CHAPTER 4 : DISCRETE PROBABILITY DISTRIBUTIONS

Probability distributions can be represented by tables or by formulas.
The simplest type of probability distribution can be displayed in a table.

Discrete Probability Distributions using PDF Tables

EXAMPLE D1: Students who live in the dormitories at a certain four year college must buy a meal plan. They must select from four available meal plans: 10 meals, 14 meals, 18 meals, or 21 meals per week. The Food and Housing Office has determined that the 15% of students purchase 10 meal plan, 45% purchase the 14 meal plan of students, 30% purchase the 18 meal plan ,10% purchase the 21 meal plan.

a. What is the random variable? X = ____

____

Notation: P(Event) = probability value

P(X = 10) is the probability that a student purchases a meal plan with 10 meals per week
P(X >14) is the probability that a student purchases a meal plan with more than 14 meals per week

b. Make a table that shows the probability distribution

This table is called the PDF
Probability Distribution Function

x =Number of Meals	Probability P(x)
10	
14	
18	
21	

We can create an extra column next to the PDF table to help calculate the mean

xP(x)

c. Find the probability that a student purchases more than 14 meals:

d. Find the probability that a student does not purchase 21 meals.

e. On average, how many meals does a student purchase per week in their meal plan?

Calculate the mean. **Mean = Expected Value: $\mu = \Sigma xP(x)$** $\mu =$ ________

f. Write a sentence that interprets the mean in the context of the problem.

NOTE that it is acceptable that the mean is not whole number; it can have a fraction or a decimal.

CHAPTER 4 : DISCRETE PROBABILITY DISTRIBUTIONS
Discrete Probability Distributions using PDF Tables

- **PDF: Probability Distribution Function**
  All probabilities are between 0 and 1, inclusive AND All probabilities must sum to 1.

- **Mean = Expected Value** = $\mu = \Sigma x P(x)$
  Interpreted as a long term average over many observations
  Formula is a "weighted" average where each value is "weighted" according to how likely is its to occur

  - **Standard Deviation** = $\sigma = \sqrt{\sum (x-\mu)^2 P(x)}$ measures variation in the probability distribution

  Formula is a "weighted" average of the squared distances between each data value and the mean

  - **Variance = (standard deviation)^2** $\sigma^2 = \sum (x-\mu)^2 P(x)$ also measures variation

  Before widespread technology, variance was easier to calculate than standard deviation
  Variance is used in some types of "statistical tests" instead of standard deviation

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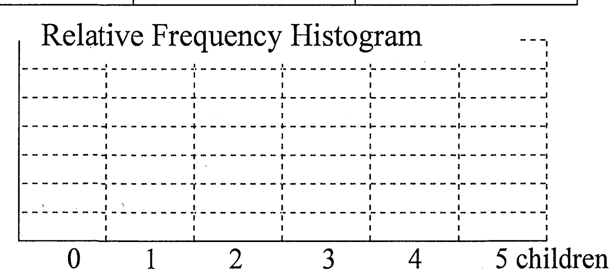
EXAMPLE D2 : A real estate developer is presenting plans to the Planning Commissioner for a development of houses and apartments he proposes to build. He needs to estimate the impact on the local schools so he must estimate the number of children expected to attend the schools. He hires a statistician who studies the demographics of the neighborhood and of similar housing developments; she provides the estimates below.
Let X = the number of school age children per household.
a. Find the probability that a household has 2 school age children.
b.Draw a relative frequency histogram of this probability distribution
c. Find the probability that a family has **at most 3** school age children.
d. Find and interpret the **expected number** of school age children per household.
e. Find the **standard deviation** for the number of school age children per household.
f. Find the expected total number of school age children in this new development if 120 housing units are built.

X	P(X)
0	0.30
1	0.20
2	
3	0.18
4	0.04
5	0.01
6 or more	0

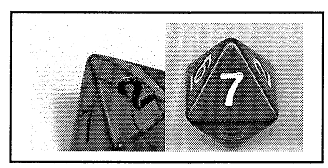


CHAPTER 4: DISCRETE PROBABILITY DISTRIBUTIONS USING PDF TABLES

EXAMPLE D3: *At the county fair, a booth has a coin flipping game.*
We are interested in the net amount of money gained or lost in one game. You pay $1 to flip **three fair coins.**
If the result contains three heads, you win $4. If the result is two heads, you win $1. Otherwise there is no prize.
a. Define the random variable and write the PDF for the amount gained or lost in one game.
b. Find the expected value for this game (Expected NET GAIN OR LOSS)
c. Find the expected total net gain or loss if you play this game 50 times.

EXAMPLE D4: Suppose you play a different game. In this game, you flip a **biased coin** twice.
A biased or unfair coin has different probabilities for landing on heads and tails.
Suppose that for this coin, P(HEAD) = 2/3 and P(TAIL) = 1/3. In this game you do not pay in order to play.
You toss the coin twice, and then win or lose according to the following:
win $3 if you toss two tails, win $1 if you toss two heads, or pay (lose) $2 if you toss one head and one tail.
We are interested in the net amount of money gained or lost in one game.
a. Define the random variable and write the PDF for the amount gained or lost in one game.
b. Find the expected value for this game (Expected NET GAIN OR LOSS)

EXAMPLE D5: In this game we roll ONE fair EIGHT SIDED DIE once.(The eight sides of the die are numbered 1, 2, 3, 4, 5, 6, 7, 8 and the die has an equal chance of landing on each side.)
Suppose that you win $6 if you roll an 8, win $2.50 if you roll a 2,
lose $2 if you roll an odd number,
and if you roll a 4 or 6 you neither win anything nor lose anything.
We are interested in the monetary outcome for one game.



a. Define the random variable and write the PDF for the amount gained or lost in one game.
b. Find the expected value for this game (Expected NET GAIN OR LOSS)

CHAPTER 4 : BINOMIAL PROBABILITY DISTRIBUTION

The Binomial Distribution is a special discrete probability distribution that arises often in problems.
A BINOMIAL probability experiment has

- a fixed number **n** of repeated trials
- each trial has outcomes that we can classify as "success or "failure"
- outcome of trials are independent (*Outcome of a trial does not influence outcome of future trials*)

The probability of success on a single trial, **p**, is constant (the same) for all trials

We are interested in the number of successes, x, in n trials **Notation: X~B(n,p)**

EXAMPLE B1: A college claims that 70% of students receive financial aid. Suppose that 4 students at the college are randomly selected. We are interested in the number of students in the sample who receive financial aid.

X = ____

p = the probability that a student receives financial aid: p = _____ q = 1−p = _____
X ~ B(4, 0.7) : Binomial with n = 4 and p = 0.7

X	P(x)
0	
1	
2	
3	
4	

Ways to get x successes in n trials

n = 4, x = 1	n = 4, x = 2	n = 4, x = 3
Abcd	**AB**cd	a**BCD**
a**B**cd	**A**b**C**d	**A**b**CD**
ab**C**d	**A**bc**D**	**AB**c**D**
abc**D**	a**BC**d	**ABC**d
	a**B**c**D**	
	ab**CD**	

a. Find the probability that AT MOST 2 of the students in the sample receive financial aid:

b. Find the probability that AT LEAST 3 of the students in the sample receive financial aid:

c. Find the **mean and the standard deviation** using the **shortcut formulas for the binomial distribution**:

$\mu = np$; $\sigma = \sqrt{npq}$ where $q = 1 - p$ **only for Binomial distribution.**

These shortcut formulas for μ *and* σ *give the same results as the definitions* $\mu = \Sigma x P(x)$, $\sigma = \sqrt{\sum (x-\mu)^2 P(x)}$

Formulas for Binomial Distribution: X~B(*n*,*p*) $P(X = x) = {}_nC_x\, p^x (1-p)^{n-x}$

$P(X = x)$ is the probability of obtaining x successes in n independent trial

$\mu = np$; $\sigma = \sqrt{npq}$ **where** $q = 1 - p$ **only for binomial distribution.**

${}_nC_x$ represents the number of ways (patterns) in which it is possible to get x successes in n trials

$${}_nC_x = \binom{n}{x} = \frac{n!}{x!(n-x)!}$$ *Where* $n! = n(n-1)(n-2)(n-3)\ldots(3)(2)(1)$ *for integers* $n > 0$

Example $4! = (4)(3)(2)(1) = 24$ $3! = (3)(2)(1) = 6$ $2! = (2)(1) = 6$ *also* $0! = 1$ *by definition*

$${}_4C_2 = \frac{4!}{2!(4-2)!} = \frac{4!}{(2!)(2!)} = \frac{(4)(3)(2)(1)}{(2)(1)(2)(1)} = 6$$

${}_nC_x$ using calculator MATH PROB nCr :
Example: 4 MATH PROB nCr 2 ENTER

CHAPTER 4: BINOMIAL PROBABILITY DISTRIBUTION

Binomial Distribution TI 83, 84 Calculator

Use binompdf or binomcdf found at 2nd Distr

binompdf : P(X = value) probability distribution function

binomcdf : P(X ≤ value) cumulative distribution function

P(X= x)	binompdf (n,p,x)
P(X ≤ x)	binomcdf (n,p,x)
P(X < x)	binomcdf (n,p,x − 1)
P(X > x)	1 − binomcdf (n,p,x)
P(X ≥ x)	1 − binomcdf (n,p,x − 1)

EXAMPLE B2: *http://www.pewresearch.org/fact-tank/2016/01/05/pew-research-center-will-call-75-cellphones-for-surveys-in-2016/?utm_source=Pew+Research+Center&utm_campaign=4a62041804-Methods_Newsletter_for_June6_24_2015*

Many survey organizations that conduct "public opinion polls" gather their data through telephone surveys. These include political polls, polls about current events, and other topics about demographics, lifestyle, economic issues, etc. In recent years, survey organizations have had to change their data gathering methods because sampling from landline phones only would exclude significant portions of the population.

Kyley McGeeney, a research methodologist at Pew Research Center, wrote that "All major survey organizations that conduct telephone surveys include cellphones in their samples. They have to, because the kinds of people who rely only on a cellphone are different from those reachable on a landline, even though being cellphone-only is becoming more mainstream. Cellphone-only individuals are considerably younger than people with a landline. They tend to have less education and lower incomes than people with a landline. They are also more likely to be Hispanic and to live in urban areas. For this reason, excluding cellphones from a poll – or not including enough of them – would provide a sample that is not representative of all U.S. adults."

The Pew Research Center cites that:
65.7% of 25- to 29-year-olds live in wireless-only households and do not have landlines.

Suppose we took a sample consisting of 100 people age 25-29 and we are interested in counting the number of people in the sample who have only cell phone service.

X = (*description*)____

p = (*description*)____

p = (*value*) _________ X ~ _________

a. Find the probability that 60 have only cell phone service

b. Find the probability that at most (≤)60 have only cell phone service

c. Find the probability that less than 60 have only cell phone service

d. Find the probability that the number who have only cell phone service exceeds (is more than) 60

e. Find the probability that at least (≥) 60 have only cell phone service

f. Find the probabilitythat exactly half of the people in the sample have only cell phone service

CHAPTER 4: BINOMIAL PROBABILITY DISTRIBUTION

P(X= x)	binompdf (n,p,x)
P(X ≤ x)	binomcdf (n,p,x)
P(X < x)	binomcdf (n,p,x – 1)
P(X > x)	1 – binomcdf (n,p,x)
P(X ≥ x)	1 – binomcdf (n,p,x – 1)

Binomial Distribution TI 83, 84 Calculator

Use binompdf or binomcdf found at 2nd Distr

binompdf : P(X = value) probability distribution function

binomcdf : P(X ≤ value) cumulative distribution function

EXAMPLE B3: Make sure that the probability of success matches the definition of a success

A recent study showed that about 60% of California voters voted by mail. Suppose we are selecting a random sample of 75 voters and we are interested in the number of people in the sample who vote in person at the polls.

X = *(description)*____

p = *(description)*____

p = *(value)* ________ X ~ ________

a. Find the probability that more than 25 of the voters in the sample voted in person at the polls.

b. How many people in the sample would you expect to vote in person at the polls.

EXAMPLE B4: PRACTICE *Try-It 4.13 Introductory Statistics from Openstax download for free at www.openstax.org*

About 32% of students participate in a community volunteer program outside of school.
Suppose that 30 students are selected at random.
X = the number of students who participate in a community volunteer program outside of school **X ~** ________

a. Find the probability that at most 14 participate in a community volunteer program outside of school

b. Find the probability that at least 15 participate in a community volunteer program outside of school

c. Find the probability that more than 20 participate in a community volunteer program outside of school

d. For many samples of 30 students, on average, what is the expected number per sample who participate in a community volunteer program outside of school?

e. Find the standard deviation

NOTE: Recognizing Scientific Notation on your Calculator:

Sometimes probabilities are very small numbers.
If the number is very close to 0, your calculator automatically uses scientific notation:

0.000068 = 6.8×10^{-5} appears on the calculator as 6.80000000E-5

4.26713E-6 = 4.26713×10^{-6} = 0.00000426713 (move decimal point left 6 places)