

◆**Example 5** A state issues a 15 year \$1000 bond that pays \$25 every six months. If the current market interest rate is 4%, what is the fair market value of the bond?

Solution: The bond certificate promises two things – an amount of \$1,000 to be paid in 15 years, and semi-annual payments of \$25 for 15 years. To find the fair market value of the bond, we find the present value of the \$1,000 face value we are to receive in 15 years and add it to the present value of the \$25 semi-annual payments for the 15 years. In this example, $nt = 2(15) = 30$.

We will let P_1 = the present value of the lump-sum \$1,000

$$P_1(1 + .04/2)^{30} = \$1,000$$

$$P_1 = \$552.07$$

We will let P_2 = the present value of the \$25 semi-annual payments is

$$P_2(1 + .04/2)^{30} = \frac{\$25[(1 + .04/2)^{30} - 1]}{(.04/2)}$$

$$P_2(1.18114) = \$1014.20$$

$$P_2 = \$559.90$$

The present value of the lump-sum \$1,000 = \$552.07

The present value of the \$30 semi-annual payments = \$559.90

Therefore, the fair market value of the bond is

$$P = P_1 + P_2 = \$552.07 + \$559.90 = \$1111.97$$

Because the market interest rate of 4% is lower than the interest rate of 5% implied by the semiannual payments, the bond is selling at a premium: the fair market value of \$1,111.97 is more than the face value of \$1,000.

To summarize:

To Find the Fair Market Value of a Bond:

Find the present value of the face amount A that is payable at the maturity date:

$$A = P_1(1 + r/n)^{nt} \quad ; \text{ solve to find } P_1$$

Find the present value of the semiannually payments of \$ m over the term of the bond:

$$P_2(1 + r/n)^{nt} = \frac{m[(1 + r/n)^{nt} - 1]}{r/n} \quad ; \text{ solve to find } P_2$$

The fair market value (or present value or price or current value) of the bond is the sum of the present values calculated above:

$$P = P_1 + P_2$$