A LINEAR PROGRAM is a problem in which we want to OPTIMIZE something (such as maximizing profit, maximizing revenue, maximizing income, minimizing cost, or minimizing calories) subject to constraints on available resources.

We need to determine the best solution while staying within the restrictions of the constraints.

Steps needed to to set up and solve a linear programming problem geometrically.

1) Set up a clear mathematical statement of the problem

   a) Write the **objective function** that describes what we want to optimize and write down whether we need to minimize or maximize it.

   b) Write the constraints as linear inequalities. Also write all the non-negativity constraints.

2) Solve graphically

   a) Graph the constraint lines.

   b) Shade the area that satisfies ALL the inequalities at the same time (this is called the feasible region). Use "test points" as needed to determine which way to shade for each inequality. Keep in mind that (0,0) is a great test point to use if it is not on the constraint line; arithmetic is easy using (0,0).

   c) Solve for intersections (vertices) to find all corners of the feasible region.

   d) Evaluate the objective function at all vertices (corners) of the feasible region to determine which corner maximizes or minimizes (optimizes) the objective function, as required by the problem.

3) Write up the solution

   a) Draw the objective function on the graph at the point where it is optimized.

   b) Write the solution in sentence for indicating the values of the variables at the optimal solution and the optimal value of the objective function. Include appropriate units in your answers. Include the context of the problem clearly stating what each variable relates to
   (such as "They should produce 600 wine racks and 400 breadboxes to achieve a maximum profit of $20,000"). Do not just state the answer in terms of x and y with no context.

We will do some but probably not all of the following problems for chapter 3 in class to learn how to set up linear programs and solve them geometrically.

Problem 1: Keisha works in an office but also has a part-time business making jewelry and selling it on Etsy. She needs to plan her jewelry work for this week.
Keisha makes and sells two styles of necklaces.
Necklace Style A uses $10 in materials and takes 2 hours of labor to make.
Necklace Style B uses $20 in materials but takes 1 hour of labor to make.
For this week, Keisha has a budget of $160 for materials and has 14 hours total in labor available.
Suppose that necklaces A and B sell for the same price, $40 each.
How many of each type of necklace should she make in order to maximize revenue subject to the constraints for her materials budget and her available labor time?

Problem 2: Library Supply Company makes superior quality bookcases and regular quality bookcases. Superior bookcases need 3 hours of shop time to construct and regular bookcases require 2 hours of shop time to construct. Both superior and regular bookcases require 1 hour of finishing.
The company has a total of 900 hours of shop time available for constructing bookcases and 400 hours of finishing time available.
If the profit is $75 for a superior bookcase and $30 for a regular bookcase, how many of each type should they make to maximize profit?

Problem 3: Library Supply Company makes superior quality bookcases and regular quality bookcases. Superior bookcases need 3 hours of shop time to construct and regular bookcases require 2 hours of shop time to construct.Both superior and regular bookcases require 1 hour of finishing.
The company has a total of 900 hours of shop time available for constructing bookcases and 400 hours of finishing time available.
Other considerations limit the number of superior quality bookcases to at most 350 bookcases. (Note we now have 3 constraint lines)
If the profit is $75 for a superior bookcase and $30 for a regular bookcase, how many of each type should they make to maximize profit?

Problem 4: From Textbook, Section 3.1
Niki holds two part-time jobs, Job I and Job II. She never wants to work more than a total of 12 hours a week. She has determined that for every hour she works at Job I, she needs 2 hours of preparation time, and for every hour she works at Job II, she needs one hour of preparation time, and she cannot spend more than 16 hours for preparation. If she makes $40 an hour at Job I, and $30 an hour at Job II, how many hours should she work per week at each job to maximize her income?

Problem 5: From Textbook, Section 3.2:
Prof. Hamer is on a low cholesterol diet. During lunch at the cafeteria he always chooses between 2 meals, Pasta or Tofu.
The Pasta meal has 8g of protein, 60 g of carbs, 2g of vitamins and 60 mg of cholesterol.
The Tofu meal has 16 g of protein, 40 g of carbs, 2 g of vitamins and 50 mg of cholesterol.
Mr Hamer needs at least 200 g of protein, 960 g of carbs and 40 g of vitamins for lunch each month.
How many days should he have the Pasta meal and how many days should he have the Tofu meal, so that he gets adequate amount of protein, cars and vitamins and at the same time minimizes his cholesterol intake?

Math 11 Chapter 3: Linear Program Geometric Solution Sheet (2 Variables)

1. WRITE THE LINEAR PROGRAM

a. <u>Definition of Variables:</u> x = ____

y = ____

b. <u>Objective Function:</u> *Clearly indicate if the problem requires minimize or maximize*

Minimize or Maximize: Z = __________________________________
(circle one/cross out other)

c. <u>Subject to Constraints</u>: (include non-negativity constraints; add more lines at right if needed)

C1: __________________________________

C2: __________________________________

C3: __________________________________

C4: __________________________________

C5: __________________________________

C6: __________________________________

2. SOLVE THE LINEAR PROGRAM

a. Graph the lines corresponding to constraints
and label the lines C1, C2, C3, . . . as appropriate
Scale and label the axes appropriately
(USE A <u>RULER</u> and draw graph in pencil –
you may need to redraw and rescale if you do not select the scale appropriately on the first try)

b. Shade feasible region

c. Identify and label all vertices (critical points) of the feasible region.
Solve <u>algebraically</u> for the intersections.
(Use a separate sheet or back of page to do the algebra)

d. Evaluate objective function at each critical point
Determine which critical point is optimal. *Show your work in the table below.*

Critical Point	Intersection of	Objective Function

3. STATE YOUR ANSWER IN A SENTENCE that describes the optimal solution.

- Explain the optimal values of each variable and the optimal value of the objective function, *stating everything in the context of the problem and including appropriate units in the answer.*