frac{24m^8}{18m^3}$ d. $\left(\frac{5x}{3}\right)^2$

3. Evaluate the algebraic expression $8(2x)^2$ given $x = -5$. Show your work.

4. Evaluate the algebraic expression $\frac{2}{3x^2}$ given $x = 4$. Show your work.

5. Write an algebraic expression that represents the total area of the figure shown below.
Simplify completely. Show your work.



Lesson 14 –Solving Quadratic Equations

As you work through this lesson, you will learn to identify quadratic functions and their graphs (called parabolas). You will learn the important parts of the parabola including the direction of opening, the vertex, intercepts, and axis of symmetry.

We will then learn several methods for solving quadratic equations. Graphing is the first method you will work with to solve quadratic equations followed by factoring and then the quadratic formula. You will get a tiny taste of something called complex numbers and then will finish up by putting all the solution methods together.

Finally, you will learn how to recognize all the important characteristics of quadratic functions in the context of a specific application. Even if a problem does not ask you to graph the given quadratic function or equation, doing so is always a good idea so that you can get a visual feel for the problem at hand.

Pay special attention to the problems you are working with and details such as signs and coefficients of variable terms. Extra attention to detail will pay off in this lesson.

Lesson Topics

Section 14.1: Characteristics of Quadratic Functions

- Identify the Vertical Intercept
- Determine the Vertex
- Domain and Range
- Determine the Horizontal Intercepts (Graphically)

Section 14.2: Quadratic Equations in Standard Form

- Horizontal Intercepts
- Number and Types of solutions to quadratic equations

Section 14.3: Solving Quadratic Equations by Factoring

Section 14.4: The Quadratic Formula

Section 14.5: Complex Numbers

Section 14.6: Complex Solutions to Quadratic Equations

Section 14.7: Applications of Quadratic Functions

- Steps to solve quadratic application problems

Main Lesson 14

Section 14.1 – Characteristics of Quadratic Functions

A QUADRATIC FUNCTION is a function which can be written the form

$$f(x) = ax^2 + bx + c$$

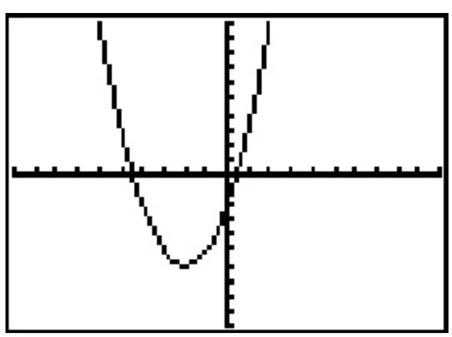
Characteristics include:

- Three distinct terms each with its own coefficient:
  - An x^2 term with coefficient a $(a \neq 0)$
  - An x term with coefficient b
  - A constant term, c
  - Note: If any term is missing, the coefficient of that term is 0
- The graph of this function is called a "parabola", is shaped like a "U", and opens either up or down
- a determines which direction the parabola opens ($a > 0$ opens up, $a < 0$ opens down)
- c is the vertical intercept with coordinates (0, c)

Problem 1

Given the quadratic function $f(x) = x^2 + 4x - 2$, complete the table below.

Identity the coefficients *a, b, c*	
Which direction does the parabola open?	
What is the vertical intercept?	



Problem 2

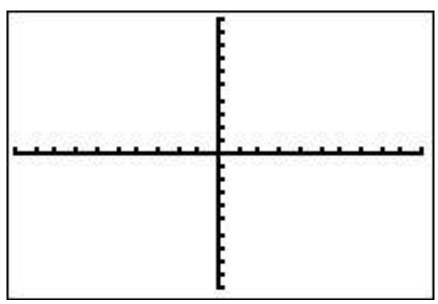
Given the quadratic function $f(x) = x^2 - 2x + 3$, complete the table below.

Identity the coefficients *a, b, c*	
Which direction does the parabola open? Why?	
What is the vertical intercept?	

Problem 3	**YOU TRY**

Given the quadratic function $f(x) = 2x^2 - 5$, complete the table below. Sketch a *possible* graph of the function.

Identity the coefficients *a, b, c*	
Which direction does the parabola open? Why?	
What is the vertical intercept? Plot and label on the graph.	



Given a quadratic function, $f(x) = ax^2 + bx + c$:

The VERTEX is the lowest or highest point (ordered pair) of the parabola

- To find the input value, identify coefficients *a* and *b* then compute $-\dfrac{b}{2a}$
- Plug this input value into the function to determine the corresponding output value, (i.e. evaluate $f\left(-\frac{b}{2a}\right)$)
- The vertex is always written as an ordered pair. Vertex = $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

Lesson 14 - Solving Quadratic Functions

The AXIS OF SYMMETRY is the vertical line that passes through the Vertex, dividing the parabola in half.

- Equation $x = -\frac{b}{2a}$

Problem 4

Given the quadratic function $f(x) = x^2 + 4x + 4$, complete the table below.

Identity the coefficients *a, b, c*	
Determine the coordinates of the vertex.	
Identify the Axis of Symmetry Equation.	
Sketch the graph	

Lesson 14 - Solving Quadratic Functions

Problem 5

Given the quadratic function $f(x) = x^2 - 2x + 3$, complete the table, generate a graph of the function, and plot/label the vertex and axis of symmetry on the graph.

Identity the coefficients *a, b, c*	
Determine the coordinates of the vertex.	
Identify the Axis of Symmetry Equation.	
Graph of the function. Plot/label the vertex and axis of symmetry on the graph.	

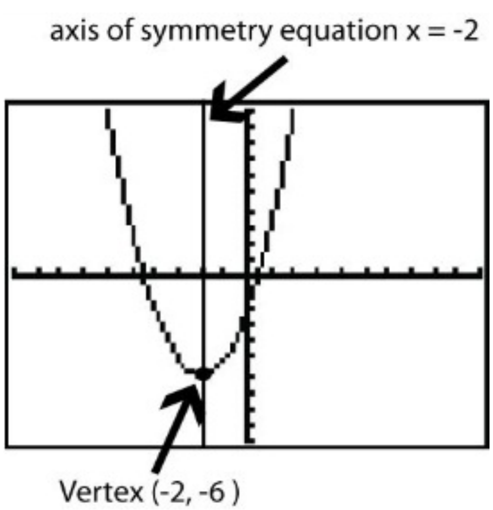
Problem 6	**YOU TRY**

Given the quadratic function $f(x) = 2x^2 - 5$, complete the table, graph the function, and plot/label the vertex and axis of symmetry on the graph. Compare your graph to **Problem 3**.

Identity the coefficients *a, b, c*	
Determine the coordinates of the vertex.	
Identify the Axis of Symmetry Equation.	
Graph of the function. Plot/label the vertex and axis of symmetry on the graph.	

Problem 7	**WORKED EXAMPLE**

Determine the domain and range of the quadratic function $f(x) = x^2 + 4x - 2$



Domain of $f(x)$:

All real numbers. $-\infty < x < \infty$ $(-\infty, \infty)$

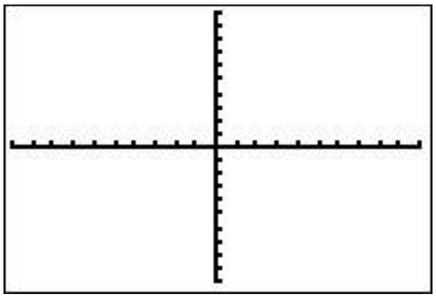
Range of $f(x)$:

Since the parabola opens upwards, the vertex $(-2, -6)$ is the *lowest* point on the graph.

The range is therefore $-6 \leq f(x) < \infty$, or $[-6, \infty)$

Problem 8

Determine the domain and range of $f(x) = -2x^2 - 6$.

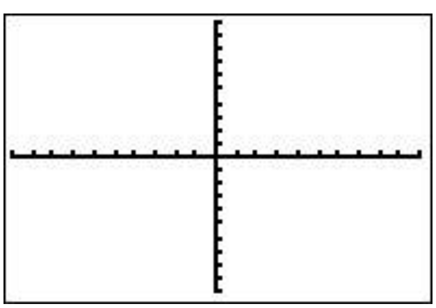


Domain of $f(x)$:

Range of $f(x)$:

Problem 9	**YOU TRY**

Determine the domain and range of $f(x) = x^2 - 4x + 3$ by first finding the vertex. Sketch the graph and label the vertex.

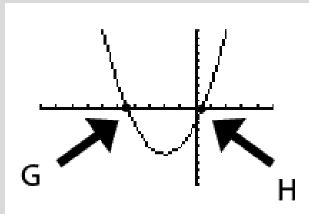


Vertex ordered pair:

Domain of $f(x)$:

Range of $f(x)$:

Finding Horizontal Intercepts of a Quadratic Function



The quadratic function, $f(x) = ax^2+bx+c$, will have horizontal intercepts when the graph crosses the x-axis (i.e. when $f(x) = 0$). These points are marked on the graph above as G and H. To find the coordinates of these points, what we are really doing is solving the equation $ax^2+bx+c = 0$. Later, we will learn methods for solving this equation.

Section 14.1 Practice Problems

1. For each of the following quadratic functions:
   - Identify the coefficients *a, b, c*
   - Determine if the parabola opens up or down and state why.
   - Draw the graph neatly.
   - Identify the vertical-intercept.
   - Mark and label the vertical intercept on the graph.

a) $f(x) = 2x^2 - 4x - 4$

a = _________ b = _________ c = _________

Vertical Intercept: __________

Which direction does this parabola open? Why?

b) $f(x) = -x^2 + 6x - 4$

a = _________ b = _________ c = _________

Vertical Intercept: __________

Which direction does this parabola open? Why?

c) $f(x) = 2x^2 - 6x + 4$

a = _________ b = _________ c = _________

Vertical Intercept: __________

Which direction does this parabola open? Why?

d) $f(x) = x^2 - 3x$

a = __________ b = __________ c = __________

Vertical Intercept: ___________

Which direction does this parabola open? Why?

e) $f(x) = \frac{x^2}{2} - 3$

a = __________ b = __________ c = __________

Vertical Intercept: ___________

Which direction does this parabola open? Why?

2. For each of the following quadratic functions (Show your work):
   - Calculate the vertex *by hand* and write it as an ordered pair.
   - Determine the axis of symmetry and write it as a linear equation (x = #).

	Function	$-\frac{b}{2a}$	$f\left(-\frac{b}{2a}\right)$	Vertex	Axis of Symmetry
a)	$f(x) = -2x^2 + 2x - 3$				
b)	$g(x) = \frac{x^2}{2} - 3x + 2$				
c)	$f(x) = -x^2 + 3$				
d)	$p(t) = 4t^2 + 2t$				
e)	$h(x) = 3x^2$				

Lesson 14 - Solving Quadratic Functions

3. Complete the table. Show your work.

	Function	Domain	Range
a)	$f(x) = -2x^2 + 2x - 3$		
b)	$g(x) = \frac{x^2}{2} - 3x + 2$		
c)	$f(x) = -x^2 + 3$		
d)	$p(t) = 4t^2 + 2t$		
e)	$h(x) = 3x^2$		

Section 14.2 – Quadratic Equations in Standard Form

A QUADRATIC EQUATION in STANDARD FORM is an equation of the form

$$ax^2 + bx + c = 0$$

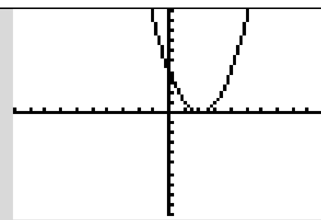
If the quadratic equation $ax^2 + bx + c = 0$ has real number solutions x_1 and x_2, then the x-intercepts of $f(x) = ax^2 + bx + c$ are $(x_1, 0)$ and $(x_2, 0)$.

Note that if a parabola does not cross the x-axis, then its solutions lie in the complex number system and we say that it has *no real x-intercepts*.

There are three possible cases for the number of solutions to a quadratic equation in standard form.

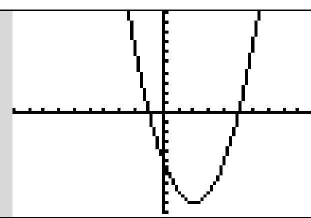
CASE 1: One, repeated, real number solution

The parabola touches the x-axis in *just one* location (i.e. only the vertex touches the x-axis)



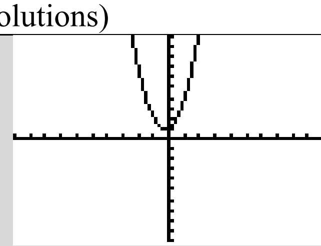
CASE 2: Two unique, real number solutions

The parabola crosses the x-axis at *two* unique locations.



CASE 3: No real number solutions (but two Complex number solutions)

The parabola does NOT cross the x-axis.

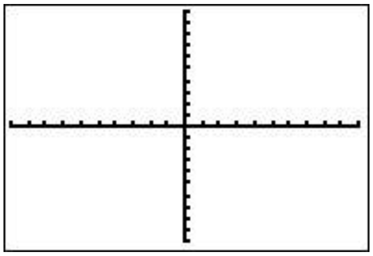


Lesson 14 - Solving Quadratic Functions

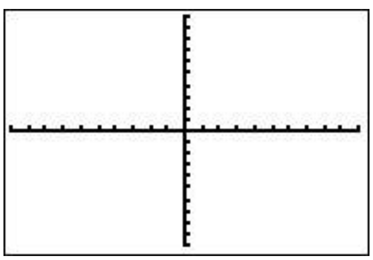
Problem 1

Determine the number and type of solutions to each of the quadratic equations below by first finding the vertex and whether the parabola opens up or down. Begin by putting the equations into standard form. Draw a rough sketch of the parabola and label the intercepts on your graph.

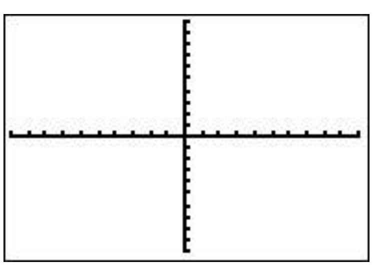
a) $x^2 - 10x + 25 = 0$



b) $-2x^2 + 8x - 3 = 0$



c) $3x^2 - 2x = -5$



Problem 2	**YOU TRY**

Determine the number and type of solutions to each of the quadratic equations below. Begin by putting the equations into standard form. Draw a rough sketch of the parabola, including the vertex, vertical intercept, and any horizontal intercepts should appear on the screen.

a) $-x^2 - 6x - 9 = 0$

Number of Real Solutions:_______________

Number of Complex Solutions: _________________

b) $3x^2 + 5x + 20 = 0$

Number of Real Solutions:_______________

Number of Complex Solutions: _________________

c) $2x^2 + 5x = 7$

Number of Real Solutions:_______________

Number of Complex Solutions: _________________

Section 14.2 Practice Problems

Find the vertex and vertical intercept for each of the quadratic functions below. Then determine the number and type of roots the function has. You do not need to solve for the roots.

a) $f(x) = x^2 - 6x + 9$

Vertex: ________________

Vertical Intercept: ________

Number of Real Roots:__________

Number of Complex Roots: ____________________

b) $f(x) = 5x^2 + 4x - 5$

Vertex: ________________

Vertical Intercept: ________

Number of Real Roots:__________

Number of Complex Roots: ____________________

Determine the number and type of solutions for each equation. You do not need to solve the equations.

c) $2x^2 - 4x = 3$

Number of Real Solutions:__________

Number of Complex Solutions: ____________________

d) $3x^2 + 6x + 4 = 0$

Number of Real Solutions:__________

Number of Complex Solutions: ____________________

e) $3x^2 + 5 = 6x$

Number of Real Solutions:__________

Number of Complex Solutions: ____________________

f) $-7x^2 = 12x - 4$

Number of Real Solutions:__________

Number of Complex Solutions: ____________________

Section 14.3 – Solving Quadratic Equations by Factoring

In this section, we will see how a quadratic equation written in standard form: $ax^2 + bx + c = 0$ can be solved *algebraically* using FACTORING methods.

The Zero Product Principle
If $a \cdot b = 0$, then $a = 0$ or $b = 0$

To solve a quadratic equation by FACTORING:

Step 1: Make sure the quadratic equation is in standard form: $ax^2 + bx + c = 0$

Step 2: Write the left side in completely factored form

Step 3: Apply the ZERO PRODUCT PRINCIPLE
Set each linear factor equal to 0 and solve for x

Step 4: Verify the result by plugging into your ORIGINAL equation.

Problem 1

Solve by factoring: $5x^2 - 10x = 0$

Step 1: This quadratic equation is already in standard form.

Step 2:

Step 3:

Step 4:

Lesson 14 - Solving Quadratic Functions

Problem 2

Solve by factoring: $x^2 - 7x + 12 = 2$

Step 1:

Step 2:

Step 3:

Step 4:

Problem 3

Solve the equations below by factoring. Show all of your work. Verify your result.

a) Solve by factoring: $-2x^2 = 8x$

b) Solve by factoring: $x^2 = 3x + 28$

c) Solve by factoring: $x^2 + 5x = x - 3$

Problem 4	YOU TRY

Use an appropriate factoring method to solve each of the quadratic equations below. Show all of your work. Be sure to write your final solutions using proper notation. Verify your answers.

a) Solve $x^2 + 3x = 10$

b) Solve $3x^2 = 17x$

Section 14.3 Practice Problems

Solve each of the following quadratic equations by factoring. Be sure to write your final solutions using proper notation. Verify your solutions.

a) $4x^2 - 8x = 0$

b) $9x^2 - 6x = 0$

c) $2x^2 = 4x$

d) $x^2 + 8x + 12 = 0$

Lesson 14 - Solving Quadratic Functions

e) $x^2 + 42 = x$

f) $x^2 – 4x = 5$

g) $x^2 - 36 = 0$

h) $9x^2 + 15x = 0$

Lesson 14 - Solving Quadratic Functions

i) $x^2 + 10x - 24 = 0$

j) $2x^2 - 4x - 30 = 0$

Section 14.4 –The Quadratic Formula

The Quadratic Formula can be used to solve quadratic equations written in standard form:

$$ax^2 + bx + c = 0$$

The Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

To solve a quadratic equation using the QUADRATIC FORMULA:

Step 1: Make sure the quadratic equation is in standard form: $ax^2 + bx + c = 0$

Step 2: Identify the coefficients *a, b,* and *c.*

Step 3: Substitute these values into the Quadratic Formula

Step 4: Simplify your result completely.

Step 5: Verify your result by plugging into the original equation.

Do you wonder where this formula came from? Well, you can actually **derive** this formula directly from the quadratic equation in standard form $ax^2 + bx + c = 0$ using a factoring method called COMPLETING THE SQUARE. Instead, we will **verify** the formula

How to Verify the Quadratic Formula

Lesson 14 - Solving Quadratic Functions

Problem 1

Solve the quadratic equation $3x^2 - 2 = -x$ by using the Quadratic Formula. Verify your result.

Step 1:

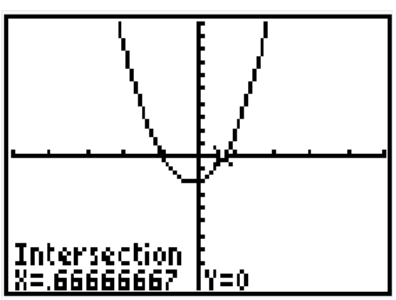
Step 2:

Step 3:

Step 4:

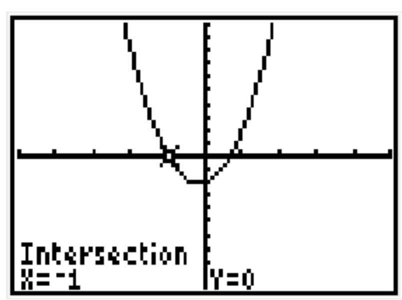
Step 5:

Graphical verification of Solution $x = \frac{2}{3}$



[Note that $\frac{2}{3} \approx .6666667$]

Graphical verification of Solution $x = -1$



You can see that this is an example of the "Case 2" possibility of two, unique real number solutions for a given quadratic equation.

Problem 2

Solve each quadratic equation by using the Quadratic Formula. Verify your result.

$$\text{Quadratic Formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a) Solve $-x^2 + 3x + 10 = 0$

b) Solve $2x^2 - 4x = 3$

Problem 3	**YOU TRY**

Solve $3x^2 = 7x + 20$ using the Quadratic Formula. Show all steps and simplify your answer. Verify your answer by plugging into the original equation.

Section 14.4 Practice Problems

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Solve each quadratic equation by using the Quadratic Formula.

- Place your given quadratic equation in standard form.
- Identify the coefficients *a, b, c*
- Substitute these values into the quadratic formula
- Simplify your result completely then check your solution graphically
- Mark and label the solutions on your graph.

a) Solve $2x^2 - 2x - 4 = 0$ (This one is a fill in the blank)

$a =$ _____, $b =$ ______, $c =$ ______

$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$$

$$x = \frac{(\quad) \pm \sqrt{(\quad) - (\quad)}}{(\quad)}$$

$$x = \frac{(\quad) \pm \sqrt{(\quad)}}{(\quad)}$$

$$x_1 = \frac{(\ )+\sqrt{(\ )}}{(\ )} \text{ and } x_2 = \frac{(\ )-\sqrt{(\ )}}{(\ )}$$

$$x_1 = \frac{(\ )+(\ )}{(\ )} \text{ and } x_2 = \frac{(\ )-(\ )}{(\ )}$$

$$x_1 = \frac{(\ )}{(\ )} \text{ and } x_2 = \frac{(\ )}{(\ )}$$

$$x_1 = 2 \text{ and } x_2 = -1$$

Final solution x = -1, 2

Lesson 14 - Solving Quadratic Functions

b) $2x^2 - 5x = 4$

c) $4x^2 - 2x = 6$

d) $6x^2 - 4x = 1$

e) $-2x^2 = 3x + 12$

Section 14.5 – Complex Numbers

Suppose we are asked to solve the quadratic equation $x^2 = -1$. Well, right away you should think that this looks a little weird. If I take any real number times itself, the result is always positive. Therefore, there is no REAL number x such that $x^2 = -1$. [Note: See explanation of Number Systems on the next page]

Hmmm…well, let's approach this using the Quadratic Formula and see what happens.

To solve $x^2 = -1$, need to write in standard form as $x^2 + 1 = 0$. Thus, $a = 1$ and $b = 0$ and $c = 1$.

Plugging these in to the quadratic formula, I get the following:

$$x = \frac{-0 \pm \sqrt{0^2 - 4(1)(1)}}{2(1)} = \frac{\pm\sqrt{-4}}{2} = \frac{\pm\sqrt{4(-1)}}{2} = \frac{\pm\sqrt{4}\sqrt{-1}}{2} = \frac{\pm 2\sqrt{-1}}{2} = \pm\sqrt{-1}$$

Well, again, the number $\sqrt{-1}$ does not live in the real number system nor does the number $-\sqrt{-1}$ yet these are the two solutions to our equation $x^2 + 1 = 0$.

The way mathematicians have handled this problem is to define a number system that is an extension of the real number system. This system is called the Complex Number System and it has, as its base defining characteristic, that equations such as $x^2 + 1 = 0$ can be solved in this system. To do so, a special definition is used and that is the definition that:

$$i = \sqrt{-1}$$

With this definition, then, the solutions to $x^2 + 1 = 0$ are just $x = i$ and $x = -i$ which is a lot simpler than the notation with negative under the radical.

When Will We See These Kinds of Solutions?

We will see solutions that involve the complex number "i" when we solve quadratic equations that never cross the x-axis. You will see several examples to follow that will help you get a feel for these kinds of problems.

Complex Numbers $a + bi$

Complex numbers are an extension of the real number system.
Standard form for a complex number is

$$a + bi$$

where a and b are real numbers,

$$i = \sqrt{-1}$$

Problem 1	**WORKED EXAMPLE – Complex Numbers**

a) $\sqrt{-9} = \sqrt{9}\sqrt{-1}$
$= 3\sqrt{-1}$
$= 3i$

b) $\sqrt{-7} = \sqrt{7}\sqrt{-1}$
$= \sqrt{7}i$ or $i\sqrt{7}$

c) $\frac{3+\sqrt{-49}}{2} = \frac{3+\sqrt{49}\sqrt{-1}}{2}$
$= \frac{3+7i}{2}$
$= \frac{3}{2} + \frac{7}{2}i$

THE COMPLEX NUMBER SYSTEM

Complex Numbers:
All numbers of the form $a + bi$ where a, b are real numbers and $i = \sqrt{-1}$
Examples: $3 + 4i$, $2 + (-3)i$, $0 + 2i$, $3 + 0i$

Real Numbers – all the numbers on the REAL NUMBER LINE – include all RATIONAL NUMBERS and IRRATIONAL NUMBERS

Rational Numbers :
- ratios of integers
- decimals that terminate or repeat
- Examples:

$0.50 = \frac{1}{2}$, $-.75 = -\frac{3}{4}$ - $\frac{3}{4}$,
$0.43 = \frac{43}{100}$ $\frac{43}{137}$, $0.33 = \frac{33}{100}$

Integers: Zero, Counting Numbers and their negatives
{…–5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, …}

Whole Numbers: Counting Numbers and Zero
{0, 1, 2, 3, 4, 5, 6, 7, …..}

Counting Numbers
{1, 2, 3, 4, 5, 6, 7, …..}

Irrational Numbers

Examples: π, e, $\sqrt{5}$, $\sqrt{47}$ $\sqrt{11}$

- Decimal representations for these numbers never terminate and never repeat

Lesson 14 - Solving Quadratic Functions

Complex numbers are an extension of the real number system. As such, we can perform operations on complex numbers. This includes addition, subtraction, multiplication, and powers.

A complex number is written in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$

Extending this definition a bit, we can define $i^2 = \left(\sqrt{-1}\right)^2 = \sqrt{-1} \cdot \sqrt{-1} = -1$

Problem 2

Preform the indicated operations. Recall that $i^2 = -1$.

a) $(8 - 5i) = (1 + i)$

b) $(3 - 2i) - (4 + 1)$

c) $5i(8 - 3i)$

d) $(2 + i)(4 - 2i)$

e) $(3 - 5i)^2$

Problem 3	**YOU TRY**

Simplify each of the following and write your answers in the form $a + bi$.

a) $\dfrac{15 - \sqrt{-9}}{3}$

b) $(10+4i)(8 - 5i)$

Section 14.5 Practice Problems

Simplify each of the following and write in the form $a + bi$.

a) $\sqrt{-81} =$

b) $\sqrt{-11} =$

c) $(4 - 2i) - (6 + 8i) =$

d) $3i(2 - 4i) =$

e) $(3 - i)(2 + i) =$

f) $(4 - 8i) - 3(4 + 4i) =$

g) $(2 + i)^2 =$

h) $\dfrac{4 - \sqrt{-8}}{6} =$

i) $\dfrac{1 + \sqrt{-36}}{3} =$

j) $\dfrac{2 - \sqrt{4 - 4(2)(5)}}{4} =$

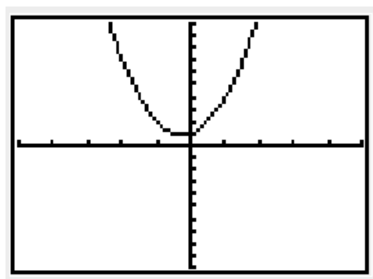
Section 14.6 – Complex Solutions to Quadratic Equations

Work through the following to see how to deal with equations that can only be solved in the complex number system.

Problem 1

Solve $2x^2 + x + 1 = 0$ for x. Leave results in the form of a complex number, $a+bi$.

The graph below shows that the graph of $y = 2x^2 + x + 1$ does not cross the x-axis at all. This is an example of our "Case 3" possibility and will result in no *real* solutions but two unique *complex* solutions.



To find the solutions, make sure the equation is in standard form (check).

Identify the coefficients:.

Insert these into the quadratic formula and simplify:

Break this into two solutions in the a+bi form:

The final solutions are:

Problem 2

Solve $x^2 + 4x + 8 = 1$ for x. Leave results in the form of a complex number, $a+bi$.

Problem 3	**YOU TRY**

Solve $2x^2 - 3x = -5$ for x. Leave results in the form of a complex number, $a+bi$.

Work through the following problem to put the solution methods of graphing, factoring and quadratic formula together while working with the same equation.

Problem 4	YOU TRY

Given the quadratic equation $x^2 + 3x - 7 = 3$, solve using the processes indicated below.

a) Determine the number of solutions by. Sketch the graph, including the vertex, vertical intercept, and any horizontal intercepts.

b) Solve by factoring. Show all steps. Clearly identify your final solutions.

c) Solve using the Quadratic Formula. Clearly identify your final solutions.

Section 14.6 Practice Problems

1. Solve the quadratic equations in the complex number system. Leave your final solution in the complex form, $a \pm bi$.

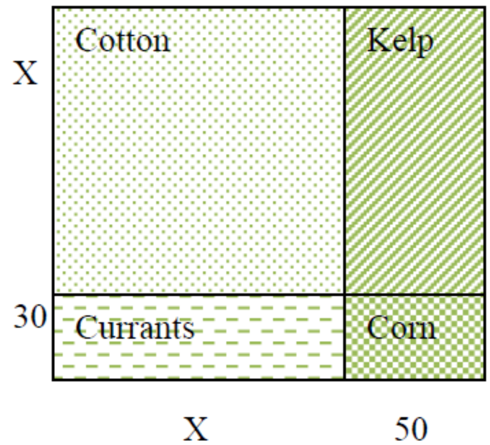
   a) $\frac{1}{2}x^2 + 5x + 17 = 0$

   b) $x^2 + 2x + 5 = 0$

   c) $4x^2 = -9$

   d) $-3x^2 + 4x - 7 = 0$

2. Farmer Treeman wants to plant four crops on his land, Cotton, Corn, Kelp and Currants. He has 40,000 square feet for planting. He needs the length and width of the property to be as shown in the picture below (measured in feet). He determines the equation for the area of his property is $x^2 + 80x + 1500 = 40000$



a) What will the length and width of the property need to be? Show your work.

b) Determine the area of each section of the land. Include units in your answers.

Cotton: ____________________ Kelp:______________________

Currants: ____________________ Corn: ______________________

Section 14.7 – Applications of Quadratic Functions

A large number of quadratic applications involve launching objects into the sky (arrows, baseballs, rockets, etc…) or throwing things off buildings or spanning a distance with an arched shape. While the specifics of each problem are certainly different, the information below will guide you as you decipher the different parts.

HOW TO SOLVE QUADRATIC APPLICATION PROBLEMS

1. Draw an accurate graph of the function using first quadrant values only. Label the x-axis with the input quantity and units. Label the y-axis with the output quantity and units.
2. Identify, plot, and label the vertical intercept.
3. Identify, plot, and label the vertex.
4. Identify, plot, and label the positive horizontal intercept(s) (usually, there is only one horizontal intercept that we care about…if both are needed for some reason, then plot them both and include negative input values in your graph for part 1).
5. Once you have done steps 1 – 4, THEN read the specific questions you are asked to solve.

Questions that involve the vertical intercept $(0, c)$:

- How high was the object at time t = 0? c
- What was the starting height of the object? c

Questions that involve the vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$:

- How high was the object at its highest point? $f\left(-\frac{b}{2a}\right)$
- What was the max height of the object? $f\left(-\frac{b}{2a}\right)$
- How long did it take the object to get to its max height? $-\frac{b}{2a}$
- What is the practical range of this function? $0 \leq f(x) \leq f\left(-\frac{b}{2a}\right)$

Questions that involve (usually) the positive horizontal intercept $(x_2, 0)$:

- When did the object hit the ground? x_2
- What is the practical domain of this function? $0 \leq x \leq x_2$
- How long did it take the object to hit the ground? x_2
- How far was the object from the center? x_2

Lesson 14 - Solving Quadratic Functions

Problem 2

The function $h(t) = -16t^2 + 80t + 130$, where $h(t)$ is height in feet, models the height of an arrow shot into the sky as a function of time (seconds).

Before even LOOKING at the specific questions asked, find the following items and plot/label the graph.

1. Identify the vertical intercept

2. Determine the vertex.

3. Determine the positive horizontal intercept

4. Draw an accurate graph of the function using first quadrant values only. Label the horizontal axis with the input quantity and units. Label the vertical axis with the output quantity and units. *Label the vertex and intercepts.*

QUESTIONS TO ANSWER NOW:

a) After how many seconds does the arrow reach its highest point?
   The input value of the vertex is 2.5. So, the arrow reaches its highest point after 2.5 seconds.

b) How high is the arrow at its highest point?
   The output value of the vertex is 230. So, the arrow is 230 feet above the ground at its highest point.

c) After how many seconds does the arrow hit the ground?
   The horizontal intercept is (6.29,0). The arrow will hit the ground after 6.29 seconds.

d) What is the practical domain of this function?
   Time starts at 0 seconds and goes until the arrow hits the ground. So, practical domain is $0 \le t \le 6.29$ *seconds.*

e) What is the practical range of this function?
   The arrow passes through all height values from 0 (when it hits the ground) to its max height of 230 ft. So, practical range is $0 \le h(t) \le 230$*feet.*

f) What does the vertical intercept represent?
   The vertical intercept represents the height of the arrow at time t = 0. Thus, the arrow starts at 130 feet off the ground.

Problem 3

A train tunnel is modeled by the quadratic function $h(x) = -0.35x^2 + 25$, where x is the distance, in feet, from the center of the tracks and $h(x)$ is the height of the tunnel, also in feet. Assume that the high point of the tunnel is directly in line with the center of the train tracks.

a) Draw a complete diagram of this situation. Find and label each of the following: vertex, horizontal intercept (positive side), and vertical intercept.

b) How wide is the base of the tunnel?

c) A train with a flatbed car 6 feet off the ground is carrying a large object that is 15 feet high. How much room will there be between the top of the object and the top of the tunnel?

Problem 4	YOU TRY

A toy rocket is shot straight up into the air. The function $H(t) = -16t^2 + 128t + 3$ gives the height (in feet) of a rocket after *t* seconds. Round answers to two decimal places as needed. All answers must include appropriate units of measure.

a) Draw a complete diagram of this situation. Find and **label** each of the following: vertex, horizontal intercept (positive side), and vertical intercept.

b) How long does it take for the rocket to reach its maximum height? Write your answer in a complete sentence.

c) What is the maximum height of the rocket? Write your answer in a complete sentence.

d) How long does it take for the rocket to hit the ground? Write your answer in a complete sentence.

e) Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.

f) How high is the rocket after 1 second?

g) At what time(s) is the rocket 150 feet off the ground?

Section 14.7 Practice Problems

1. The function $h(t) = -0.2t^2 + 1.3t + 15$, where *h(t)* is height in feet, models the height of an "angry bird" shot into the sky as a function of time (seconds).

   a) Draw a complete diagram of this situation. Find and label each of the following: vertex, horizontal intercept (positive side), and vertical intercept.

   b) How high above the ground was the bird when it was launched?

   c) After how many seconds does the bird reach its highest point?

   d) How high is the angry bird at its highest point?

   e) After how many seconds does the angry bird hit the ground?

   f) How far has the angry bird traveled after 6 seconds?

   g) After how many seconds if the angry bird 10 feet off the ground?

2. A company's revenue earned from selling x items is given by the function $R(x) = 680x$, and cost is given by $C(x) = 10000 + 2x^2$.

   a) Write a function, $P(x)$, that represents the company's *profit* from selling x items.

   b) Identify the vertical intercept of $P(x)$. Write it as an ordered pair and interpret its meaning in a complete sentence.

   c) How many items must be sold in order to maximize the profit?

   d) What is the maximum profit?

   e) How many items does this company need to sell in order to break even?

   f) Determine the practical domain and practical range of this function.

3. An arrow is shot straight up into the air. The function $H(t) = -16t^2 + 90t + 6$ gives the height (in feet) of an arrow after t seconds. Round answers to two decimal places as needed. All answers must include appropriate units of measure.

   a) How long does it take for the arrow to reach its maximum height? Write your answer in a complete sentence.

   b) Determine the maximum height of the arrow. Write your answer in a complete sentence.

   c) How long does it take for the arrow to hit the ground? Write your answer in a complete sentence.

   d) Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.

e) At what time(s) is the arrow 100 feet off the ground?

f) How high is the arrow after 5 seconds?

g) Determine $H(3)$. Write a sentence explaining the meaning of your answer in the context of the arrow.

h) Solve the equation $H(t) = 80$. Write a sentence explaining the meaning of your answer in the context of the arrow

Lesson 14 Review

1. Fill out the following table. Intercepts must be written as ordered pairs. Always use proper notation. Round to two decimal places.

	$f(x) = 2x^2 - 4x - 30$	$g(x) = 5 - x^2$	$y = 5x^2 - 4x + 17$
Opens Upward or Downward?			
Vertical Intercept			
Vertex			
Domain			
Range			
Axis of Symmetry (Equation)			

Lesson 14 - Solving Quadratic Equations

2. Simplify each of the following and write in the form $a + bi$.

a) $\sqrt{-9} =$

b) $\dfrac{8 - \sqrt{-49}}{8} =$

3. By definition, $i = \sqrt{-1}$ and $i^2 =$ ________

4. Solve the following equations *algebraically* (Factoring or Quadratic Formula). You must show all algebraic steps for full credit. Write complex solutions in the form $x = a + bi$ and $x = a - bi$. Check your answers.

a) $3x^2 + 2x + 3 = 8$

b) $x^2 + 9x + 11 = x - 5$

c) $x^2 + 3x + 7 = 2$

5. The function $H(t) = -16t^2 + 88t$ gives the height (in feet) of golf ball after t seconds. Round answers to two decimal places as needed. All answers must include appropriate units of measure.

   a) Determine the maximum height of the golf ball. Show your work. Write your answer in a complete sentence.

   b) How long does it take for the ball to hit the ground? Show your work. Write your answer in a complete sentence.

   c) Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.

   d) Determine the practical domain of $H(t)$. Use proper notation and include units.

   e) Determine the practical range of $H(t)$. Use proper notation and include units.

Chapter G – Factoring Quadratic Expressions and Solving Quadratic Equations

We saw in Section 8.1 that the x-intercepts of a quadratic function $f(x) = ax^2 + bx + c$ are the solution(s) to the equation $f(x) = 0$. That is the goal of this supplemental chapter.

G.1: Review of solving functions

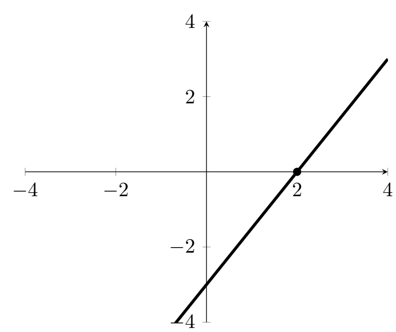
When we worked with **linear** functions, i.e. $f(x) = mx + b$, there were times when we were interested in determining what values of x would result in $f(x) = 0$. We said that this value of x was the **solution** to the equation $mx + b = 0$. We found the solutions to these linear equations by **isolating** the variable x. We also recognized that this is the x-intercept of the equation $y = mx + b$ since it is the x value when get when $y = 0$.

Example 1 : Given $f(x) = \frac{3}{2}x - 3$, find the solution to $f(x) = 0$

We can solve this algebraically:

$$\frac{3}{2}x - 3 = 0$$
$$\frac{3}{2}x = 3$$
$$3x = 6$$
$$x = 2$$

Or we can solve this by graphing the line $y = \frac{3}{2}x - 3$ *and seeing the* x*=intercept is* $(2, 0)$*, so when* $y = 0$*, the solution is* $x = 2$*.*



Before we get to quadratic equations, let's look at one more concrete example of **solving** a linear equation algebraically.

You Try It: Given $f(x) = 2x + 6$, find the solution to $f(x) = 0$.

We now turn our attention to **quadratic** functions $f(x) = ax^2 + bx + c$ and determine what values of x will yield $f(x) = 0$. Unfortunately, we cannot use the same ideas as before with linear equations to find solutions to $ax^2 + bx + c = 0$. Let's see why.

Example 2 : Given $f(x) = x^2 - 3x + 2$, find the solutions to $f(x) = 0$.

Attempted Solution: Set $x^2 - 3x + 2 = 0$ *and try to solve for* x.

$$x^2 - 3x + 2 = 0$$
$$x^2 - 3x = -2$$

At this point, we are not sure how to isolate the variable x by itself. The fact that there is both an x^2 and $-3x$ term, which are **not** like terms, means that we cannot simply combine the two terms and end up with a single x or x^2 term in our equation. This means that we are stuck and unable find any solutions to this equation using this method. However, $f(x) = 0$ does, indeed, have two solutions, -2 and -1. Use the space below to **verify** these two solutions.

Example 3 : Given $g(x) = x^2 + 7x$, find the solutions to $g(x) = 0$.

Attempted Solution: Set $x^2 + 7x = 0$ and try to solve for x.

$$x^2 + 7x = 0$$
$$x^2 = -7x$$
$$x = -7$$

Check the work above by verifying that $x = -7$ is a solution to $g(x) = 0$

At this point it appears we are finished, but is $x = -7$ the only solution to $g(x) = 0$? No! Use the space below to show that $x = 0$ is also a solution to the equation $g(x) = 0$.

Important: When we are solving quadratic equations, we cannot divide both sides by x or else we will lose solutions(s)!

Think About It: Examples 2 and 3 each had two solutions. Do you think all quadratic equations have 2 solutions? Why or why not?

It may not seem very clear how we knew that -2 and -1 were the solutions to Example 2, but soon you will learn how to find these solutions by using a technique known as **factoring**. Before we discuss the details of factoring, you will work through a few more exercises where you will **verify** that a given set of numbers are solutions to a particular quadratic equation.

You Try It: Given $f(x) = x^2 - 6x$, verify that $x = 0$ and $x = 6$ are solutions to $f(x) = 0$.

Given $h(x) = 6x^2 + 24x + 18$, verify that $x = -3$ and $x = -1$ are solutions to $h(x) = 0$.

G.2: Prime Factorization and GCF

Before we show you how to factor polynomials, let's first recall how we can find the **prime factorization** of a given number.

Example 4 *:* Find the prime factorization of 36.

You Try It: Find the prime factorizations of the following.

14 : __________

40 : __________

We can use this same prime factorization idea if we include variables with our numbers.

Example 5 *:* Find the prime factorization of $4x^2$.

You Try It: Find the prime factorizations of the following.

$6x$: __________

$17x$: __________

$18x^2$: __________

$75x^2$: __________

Next, we want to find the **greatest common factor**, abbreviated GCF, for two polynomial terms. Later we will use the greatest common factor to factor quadratic expressions and equations.

Example 6 : Compute the GCF for $4x^2$ and $12x$.

Example 7 : Compute the GCF for $6x^2$ and $24x$.

You Try It: Compute the GCF for the following.

x^2 and $6x$

$8x^2$ and $12x$

G.3: Factoring Quadratic Expressions of the Form $ax^2 + bx$ Using GCF

We are now in a position to learn how to factor quadratic equations of the form $ax^2 + bx$.

Example 8 : Factor $4x^2 + 12x$.

Example 9 : Factor $6x^2 - 24x$.

Example 10 : Factor $x^2 - 6x$.

You Try It: Factor the following quadratic expressions.

$4x^2 + 16x$: ____________

$3x^2 - 18x$: ____________

Now, the factored forms we found above will tell us what the solutions are to equations of the form $ax^2 + bx = 0$ by using the **Zero Product Principle**.

Zero Product Principle

If $a \cdot b = 0$, then $a = 0$ or $b = 0$

Using this, we are finally able to see how factoring allows us to find the solutions to certain quadratic equations.

Example 11 *:* Given $g(x) = 4x^2 + 12x$, find the solutions to $g(x) = 0$.

Example 12 *:* Given $f(x) = x^2 - 6x$, find the solutions to $f(x) = 0$.

You Try It: Find the solutions to the following quadratic equations.

$4x^2 + 16x = 0$: ____________

$x^2 + x = 0$: ____________

$5x^2 + 20x = -5x$: ____________

G.4: Factoring Quadratic Expressions of the form $x^2 + bx + c$

We know how to factor quadratic expressions of the form $ax^2 + bx$ in order to solve equations of the form $ax^2 + bx = 0$. Now, we will look at how to solve general quadratic equations of the form $ax^2 + bx + c = 0$. Once we add the constant c term to the equation it makes our job of finding the solutions to $ax^2 + bx + c = 0$ a little bit harder. Let's start with the case where $a = 1$.

Example 13 *:* Given $f(x) = x^2 - 3x + 2$, find the solutions to $f(x) = 0$.

We set $x^2 - 3x + 2 = 0$ *and try our previous factoring techniques. The GCF of* x^2, $3x$, *and* 2 *is* 1 *and means we cannot factor an* x *term out of the equation. So, we are again stuck and unable to find the solutions to* $x^2 - 3x + 2 = 0$ *using the factoring techniques we presented above. However, we want to use the factoring idea from before, but in a slightly different way.*

First, **verify** that $x^2 - 3x + 2 = (x - 1)(x - 2)$

Now use the **Zero Product Principle** to solve $(x - 1)(x - 2) = 0$

You Try It: Given $f(x) = x^2 + 4x + 3$, first verify $x^2 + 4x + 3 = (x + 1)(x + 3)$, the use this to solve $f(x) = 0$.

You Try It: Given $h(x) = x^2 - 4$, first verify $x^2 - 4 = (x-2)(x+2)$, the use this to solve $h(x) = 0$.

In each of the previous exercises, you were able to see what the factored form was for each of the quadratic functions. Our next step is to show you how we can find the factored form for a given quadratic equation $x^2 + bx + c = 0$. (Notice that a was equal to 1 in each of the above examples. We will eventually deal with the case when $a \neq 1$.) There is a little bit of trial and error that goes into finding the factored form of a quadratic equation. We revisit the examples we started with to show how we can obtain the factored form of the given quadratic functions using a method called **factor by grouping**.

Example 14 *:* Factor the quadratic function $f(x) = x^2 - 3x + 2$.

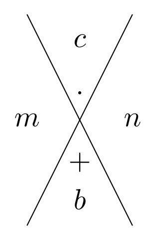
$x^2 - 3x + 2$	*Write the expression as $x^2 + bx + c$ if it isn't already given in that form.*
$x^2 - 2x - x + 2$	*Split up the bx term.*
$(x^2 - 2x) + (-x + 2)$	***Group** the first two terms and the last two terms. Make sure there is a + between the groups*
$x(x-2) - 1(x-2)$	*Find the GCF of each group (±). What's left in the parentheses should match!*
$(x-2)(x-1)$	*Factor out the matching groups to make a **product***

Now FOIL $(x-2)(x-1)$ to make sure we factored correctly.

Example 15 *:* Given $f(x) = x^2 + 4x + 3$, find the solutions to $f(x) = 0$ using factoring by grouping. The first step is done for you.

$$x^2 + 4x + 3 = x^2 + 3x + x + 3$$

But how do we know how to split up the bx? If you look at the last two examples, you'll see that we need coefficients m and n that add up to b and multiply to give c. We can visualize this using the following diagram.



Because of this diagram, we call this type of factoring the **Big X Method**. When filling in the "Big X", we know b and c from the equation $f(x) = x^2 + bx + c$, and it's your job to find m and n using a bit of guessing and checking. There are a couple hints that will make finding m and n a little easier.

Hint 1:

- If $c > 0$, then m and n are either both positive or both negative. If $b > 0$ they are positive, and if $b < 0$ they are negative.
- If $c < 0$, then one number is positive and one number is negative.
- What should we do if $c = 0$?

Hint 2: If you get stuck, list all the pairs of whole numbers whose product is $|c|$.

Example 16 *:*

Given $g(x) = x^2 - x - 12$, find the solutions to $g(x) = 0$ using factoring.

Example 17 *:* Given $h(x) = x^2 - 4$, find the solutions to $h(x) = 0$ using factoring.

Solution 1:

Solution 2:

Remark: When $b = 0$ we always have the choice to use either the **Big X Method** or the **Square Root Property**.

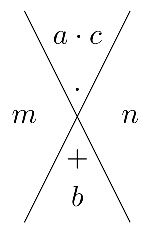
You Try It: Given $f(x) = x^2 - 5x + 4$, find the solutions to $f(x) = 0$ using factoring.

You Try It: Given $g(x) = x^2 + 8x + 15$, find the solutions to $g(x) = 0$ using factoring.

You Try It: Find the solutions to $x^2 - 16 = 0$.

G.5: Factoring Quadratic Expressions of the Form ax^2+bx+c

In the previous section, we learned how to factor equations of the form $x^2 + bx + c$ in order to help us find solutions to equations of the form $x^2 + bx + c = 0$. We are now in a position to learn how to factor the most general form of quadratic functions, $f(x) = ax^2 + bx + c$ when a is not necessarily equal to one. We use the same **Big X Method** with **grouping**, but now our diagram looks like this.



Example 18 *:* Given $f(x) = 2x^2 - 3x - 2$, find the solutions to $f(x) = 0$ using factoring.

Notice that this is actually *exactly the same method* as we used last section since $a \cdot c = c$ in the case when $a = 1$. Let's do one more example together.

Example 19 : Given $g(x) = 6x^2 + 8x - 8$, find the solutions to $g(x) = 0$.

You Try It: Given $h(x) = 3x^2 + 2x - 1$, find the solutions to $h(x) = 0$.

You Try It: Given $f(x) = 9x^2 - 3x - 6$, find the solutions to $f(x) = 0$

Practice Problems: Part 1

(i) **Verify** that $x = -4$ and $x = 1$ are solutions to the equation $x^2 + 3x - 4 = 0$.

(ii) **Verify** that $x = \frac{1}{2}$ and $x = -2$ are solutions to the equation $2x^2 + 3x - 2 = 0$.

(iii) Find the prime factorization of each of the following.

(a) $50x^2$

(b) $48x$

(iv) Find the greatest common factor of the following.

(a) x^2 and $2x$

(b) $3x^2$ and $9x$

(c) $4x^2$ and $10x$

(v) Factor the following quadratic expressions

(a) $5x^2 + x$

(b) $18x^2 - 9x$

(c) $12x^2 - 36x$

(vi) Find the solutions to the following quadratic equations.

(a) $3x^2 - 18x = 0$

(b) $-13x + 7x^2 = 0$

(c) $x^2 + x = 0$

(vii) Find the solutions to each of the following

(a) Given $f(x) = x^2 + 2x - 24$, find the solutions to $f(x) = 0$.

(b) Given $h(x) = x^2 - 3x - 15$, find the solutions to $h(x) = 3$.

(c) Given $g(x) = x^2 - 9$, find the solutions to $g(x) = 0$.

(viii) Find the solutions to each of the following

(a) Given $f(x) = 15x + 9x^2$, find the solutions to $f(x) = 0$.

(b) Find the solutions to $9x^2 - 6x - 8 = 0$.

(c) Find the solutions to $-2x^2 + 10x = 0$.

(d) Given $g(x) = 2x^2 - 9x + 10$, find the solutions to $g(x) = 0$.

(e) Given $h(x) = 4x^2 + 10x - 5$, find the solutions to $h(x) = 1$.

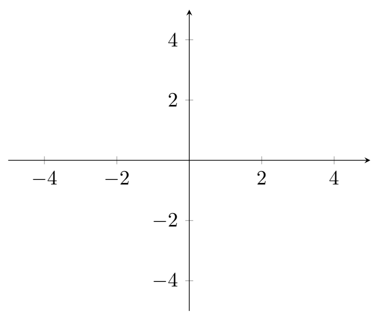
G.6: Vertex Form of a Quadratic Equation

Standard form $f(x) = ax^2 + bx + c$ is not the only way to write a quadratic equation. We could instead write a quadratic equation as $f(x) = a(x-h)^2 + k$. Before we go any further, let's take a moment to multiply this out and make sure that it is, indeed, a quadratic equation.

Quadratic functions written in the form $f(x) = a(x-h)^2 + k$ are said to be written in **vertx form** because the point (h, k) is the **vertex** of the parabola graphed by $y = f(x)$. When a quadratic function is written in vertex form, it is very easy to graph.

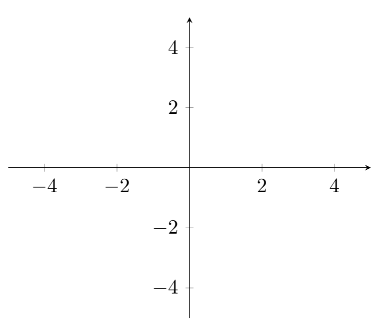
- We know the vertex is at (h, k).
- Since the a in $f(x) = a(x-h)^2 + k$ is the same as the a in $f(x) = ax^2 + bx + c$, we know the parabola opens up if $a > 0$ and down if $a < 0$.
- Since the axis of symmetry is always the vertical line through the vertex, we know the axis of symmetry is $x = h$.
- By plotting the vertex and one other point, we can use the axis of symmetry to plot a third point, and then just connect the dots!

Example 20 For the quadratic function $f(x) = (x+2)^2 - 4$, sketch a graph and label the vertex, axis of symmetry, y-intercept and at least one additional point.

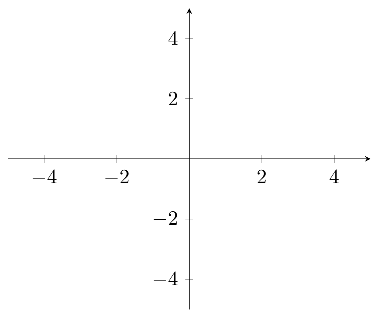


Important: Note that there is a **minus** sign in front of the h in the formula $f(x) = a(x-h)^2 + k$. So in Example 20 above, we can think of the function as $f(x) = (x-(-2))^2 - 4$ to see that the vertex (h,k) is $(-2,-4)$, not $(2,-4)$.

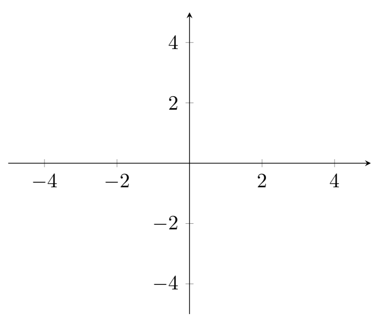
Example 21 For the quadratic function $f(x) = -2(x-1)^2 - 2$, sketch a graph and label the vertex, axis of symmetry, y-intercept and at least one additional point.



You Try It: For the quadratic function $f(x) = (x-1)^2 - 3$, sketch a graph and label the vertex, axis of symmetry, y-intercept and at least one additional point.

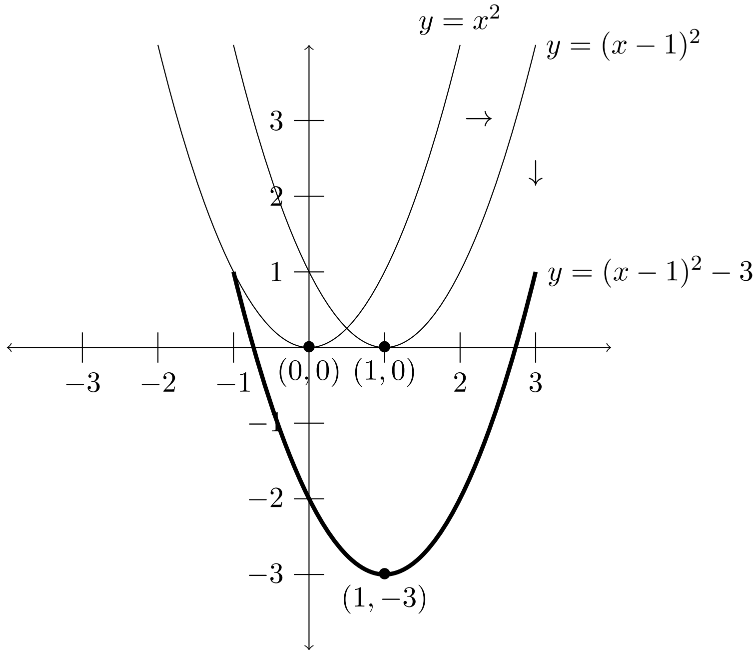


You Try It: For the quadratic function $f(x) = -(x-1)^2$, sketch a graph and label the vertex, axis of symmetry, y-intercept and at least one additional point.



Now that we've had some practice at graphing quadratic equations in vertex form $f(x) = a(x-h)^2 + k$, you may be wondering *why* these parabolas have the vertex (h, k). We know that the graph of $y = x^2$ is a parabola that opens upward with a vertex at $(0, 0)$. To see what happens when we start adding, subtracting and multiplying, let's use the concrete example $y = (x-1)^2 - 3$. The table and graph on the next page show the steps we can take to transform $y = x^2$ to $y = (x-1)^2 - 3$.

x	$y = x^2$	$y = (x-1)^2$	$y = (x-1)^2 - 3$
3	9	4	1
2	4	1	-2
1	1	0	-3
0	0	1	-2
-1	1	4	1
-2	4	9	6



As you can see, the $h = 1$ *shifts* the graph to the right by 1, and the $k = -3$ *shifts* the graph down by 3, moving the vertex from $(0, 0)$ to $(1, -3)$. These moves are called graph *translations* and you can learn more about them by taking a class in Pre-Calculus.

G.7: Converting Between Standard Form and Vertex Form

At the beginning of the last section, we saw that to change a quadratic function in *vertex form* $f(x) = a(x-h)^2 + k$ to *standard form* $f(x) = ax^2 + bx + c$, all we need to do is multiply it out and combine like terms. Converting from vertex form to standard for can be very useful if, for example, we wanted to use the quadratic formula to find the zeros the of the function.

You Try It: Convert the following quadratic equations into standard form $f(x) = ax^2 + bx + c$, then use the quadratic formula to find the zeros of the functions, if they exist.

- $f(x) = 2(x+1)^2 - 2$

- $g(x) = -3(x-2)^2 - 1$

Converting a quadratic function from standard form $f(x) = ax^2 + bx + c$ to vertex form $f(x) = a(x-h)^2 + k$ can also be very useful. Why might we want to do this?

Unfortunately, converting from standard form to vertex form is a bit trickier than converting from vertex form to standard form. The steps of the process are outlined below.

$$
\begin{aligned}
f(x) =& \; ax^2 + bx + c & \\
=& \; a\left(x^2 + \tfrac{b}{a}x\right) + c & \text{Factor out the } a \text{ from the } x \text{ terms only} \\
=& \; a\left(x^2 + \tfrac{b}{a}x + \tfrac{b^2}{4a^2} - \tfrac{b^2}{4a^2}\right) + c & \text{We can add and subtract } \tfrac{b^2}{4a^2} \text{ since this does} \\
& & \text{not change the equation} \\
=& \; a\left(\left(x + \tfrac{b}{2a}\right)^2 - \tfrac{b^2}{4a^2}\right) + c & \text{You can check by FOILing that} \\
& & x^2 + \tfrac{b}{a}x + \tfrac{b^2}{4a^2} = \left(x + \tfrac{b}{2a}\right)^2 \\
=& \; a\left(x + \tfrac{b}{2a}\right)^2 - \tfrac{b^2}{4a} + c & \text{Distribute the } a \\
=& \; a\left(x + \tfrac{b}{2a}\right)^2 + \left(-\tfrac{b^2}{4a} + c\right) & \text{Group the constant terms.}
\end{aligned}
$$

These calculations show that when converting from standard form $f(x) = ax^2 + bx + c$ to vertex form $f(x) = a(x-h)^2 + k$, $h = -\dfrac{b}{2a}$ and $k = -\dfrac{b^2}{4a} + c$. So the vertex of the function $f(x) = ax^2 + bx + c$ is $\left(-\dfrac{b}{2a}, -\dfrac{b^2}{4a} + c\right)$. Since the formula for k is very hard to remember, and the calculations above take a long time if we do them every time we want to convert from standard form to vertex form, it's best to remember the following.

Converting to Vertex Form

The graph of $f(x) = ax^2 + bx + c$ has the vertex

$$(h, k) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right),$$

which you can use to write $f(x)$ in the vertex form $f(x) = a(x-h)^2 + k$.

Example 22 Use the formulas $h = -\frac{b}{2a}$ and $k = f(-\frac{b}{2a})$ to convert the function $f(x) = 2x^2 + 10x - 1$ into vertex form $f(x) = a(x-h)^2 + k$.

Example 23 Answer the following for the quadratic function $f(x) = 3x^2 + 12x + 8$.

(i) Use the formulas $h = -\frac{b}{2a}$ and $k = f(-\frac{b}{2a})$ to write $f(x)$ in vertex form $f(x) = a(x-h)^2 + k$.

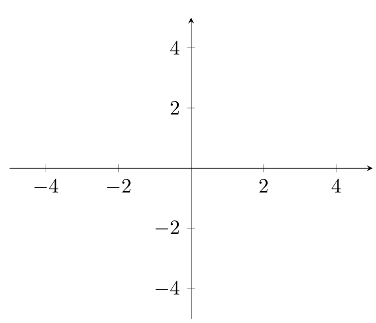
(ii) Does the graph of $f(x)$ open upward or downward?

(iii) What is the vertex of $f(x)$? Is this a maximum or a minimum?

(iv) Does $f(x)$ have any x-intercepts? If so, find them and round your answers to 1 decimal place.

(v) What is the y-intercept of $f(x)$?

(vi) Sketch a graph of $f(x)$. Label the vertex, axis of symmetry and x-intercepts.



You Try It: Answer the following for the quadratic function $f(x) = -2x^2 - 4x + 1$.

(i) Use the formulas $h = -\frac{b}{2a}$ and $k = f(-\frac{b}{2a})$ to write $f(x)$ in vertex form $f(x) = a(x-h)^2 + k$.

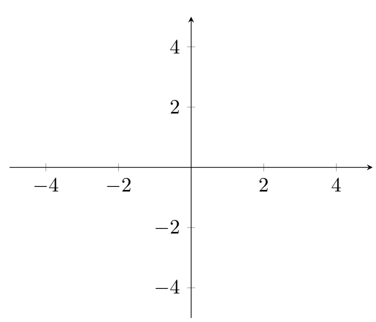
(ii) Does the graph of $f(x)$ open upward or downward?

(iii) What is the vertex of $f(x)$? Is this a maximum or a minimum?

(iv) Does $f(x)$ have any x-intercepts? If so, find them and round your answers to 1 decimal place.

(v) What is the y-intercept of $f(x)$?

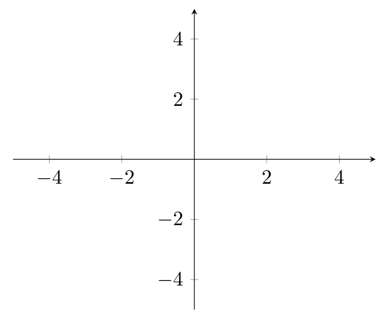
(vi) Sketch a graph of $f(x)$. Label the vertex, axis of symmetry, x-intercepts and y-intercept.



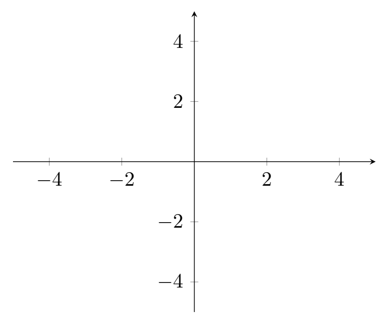
Practice Problems: Part 2

(i) For the quadratic functions below, sketch a graph and label the vertex, axis of symmetry, y-intercept and at least one additional point.

(a) $f(x) = 2(x+3)^2 - 4$



(b) $f(x) = -x^2 + 2$



(ii) Convert the following quadratic equations into standard form $f(x) = ax^2 + bx + c$, then use the quadratic formula to find the zeros of the functions, if they exist.

(a) $-(x+3)^2 + 4$

(b) $4(x-1)^2 - 1$

(iii) Answer the following for the quadratic function $f(x) = -x^2 + 2x + 4$.

(a) Use the formulas $h = -\frac{b}{2a}$ and $k = f(-\frac{b}{2a})$ to write $f(x)$ in vertex form $f(x) = a(x-h)^2 + k$.

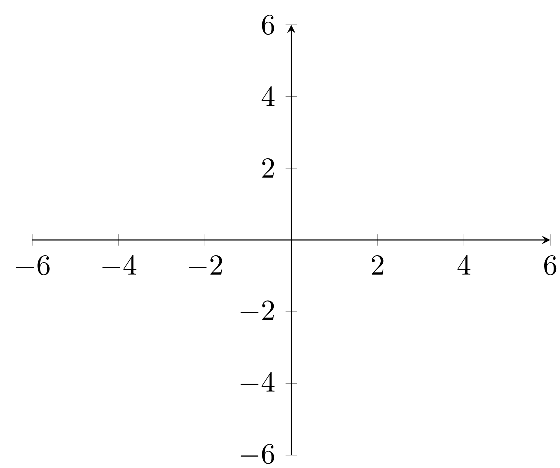
(b) Does the graph of $f(x)$ open upward or downward?

(c) What is the vertex of $f(x)$? Is this a maximum or a minimum?

(d) Does $f(x)$ have any x-intercepts? If so, find them and round your answers to 1 decimal place.

(e) What is the y-intercept of $f(x)$?

(f) Sketch a graph of $f(x)$. Label the vertex, axis of symmetry, x-intercepts and y-intercept.



(iv) Answer the following for the quadratic function $f(x) = 2x^2 + 8x + 5$.

(a) Use the formulas $h = -\frac{b}{2a}$ and $k = f(-\frac{b}{2a})$ to write $f(x)$ in vertex form $f(x) = a(x - h)^2 + k$.

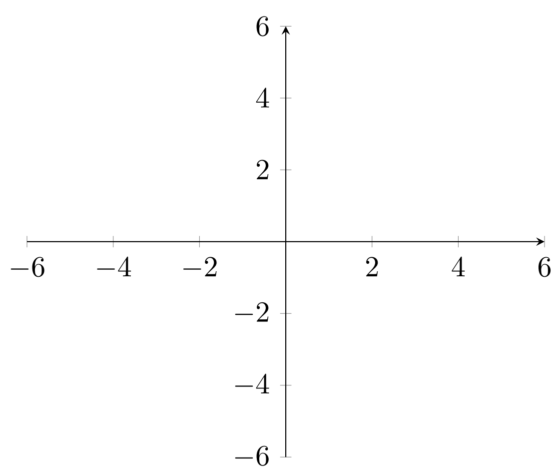
(b) Does the graph of $f(x)$ open upward or downward?

(c) What is the vertex of $f(x)$? Is this a maximum or a minimum?

(d) Does $f(x)$ have any x-intercepts? If so, find them and round your answers to 1 decimal place.

(e) What is the y-intercept of $f(x)$?

(f) Sketch a graph of $f(x)$. Label the vertex, axis of symmetry, x-intercepts and y-intercept.



Supplemental Worksheets for MPS

Cheryl Jaeger Balm

Fall 2016

Inequalities

Practice Worksheet

1. For each of the following pairs of numbers, draw a number line diagram comparing them and then determine if $a < b$ or $a > b$.

   (a) $a = 5$ and $b = -1$

   (b) $a = -10$ and $b = -7$

2. Complete the table.

Inequaltiy	**Set-builder**	**Interval**	**Number line**
$x > 3$	$\{x \| x > 3\}$	$(3, \infty)$	
$x \leq 2$			
		$(-2, 5]$	
$-2 < x \leq 0$			
	$\{x \| x \leq 4\}$		
		$[4, \infty)$	

Solving Linear Inequalities

Practice Worksheet

1. Solve each inequality and graph the solution on a number line.

   (a) $5x + 43 > 98$

   (b) $7x + 24 < 3x - 15$

   (c) $-2(x - 3) > 3x - 24$

2. A car can be rented from Company A for \$150 plus 10 cents per mile. Company B charges \$80 per week plus 30 cents per mile to rent the same car. How many miles must be driven in a week to make the rental cost for Company A a better deal than Company B?

   (a) Write expressions representing the cost of renting a car form each company.

   (b) Using your expressions from part (a), write an inequality which represent Company A being a better deal than Company B.

   (c) Solve your inequality from part (b).

Relations and Functions

Practice Worksheet

1. Determine whether each of the following relations is a function. Justify your answers.

   (a) $\{(1, 2),\ (12, 2),\ (2, 5),\ (7, 9),\ (9, 9)\}$

   (b) $\{(7, \text{dog}),\ (5, \text{bird}),\ (2, \text{turtle}),\ (0, \text{tree})\}$

   (c) $\{(5, 5),\ (6, 0),\ (0, 6),\ (7, 9),\ (9, 9),\ (0, 5)\}$

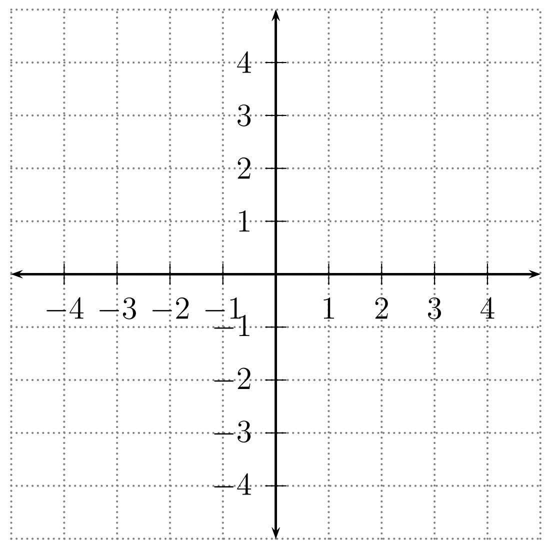
   (d) $\{(\text{CA}, \text{Cupertino}),\ (\text{CA}, \text{Sunnyvale}),\ (\text{US}, \text{CA})\}$

   (e) $\{(1, 1)\}$

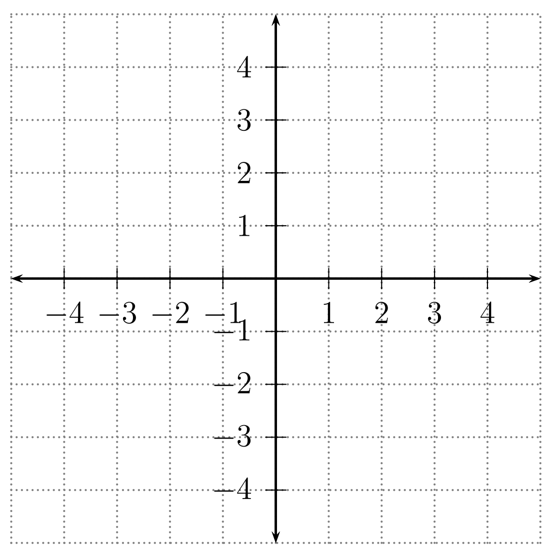
   (f) $\{(-2, 4),\ (-1, 1),\ (0, 0),\ (1, 1),\ (2, 4)\}$

2. Decide if the following relations are functions by plotting the points and determining if they satisfy the Vertical Line Test.

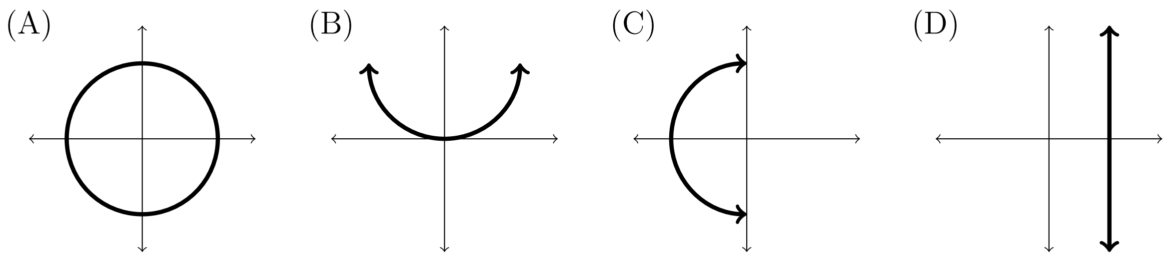
   (a) $\{(2,1),\ (2,5),\ (1,3)\}$



   (b) $\{(2,4),\ (3,4),\ (-1,4)\}$



3. Determine whether each graph below is the graph of a function. Explain why or why not using input and output terminology.



- Graph (A)

- Graph (B)

- Graph (C)

- Graph (D)

Function Notation

Practice Worksheet

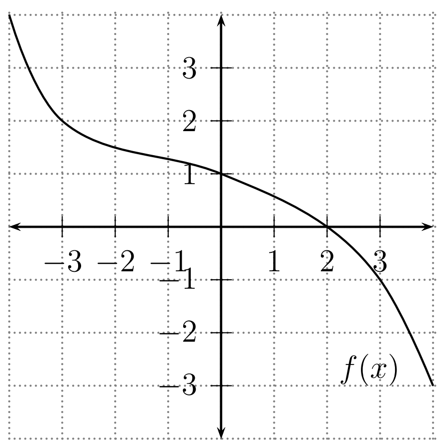
1. Use the table below to find the following values.

x	$g(x)$
0	1
1	2
2	4
3	3
4	0

(a) Find $g(2)$

(b) Find x when $g(x) = 2$

2. Use the graph below to find the following values.



(a) Find $f(0)$

(b) Find $f(-3)$

(c) Find x when $f(x) = 0$

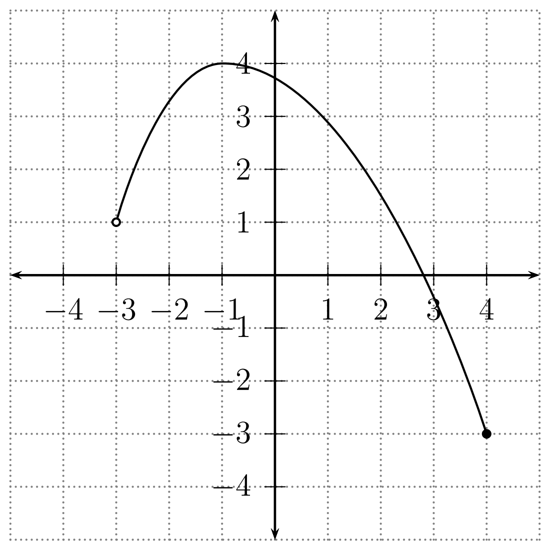
(d) Find x when $f(x) = -1$

Domain and Range

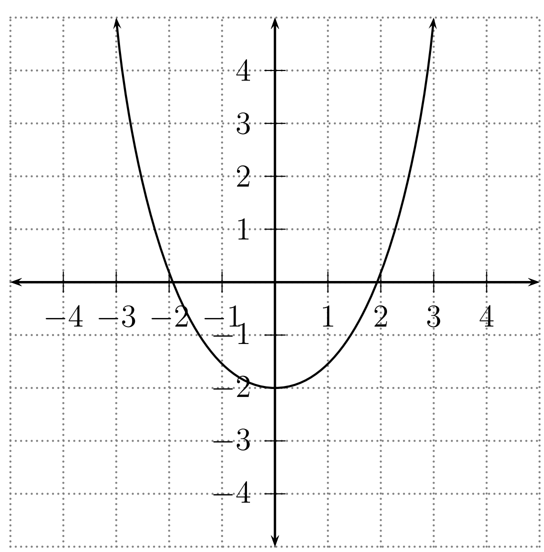
Practice Worksheet

Determine the domain and range of the following graphs of functions.

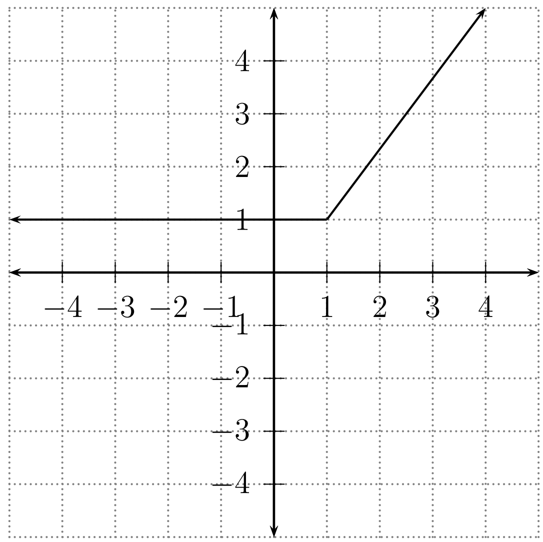
1.
   - Domain:
   - Range:



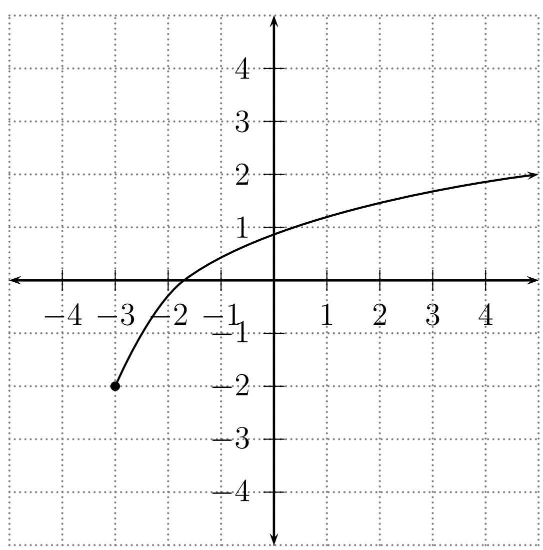
2.
   - Domain:
   - Range:



3. • Domain:
   • Range:



4. • Domain:
   • Range:



Formulas in Function Notation

Practice Worksheet

1. Evaluate each function at the given value.

   (a) Find $f(1)$ for $f(x) = (5x-3)^2+7$

   (b) Find $f(2)$ for $f(x) = 3x^3+(3x)^3$

   (c) Find $f(0)$ for $f(x) = x^2-5x-9$

2. Find each of the following for $f(x) = 3x^2-7$

   (a) $f(-3)$

   (b) $f(0)$

3. Find each of the following for $g(x) = \dfrac{2x-5}{3x+6}$

   (a) $g(2)$

   (b) $g(-2)$

4. Find each of the following for $f(x) = 2 - 2x^2$

   (a) $f(-1)$

   (b) $f(2)$

   (c) $f(-2)$

5. Find the domain of each of the following functions.

   (a) $f(x) = 100x$

   (b) $g(x) = \dfrac{1}{x}$

   (c) $h(x) = \dfrac{2x - 7}{x + 4}$

   (d) $f(x) = \dfrac{3x}{x - 1} + \dfrac{2 - x}{x + 10}$

6. The function $P(x) = 2x - 50$ gives the profit, in dollars, for a roadside lemonade stand after they have sold x cups of lemonade.

   (a) Find $P(48)$ and write a complete sentence explaining your answer.

   (b) How many cups of lemonade must be sold for the stand to break even. In other words, when is $P(x) = 0$? Answer in a complete sentence.

7. The function $d(t) = 4.9t^2$ gives the distance, in meters, that an object has traveled t seconds after it had been dropped form a tall building.

   (a) Find $d(4)$ and write a complete sentence explaining your answer.

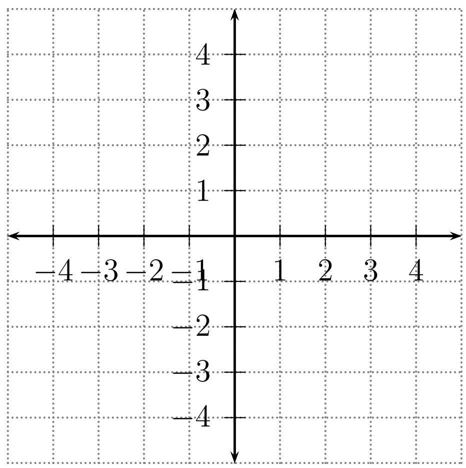
   (b) Approximaetely when does $d(t) = 100$? Write a complete sentence explaining your answer.

Horizontal and Vertical Lines

Practice Worksheet

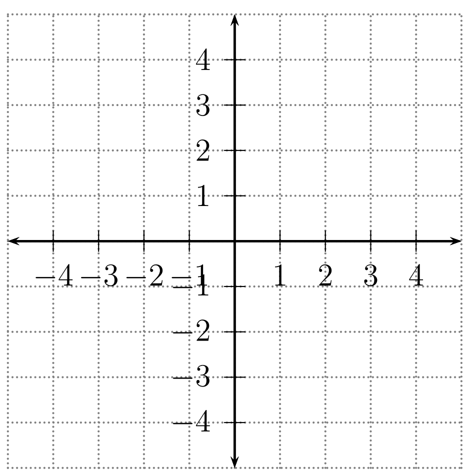
1. Graph each of the following lines on the same axes and label each one with its equation. Then decide whether each is horizontal or vertical.

   - $x = 2$
   - $y = -3$



2. Graph each of the following lines on the same axes and label each one with its equation. Then decide whether each is horizontal or vertical.

   - $y = 0$
   - $x = -3$



Graphing Linear Functions

Practice Worksheet

$$m = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

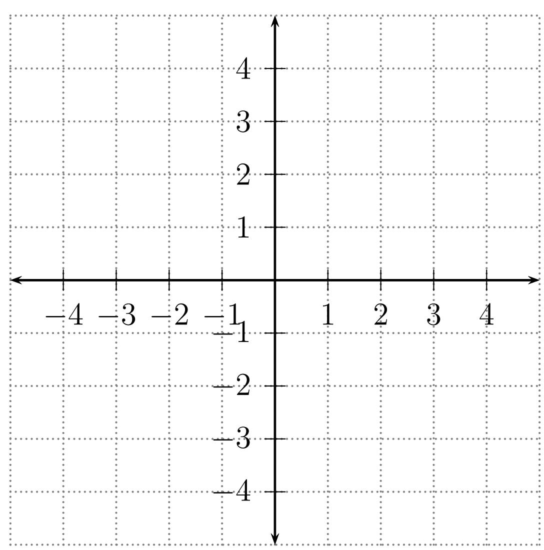
1. Find the slope of the line passing through each of the following pairs of points, then state whether the line is increasing, decreasing, horizontal or vertical.

   (a) $(3, 5)$ and $(6, 1)$

   (b) $(-3, -2)$ and $(-1, -4)$

   (c) $(4, -3)$ and $(8, -1)$

2. Graph the linear equation $2x - 3y = 6$ by first finding the x-intercept and y-intercept of the line. Check your work by finding two more points on the line.

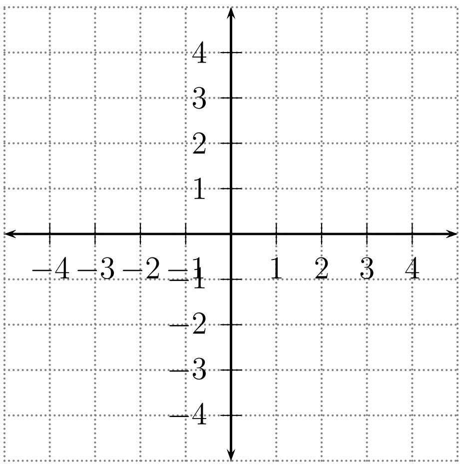


Slope-Intercept Form

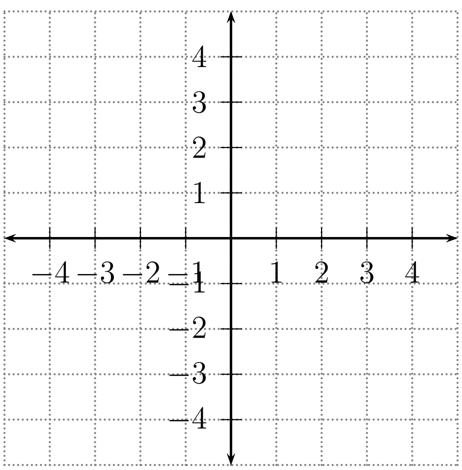
Practice Worksheet

1. For each of the following linear equations in **slope-intercept form**, find the slope and the y-intercept of the line and then graph the function.

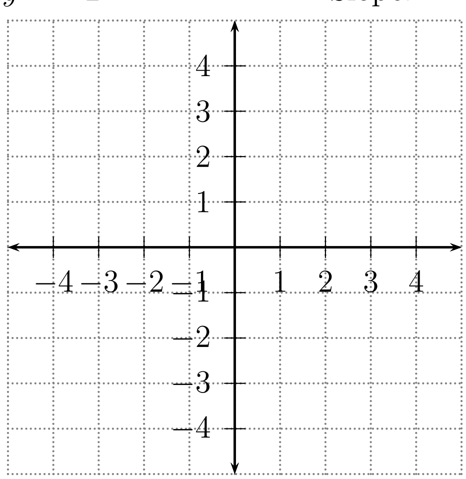
(a) $y = -2x + 4$ Slope: y-intercept:



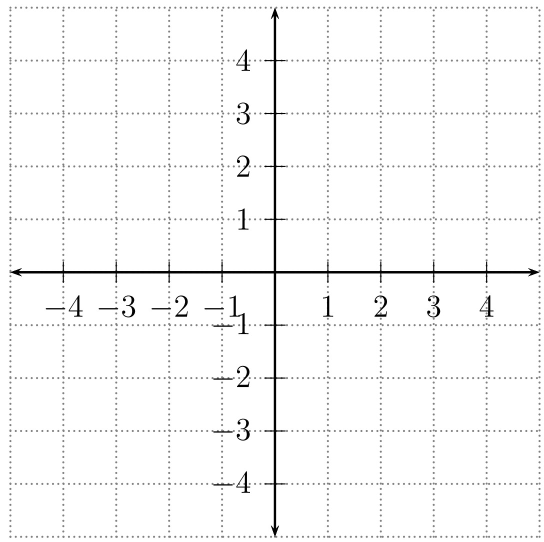
(b) $y = \frac{1}{3}x - 2$ Slope: y-intercept:



(c) $y = -2$ Slope: y-intercept:



2. Write the linear equation $x+2y = 8$ in slope-intercept form. Then use the slope and the y-intercept to graph the function.



3. Write each of the following linear equations in slope-intercept form.

   (a) $2x = y - 7$

   (b) $x + y + 1 = 0$

   (c) $y - 2 = 7$

4. Suppose that the linear function $y = 0.01x + 57.7$ models the global average temperature of the Earth in degrees Fahrenheit x years after 1995.

   (a) What is the **slope** of this linear function?

   (b) What does x represent? What are the units of x?

   (c) What does y represent? What are the units of y?

   (d) What **rate of change** does the slope of this function represent? What are the units of this **rate of change**?

   (e) What are an appropriate **domain** and **range** for this function in the context of this problem?

5. Suppose that the linear function $y = -0.28x + 1.7$ models the percentage of US taxpayers that were audited x years after 1998.

   (a) What is the **slope** of this linear function?

   (b) What does x represent? What are the units of x?

   (c) What does y represent? What are the units of y?

   (d) What **rate of change** does the slope of this function represent? What are the units of this **rate of change**?

   (e) What are an appropriate **domain** and **range** for this function in the context of this problem?

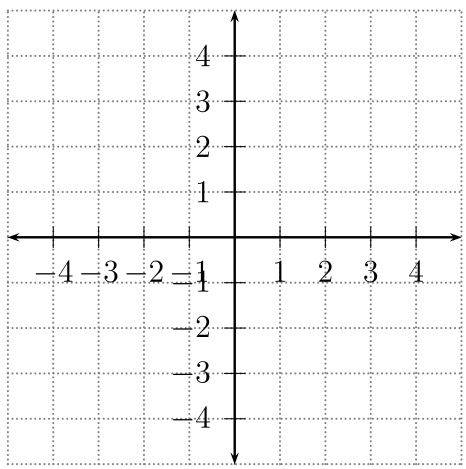
Parallel and Perpendicular Lines

Practice Worksheet

For each of the following, find the slope of the line, an equation for the line in slope-intercept form, the x-intercept and the y-intercept, then graph the line.

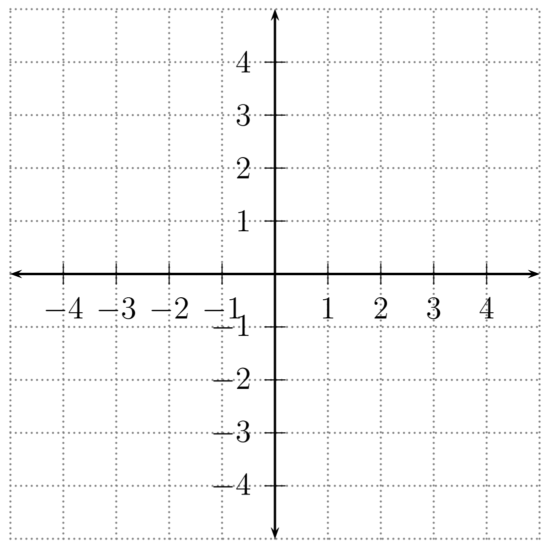
1. The line passing through the point $(-2, -1)$ and **parallel** to the line $2x - 5y = 0$. *(Please graph both lines and label them.)*

   - Slope:
   - Slope-intercept equation:
   - x-intercept:
   - y-intercept:



2. The line passing through the point $(-2, -1)$ and **perpendicular** to the line $2x - 5y = 0$. *(Please graph both lines and label them.)*

   - Slope:
   - Slope-intercept equation:
   - x-intercept:
   - y-intercept:



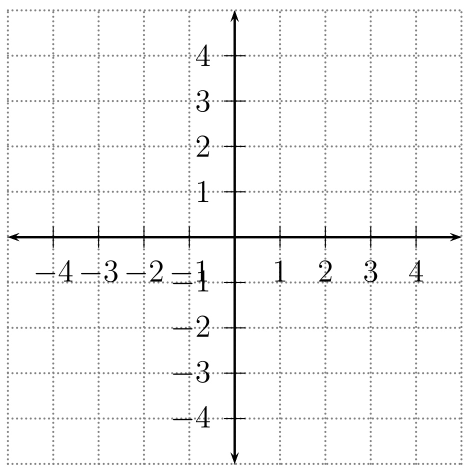
Point-Slope Form

Practice Worksheet

For each of the following, find the slope of the line, an equation for the line in **point-slope form**, an equation for the line in **slope-intercept form**, the x-intercept and the y-intercept, then graph the line.

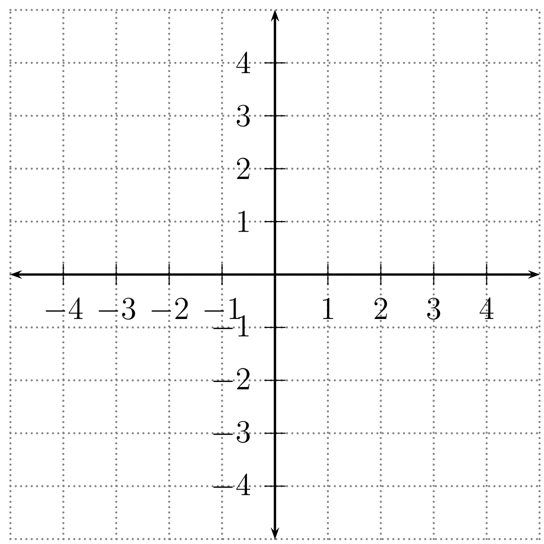
1. The line with slope $= -1$ passing through the point $(0, -1)$.

   - Slope:
   - Point-slope equation:
   - Slope-intercept equation:
   - x-intercept:
   - y-intercept:



2. The line passing through the points $(1, 2)$ and $(3, 2)$.

   - Slope:
   - Point-slope equation:
   - Slope-intercept equation:
   - x-intercept:
   - y-intercept:

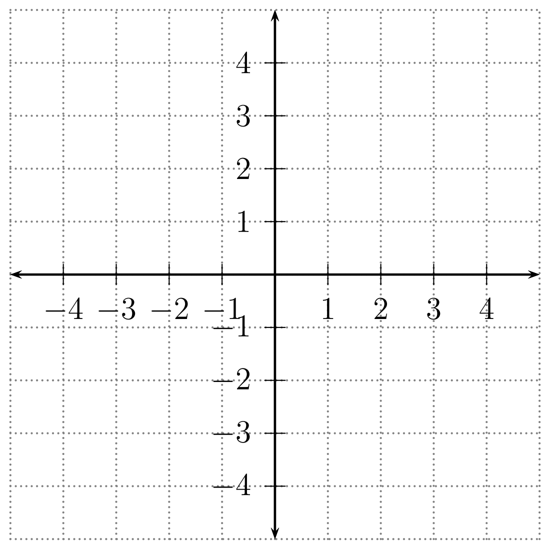


Systems of Linear Equations

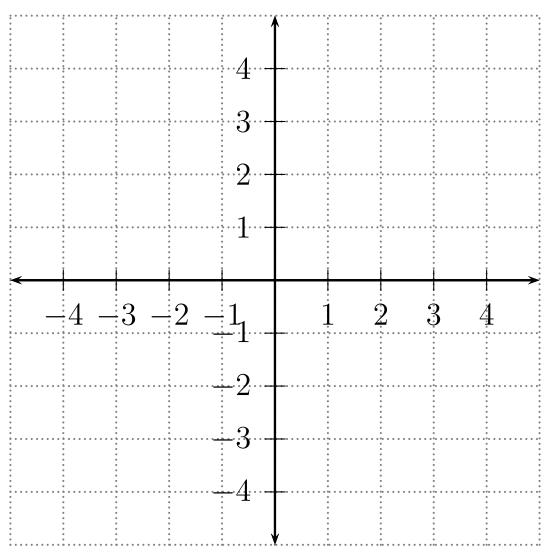
Practice Worksheet

Solve each system of linear equations by graphing, then check that your solution satisfies both equations.

1. $\begin{cases} 2x - 3y = -6 \\ x + 3y = -3 \end{cases}$



2. $\begin{cases} 2x - y = 0 \\ x + y = -3 \end{cases}$



The Substitution Method

Practice Worksheet

Solve each system of linear equations by **substitution**.

1. $\begin{cases} x = -2y + 5 \\ 7x + 2y = -13 \end{cases}$

2. $\begin{cases} y = -1.45x - 6.18 \\ y = 2.63x - 2.73 \end{cases}$

3. $\begin{cases} y = 4 - 2x \\ 4x + 2y = -6 \end{cases}$

4. $\begin{cases} -4x - 2y = -10 \\ y = -2x + 5 \end{cases}$

The Addition Method

Practice Worksheet

Solve each system of linear equations by **addition**.

1. $\begin{cases} 3x + 4y = -5 \\ -5x + 2y = 17 \end{cases}$

2. $\begin{cases} 5x - 2y = 8 \\ 2x - 3y = 1 \end{cases}$

3. $\begin{cases} x + 10y = 8 \\ 5x - 2y = -12 \end{cases}$

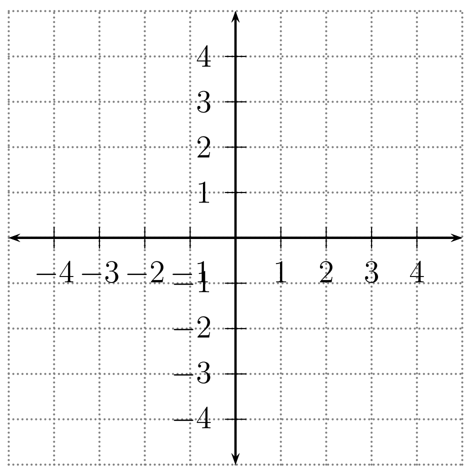
Applications of Systems of Linear Equation

Practice Worksheet

1. You are trying to decide between two gym memberships. Gym A charges an $85 initiation fee and $45 in monthly dues. Gym B charges a $15 initiation fee and $50 in monthly dues.

   (a) Write functions that represent the cost of each gym.

   (b) Graph your equations from part (a) on the coordinates below.



   (c) After how many months will the total cost of membership at the two gyms be the same?

   (d) If you know you will be going to the gym that you choose for at least 18 months, which gym is the better deal?

2. A person plans to invest a total of \$3,500 in an annuity at 3% annual interest and a mutual fund at 16% annual interest. How much should be invested in each account so that the total interest earned in one year is \$200?

3. A person plans to invest a total of \$6,500 in a CD account at 3% annual interest and an investment account at 14% annual interest. How much should be invested in each account so that the total interest earned in one year will be \$400?

4. How many quarts of a 30% acid solution and a 50% acid solution should be mixed to make 10 quarts of a 42% acid solution?

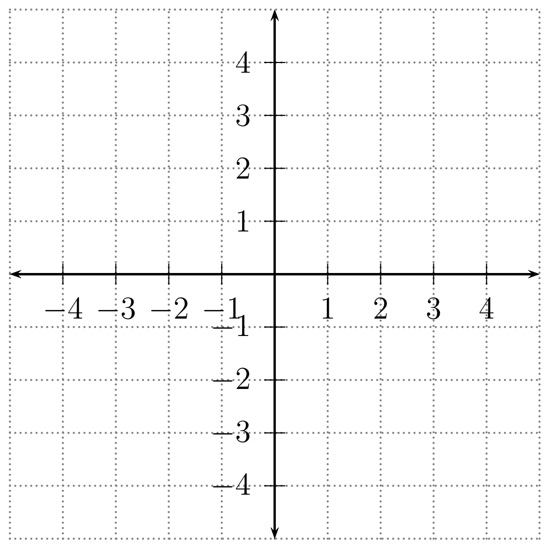
5. How many liters of a 25% antifreeze solution and a 40% antifreeze solution need to be mixed to make 6 liters of a 30% antifreeze solution?

Graphing Linear Inequalities in Two Variables

Practice Worksheet

Shade the region that represents the solution set to the linear inequality below.

$y < \frac{2}{3}x - 5$

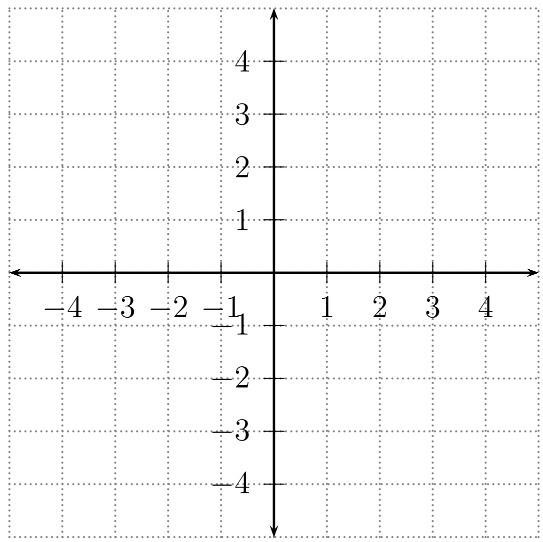


Graphing Systems of Linear Inequalities

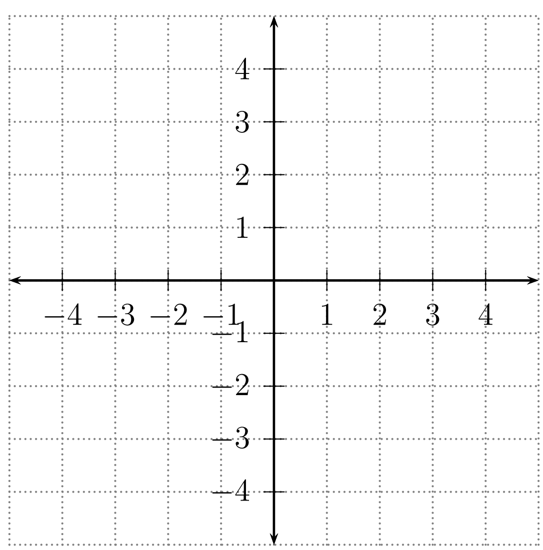
Practice Worksheet

Shade the region that represents the solution set to each system of linear inequalities below.

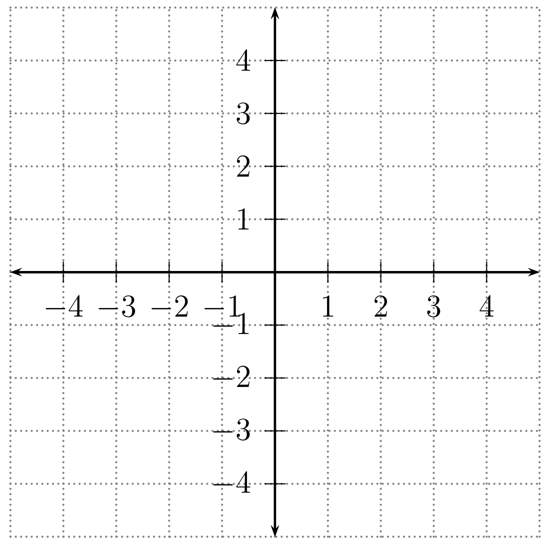
1. $\begin{cases} y \geq -\frac{4}{3}x + 3 \\ x < 1 \end{cases}$



2. $\begin{cases} 3x - 2y \geq 6 \\ x + y < 0 \end{cases}$



3. $\begin{cases} y \leq -\dfrac{2}{3}x + 1 \\ y > -4x - 2 \end{cases}$



Properties of Exponents

Practice Worksheet

Simplify each of the following expressions. Your answers should not include any negative exponents.

1. $(-x^7y)(3xy^5)$

2. $(-4x)^0$

3. $5x^2y^{-4}$

4. $3x^2 \cdot 2x^5$

5. $2y^0$

6. -3^{-2}

7. $(-2x^2y^7)(-4xy^3)$

8. -5^0

9. $(2x^2y)^3$

10. $(-xy^2z^5)^{10}$

11. $-7w^4(w^3)^6$

Division Properties of Exponents

Practice Worksheet

Simplify each of the following expressions. Your answers should not include any negative exponents.

1. $\dfrac{21x^9y^2}{7x^3y}$

2. $\dfrac{3x^5z^5}{18x^5z^3}$

3. $\left(\dfrac{x^5z}{y^2}\right)^3$

4. $\left(-\dfrac{3x^7}{2y}\right)^2$

5. $\dfrac{(x^3y^7)^2}{y^{10}}$

6. $\left(\dfrac{7b^4c^5}{14b^3c^2}\right)^0$

7. $\left(\dfrac{5x^5y^4}{2x^2y^3}\right)^2$

Multiplication of Linear Expressions

Practice Worksheet

Find the following products.

1. $6y(y+1)$

2. $(x+3)(x+7)$

3. $(a-6)(a-5)$

4. $(6a+3)(2a-4)$

5. $(4x-3)(4x+3)$

6. $(x-3)^2$

7. $(3x+7)^2$

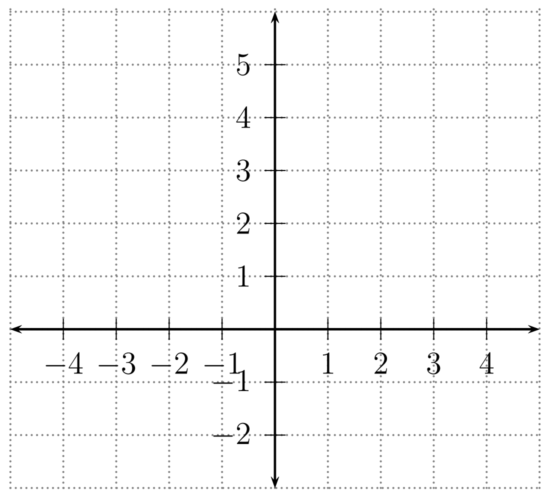
8. $(4y+5)(4y-5)$

Characteristics of Quadratic Functions

Practice Worksheet

1. Sketch a graph of the function $f(x) = x^2 - x$ by first completing the table and plotting points.

x	-2	-1	0	1	2	3
y						



2. Now answer the following questions about the function $f(x) = x^2 - x$.

   (a) What are the coefficients a, b and c?

   (b) Does this parabola open up or down?

   (c) What is the vertex of the parabola? Label it on your graph above.

   (d) What is the vertical intercept of the parabola? Label it on your graph above.

   (e) What are the horizontal intercepts of the parabola? Label them on your graph above.

   (f) What is the axis of symmetry? Draw it on your graph above.

   (g) What is the domain of the function?

   (h) What is the range of the function?

Factoring with GCF

Practice Worksheet

1. Factor out the GCF for each of the following polynomials.

   (a) $7x^2 + 49$

   (b) $28p^2 + 21p$

   (c) $-8x - 6x^2$

   (d) $9r^2 - 12r$

   (e) $12x^2 - 20x$

   (f) $-2x^4 + 50x^2$

2. Refer to the previous page to **solve** each of the following equations. Then check your answers by plugging in.

   (a) $7x^2 + 49 = 0$

   (b) $28p^2 + 21p = 0$

   (c) $-8x - 6x^2 = 0$

   (d) $9r^2 - 12r = 0$

   (e) $12x^2 - 20x = 0$

   (f) $-2x^4 + 50x^2 = 0$

Factoring Quadratic Expressions With Leading Coefficient 1

Practice Worksheet

Factor the following trinomials with leading coefficient 1. Check your answer by multiplying.

1. $x^2 + 13x + 12$

2. $m^2 - 4m - 21$

3. $x^2 + 7x + 10$

4. $t^2 - 5t + 6$

5. $a^2 - 12a + 27$

6. $x^2 - 12x + 20$

Factoring using Big X

Practice Worksheet

Factoring using Big X and Grouping together for $ax^2 + bx + c$
– Multiply $a \cdot c$ *(Top of Big X)*
– Break the product up into two factors that add to b *(Bottom of Big X)*
– Break b up into the two factors and use the **grouping method** to factor the result

1. A student submitted the following **incorrect** work to simplify an expression. Find where the student made their error(s), then redo the problem correctly.

$$\begin{aligned} 6x^2 - 5x - 4 &= 6x^2 + 3x - 8x - 4 \\ &= 3x(2x+1) - 4(2x-1) \\ &= (2x+1)(2x-1)(3x-4) \end{aligned}$$

Factor each trinomial with leading coefficient $\neq 1$.

2. $2x^2 + 9x + 7$

3. $5y^2 - 16y + 3$

4. $8x^2 - 18x + 9$

5. $6x^2 - 5x - 6$

Completely factor each trinomial. Make sure you factor out the GCF first if there is one.

6. $5p^2 - 5p - 60$

7. $3b^2 - 10b - 8$

8. $-x^2 - 2x + 35$

9. $2t^3 - 12t^2 - 32t$

10. $60y - 40y^2 + 5y^3$

Difference of Squares

Practice Worksheet

Factor each difference of squares.

1. $x^2 - 49$

2. $x^2 - 100$

3. $x^2 - 4$

4. $x^2 - 1$

5. $2x^2 - 50$

6. $9x^2 - 16$

Factoring in General

Practice Worksheet

Factor each trinomial.

1. $y^2 + 16y + 64$

2. $x^2 - 14x + 49$

3. $x^2 + 2x + 1$

4. $25x^2 + 20x + 4$

5. $16z^2 - 24z + 9$

Solving Quadratic Equations by Factoring

Practice Worksheet

1. A student submitted the following **incorrect** work to solve a quadratic equation. Find where the student made their error(s), then redo the problem correctly.

$$\begin{aligned} 3x^2 - 39x + 120 &= 0 \\ 3(x^2 - 13x + 40) &= 0 \\ 3(x-5)(x-8) &= 0 \\ x &= 3, 5, 8 \end{aligned}$$

Solve the following quadratic equations by **factoring**.

2. $x^2 + 5x + 6 = 0$

3. $64y^2 - 9 = 0$

4. $x^2 = 11x + 12$

5. $3r + r^2 - 28 = 0$

6. $2x^2 + 6x - 80 = 0$

7. $5x^2 = 35x$

8. $x^2 - 4x - 45 = 0$

9. $x^2 - 12 = -36$

10. $x^2 - 6x = 0$

11. $x^2 - 49 = 0$

12. $16x^2 = 25$

13. $(x - 1)(x + 4) = 14$

14. $2x(x + 3) = -5x - 15$

Quadratic Formula

Practice Worksheet

Solve each quadratic equation using the **quadratic formula**.

1. $2x^2 + 3x + 1 = 0$

2. $x^2 - 3x + 15 = 0$

3. $x^2 + 7x - 2 = 0$

4. $2x^2 - 5x = -6$

Complex Numbers

Practice Worksheet

Completely simplify the following square roots using **complex numbers** when appropriate. No decimal approximations.

1. $\sqrt{-64}$

2. $-\sqrt{64}$

3. $-\sqrt{-64}$

4. $\sqrt{-25}$

5. $\sqrt{-16}$

6. $-\sqrt{-16}$

7. $\sqrt{-500}$

8. $\sqrt{-2}$

9. $\sqrt{-49}$

10. $-\sqrt{-72}$

11. $4 + \sqrt{-49}$

12. $10 - \sqrt{-1}$

13. $\sqrt{-100} + 100$

14. $3 + \sqrt{-64}$

15. $20 - \sqrt{-5}$

16. $1 + \sqrt{-10}$

Quadratic Equations with Complex Solutions

Practice Worksheet

For each of the following, first decide how many real solutions and how many complex solutions the equation has, then use the quadratic formula to solve the equation.

1. $-x^2 = 7 - 2x$
   - Number of real solutions:
   - Number of complex solutions:
   - Solve:

2. $3x^2 + 3x + 4 = 0$
   - Number of real solutions:
   - Number of complex solutions:
   - Solve:

3. $4 = x^2 - x$
   - Number of real solutions:
   - Number of complex solutions:
   - Solve:

4. $2x^2 - 6x = -5$
   - Number of real solutions:
   - Number of complex solutions:
   - Solve:

Applications of Quadratic Equations

Practice Worksheet

1. A person throws a ball into the air. The height (in feet) of the ball after t seconds is given by

$$f(t) = -16t^2 + 64t + 4.$$

   (a) When is the ball at a height of 4 feet? Explain why there are two such times.

   (b) When is the ball at a height of 68 feet? Explain why there is just one such time.

2. The percentage of Americans who think that the environment should be protected can be modeled by

$$f(t) = t^2 - 8t + 64,$$

   where t is years since 2000.

   (a) Find $f(10)$. What does it mean in this situation?

   (b) Find t so that $f(t) = 73$. What does it mean in this situation?

3. A rectangular garden has a length that is 3 feet greater than the width. The area of the garden is 180 square feet. Find the length and the width.

Mixed Review

1. Find $f(1)$ for $f(x) = (x+1)^3 - 7x^5$.

2. Company A charges \$9 to rent a movie online for one week. Although only members can rent from the website, membership is free. Company B charges only \$4 to rent a movie online for one week, but there is a one-time membership fee of \$50. After how many movie rentals will the cost be the same at both websites, and what would be the total spent at each site?

3. A painting has a rectangular shape 60 inches high by 40 feet wide. There is a frame of equal width around the perimeter of the painting. The perimeter of the rectangle formed by the outside of the frame is 248 inches. Find the dimensions of the painting. *(Start by drawing a diagram!)*

4. Completely simplify each of the following.

   (a) $(-4x^3yz^5)^2$

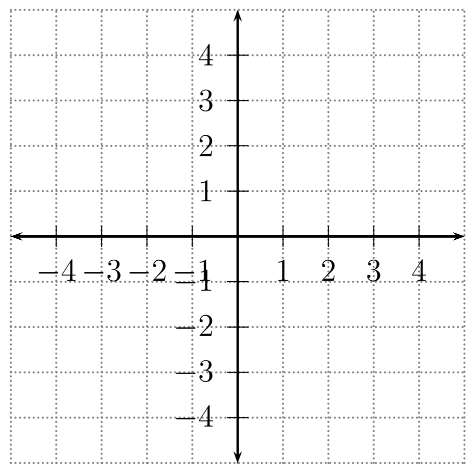
   (b) $\left(\dfrac{x^2y^5}{xy^7}\right)^5$

   (c) $\dfrac{4x^8y^2z^5}{-2x^5y^2z^3}$

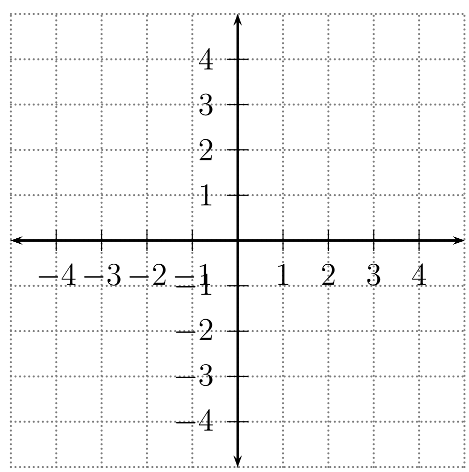
   (d) $(x^2y^3)^4(xy^7z^2)^3$

5. Find the slope of the line passing through the points $(1, 1)$ and $(3, 0)$.

6. Find the slope and the y-intercept of the line $y = 2x - 1$ and use them to graph the line.



7. Graph the line $2x - 4y = 6$ by first finding the x-intercept and y-intercept.



8. Write each of the following equations in **slope-intercept** form.

   (a) $4x - y = 0$

   (b) $x + 2y - 4 = 0$

9. Solve each system of linear equations.

   (a) $\begin{cases} x - 2y = 5 \\ 7x + 2y = -13 \end{cases}$

   (b) $\begin{cases} 3x + 4y = 5 \\ -5x + 2y = 7 \end{cases}$

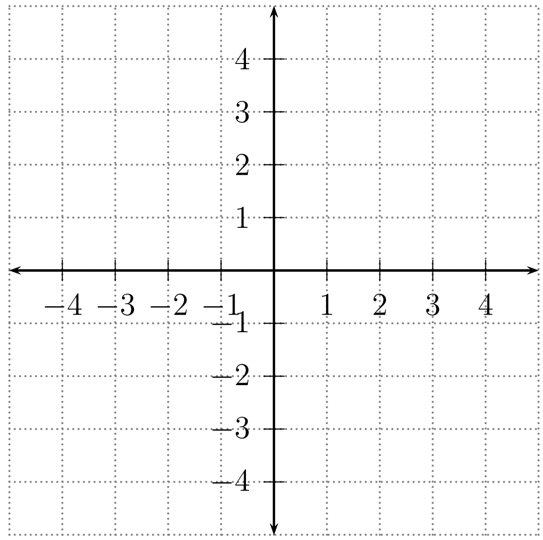
10. How many liters of a 10% acid solution and a 30% acid solution need to be mixed to make 5 liters of a 25% acid solution?

11. Solve each inequality.

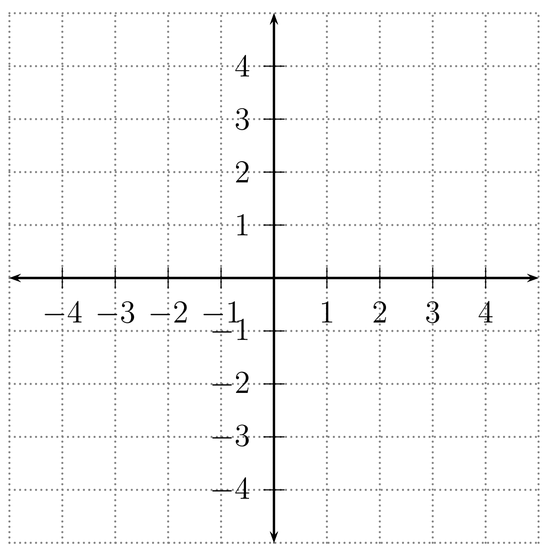
    (a) $-2(4x - 32) \leq -40$

    (b) $\dfrac{x+1}{3} + \dfrac{1}{2} \geq \dfrac{3x+6}{2}$

12. $\begin{cases} 3x - 2y > 6 \\ x + y > 2 \end{cases}$



13. $\begin{cases} y \leq -\dfrac{2}{3}x + 1 \\ y \leq -4x - 2 \end{cases}$



14. Factor using any method. Don't forget to factor out the GCF first!

    (a) $x^2 + x - 12$

    (b) $6x^2 - 5x - 4$

    (c) $10x^2 - 32x + 6$

    (d) $x^2 - 49$

    (e) $5x^2 + 20$

    (f) $x^2 - 6x + 9$

15. Solve each quadratic equation by **factoring**.

   (a) $36x^2 - 9 = 0$

   (b) $x^2 + 3x - 28 = 0$

   (c) $2x^2 = 16x$

   (d) $(x - 1)(x + 4) = 14$