# Math 1C: Quick (*but not complete*) Refresher on Circles, Ellipses, Hyperbolas

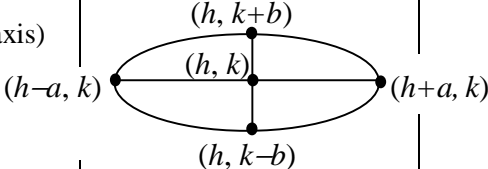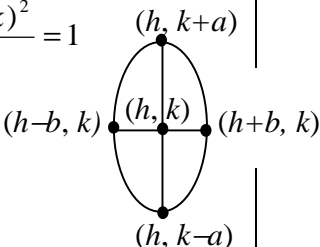**Circle**: Center at point $(h,k)$ with radius $r$

$$(x-h)^2 + (y-k)^2 = r^2$$

or equivalently $\dfrac{(x-h)^2}{r^2} + \dfrac{(y-k)^2}{r^2} = 1$

**Ellipse**: Center at point $(h, k)$

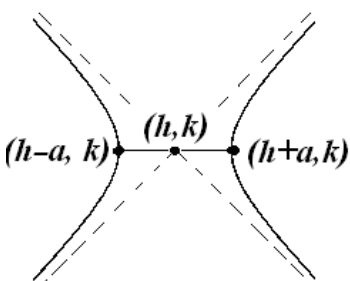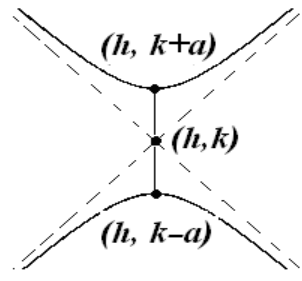Distance from center to a vertex (on major axis) is $a$
Distance from center to a covertex (on minor axis) is $b$
Since major axis is longer than minor axis, we always have $a > b$

| | Horizontal Major Axis | Vertical Major Axis |
|---|---|---|
| Equation and Shape $a > b$ determines the orientation of the ellipse on the plane (horizontal or vertical major axis) | $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$  | $\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$  |
| Center | $(h, k)$ | $(h,k)$ |
| Vertices ( $a$ units from center) | $(h-a, k)$ and $(h+a, k)$ | $(h, k-a)$ and $(h, k+a)$ |
| Covertices ($b$ units from center) | $(h, k-b)$ and $(h, k+b)$ | $(h-b, k)$ and $(h+b, k)$ |
| Length of Major Axis | $2a$ | $2a$ |
| Length of Minor Axis | $2b$ | $2b$ |
| Foci where $c^2 = a^2 - b^2$ | $(h-c, k)$ and $(h+c, k)$ | $(h, k-c)$ and $(h, k+c)$ |
| Eccentricity $c/a$ | $0 < c/a < 1$ | $0 < c/a < 1$ |
| *If eccentricity closer to 0, the ellipse is rounder; as eccentricity gets closer to 1 the ellipse is more elongated. | | |

**Hyperbola**: Center at point $(h, k)$ ; Distance to from center to a vertex (on transverse axis) is $a$
Transverse axis determines the direction (horizontal or vertical) that the hyperbola opens.

| | Horizontal Transverse Axis | Vertical Transverse Axis |
|---|---|---|
| Equation and Shape<br><br>The "positive" term indicates the direction of the transverse axis, and the direction in which the hyperbola opens.<br><br>The "negative" term indicates the direction of the conjugate axis (*not shown in this diagram*). | $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$  | $\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$  |
| Center | $(h, k)$ | $(h, k)$ |
| Vertices | $(h-a, k)$ and $(h+a, k)$ | $(h, k-a)$ and $(h, k+a)$ |
| Covertices (*endpoints of conjugate axis, BUT not on the curve of the hyperbola*) | $(h, k-b)$ and $(h, k+b)$ | $(h-b,k)$ and $(h+b,k)$ |
| Length of Transverse Axis | $2a$ | $2a$ |
| Length of Conjugate Axis | $2b$ | $2b$ |
| Foci where $c^2 = a^2 + b^2$ | $(h-c, k)$ and $(h+c, k)$ | $(h, k-c)$ and $(h, k+c)$ |
| Asymptotes | $y - k = \pm (b/a)(x-h)$ | $y - k = \pm (a/b)(x-h)$ |
| Eccentricity $c/a$ | $c/a > 1$ | $c/a > 1$ |