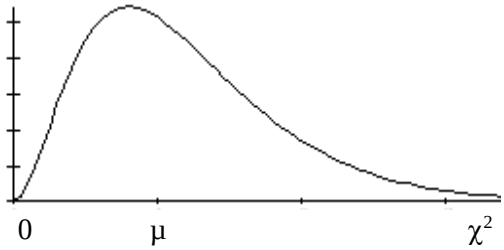
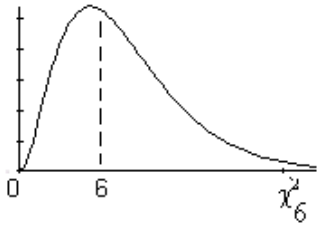
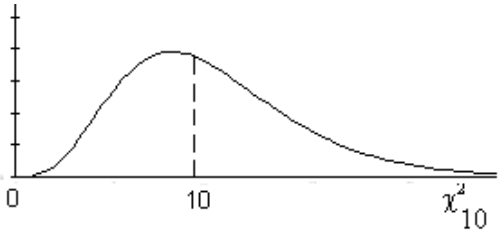
Chapter 11 χ^2 "chi-square" probability distribution

- continuous probability distribution
- shape is skewed to the right
- variable values on horizontal axis are ≥ 0
- area under the curve represents probability
- horizontal asymptote – extends to infinity along positive horizontal axis - curve gets closer to horizontal axis but does not touch it as X gets large
- shape depends on "degrees of freedom" (d.f.)
- when d.f. = 2 only, curve is an exponential distribution
- for higher degrees of freedom, the peak shifts to the right
- when d.f. > 90, curve begins to look more like normal distribution (even then χ^2 is still <u>always</u> somewhat skewed right)
- mean is located a little to the right of the peak
- Mean and Standard Deviation are determined by the degrees of freedom:

mean = d.f. standard deviation = $\sqrt{2\,(\text{d.f.})}$







χ^2 with 6 degrees of freedom χ^2 with 10 degrees of freedom

Finding Probabilities: TI –83, 84: χ^2 cdf (lower bound, upper bound, degrees of freedom)

We will be using the right tail for hypothesis tests for Goodness of Fit orfor Independence:
TI –83, 84: χ^2 cdf (lower bound, 10^99, degrees of freedom)

χ^2 "chi-square" probability distribution is used for 3 things in Chapter 11
We will study the first two of these topics listed below.

Test of Goodness of Fit:
Hypothesis: Population fits an assumed distribution (theory)
Sample data is collected from a population
Hypothesis test is performed to see if the sample data supports the theory that the population fits this assumed distribution, or not.

Test of Independence:
Hypothesis: Two qualitative variables are independent of each other
Sample data is collected from a population
Hypothesis test is performed to see if the sample data supports the theory that these two variables are independent or not

Hypothesis Test (and confidence intervals) for an unknown population standard deviation.

χ^2-Goodness of Fit Test

Null hypothesis states a probability distribution that we think the population follows.
We look at sample data to decide if the population follows that distribution or not.

- Write the N**ull and Alternative Hypothesis (**as sentences**)**
  Ho: The data fit the expected (*hypothesized*) distribution
  (Describe the expected hypothesized distribution either in a sentence, by using its appropriate name, by showing a formula or referring to a nearby visible table or list)

  Ha: The data do not fit the expected (*hypothesized*) distribution

- Decide upon α

- Collect observed data AND calculate expected data based on the table of percentages given in the problem or based on the probability distribution named or described in the problem

- Find the test statistic and pvalue and draw the graph

  Test statistic = $\sum_{\substack{all\ cells \\ in\ table}} \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$

  Test statistic is a number measuring the size of the differences between observed sample data and expected data

  χ^2 distribution , right tailed test ; df = # of CELLS – 1

  2nd DISTR χ^2cdf (test statistic, 10^99, df)

  Draw the graph ; shade pvalue; label axis, test statistic & pvalue

- Make a Decision by comparing pvalue to significance level α
  - If test statistic is large indicating sample data differ from expected, then area in right tail is small. If pvalue < α, Reject Ho
  - If test statistic is small, indicating sample data are similar to expected, then area in right tail is large. If pvalue > α, Reject Ho
  -
- Write a conclusion in the context of the problem.

Calculator Instructions for test statistic and pvalue

STAT EDIT: Enter Observed Data frequencies in list L1
Enter Expected Theoretical frequencies in list L2
(Do NOT put totals in either list)
Arrow up to the very top (title line) in list L3 and input $(L1 - L2)^2 / L2$; press Enter

STAT CALC: 1 variable stats L3

Test Statistic is ΣX = total of list L3 = $\sum_{\substack{all\ cells \\ in\ table}} \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$

2nd DISTR χ^2 cdf (lower bound, 10^99, degrees of freedom) = **pvalue**

EXAMPLE 1: Hypothesis Test for Goodness of Fit (GOF)

Student Demographics for all California Community Colleges by Age for 2014-15 http://californiacommunitycolleges.cccco.edu/PolicyInAction/KeyFacts.aspx			Ages for a Sample of 227 De Anza College Students	
19 or less	25%		19 or less	48
20 – 24	32%		20 – 24	107
25 – 29	14%		25 – 29	30
30 – 34	8%		30 – 34	14
35 and over	21%		35 and over	28
Total	100%		Total	227

We want to know if the age distribution of De Anza Colletge Students fits the age distribution of community college students statewide.

Null Hypothesis Ho: _____

_____

Alternate Hypothesis H_A: _____

_____

α = _____

	L1	L2	L3= $(L1-L2)^2/L2$
	Observed Data	Expected Data	$\frac{(Observed - Expected)^2}{Expected}$
19 or less			
20 – 24			
25 – 29			
30 – 34			
35 and over			

Total = Total = **Test Statistic = Sum =** _____

The test statistic measures the size of the differences between the distributions

Calculate the p-value

Use_____distribution with

_____degrees of freedom

Draw the graph

pvalue = _____

Decision: _____

Conclusion: _____

_____

_____

Example 2: Are calls for emergency medical services uniformly distributed by day of week?
A city is reviewing staffing levels and staffing schedules for their emergency medical response team. To determine if calls are uniformly distributed by day of the week, they analyzed data for a sample of 575 EMS calls.

Day of the Week	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
Number of EMS calls	88	79	84	85	77	75	87

Null Hypothesis Ho: _____

_____

Alternate Hypothesis H_A: _____

_____

α = _____

	L1	L2	L3= $(L1-L2)^2/L2$
	Observed Data	Expected Data	$\frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$
Sunday			
Monday			
Tuesday			
Wednesday			
Thursday			
Friday			
Saturday			

Total = Total = **Test Statistic = Sum =** _____

The test statistic measures the size of the differences between the distributions

Calculate the p-value
Use_____distribution with
_____degrees of freedom

Draw the graph

pvalue = _____

Decision: _____

Conclusion: _____

_____

_____

Practice Examples for Goodness of Fit Test ***Reminder: If alpha is not stated in the problem, use 5%.***

Example 3. Are births uniformly distributed by day of the week? The Health Commissioner in River County wants to know whether births in the county are uniformly distributed by day of week. A sample of 434 randomly selected births at hospitals in the county yield the following data.

Day of the Week	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
Number of Births in sample	41	64	71	72	71	69	46

Based on information in the National Vital Statistics Report January 7, 2009
http://www.cdc.gov/nchs/data/nvsr/nvsr57/nvsr57_07.pdf

Example 4. At a cell phone company, the marketing staff conducts a market research survey of 225 customers who bought a new model of phone and asks about the time period in which they plan to replace their phones

Phone ownership Time Period	<1 year	1 to <2 years	2 to <3 years	3 to <4 years	≥4 years
Number planning to replace phone	0	60	106	42	17

Do the observed data for planned phone replacement fit a uniform distribution by years of ownership?

Example 5. At a cell phone company, in the past 5 years the frequency with which their US customers replaced their phone by a newer model is:

Phone Ownership Time Period	<1 year	1 to <2 years	2 to <3 years	3 to <4 years	≥4 years
Percent Replaced	3%	25%	45%	18%	9%

Their marketing staff conducts a market research survey of 225 customers who bought a new model of phone and asks about the time period in which they plan to replace their phones

Phone ownership Time Period	<1 year	1 to <2 years	2 to <3 years	3 to <4 years	≥4 years
Number planning to replace phone	0	60	106	42	17

Do the observed data for planned phone replacement fit the past distribution of phone replacement times or does the distribution for planned phone replacement differ from the past distribution?

Example 6. A genetics lab experiment investigates feather color in a species of bird. Genetic theory predicts that when mating pairs of green birds of this species, the expected outcomes for probabilities of the offspring's feather colors will follow the pattern for a "dihybrid cross":

56.25% green, 18.75% yellow, 18.75% blue, 6.25% white.

Conduct a hypothesis test to determine if feather color distribution fits or does not fit this distribution. For a sample of pairs of green birds mated in the lab, the offspring's observed color distribution was:

41 green, 12 blue, 20 yellow, 7 white.

Example 7. In the fourth week of class we used the binomial distribution B(4, 0.70) to find the probabilities for X = the number of students receiving financial aid in a group of 4 students, if 70% of students at the college get financial aid.

X = Number of Students getting financial aid	0	1	2	3	4
Probability	0.0081	0.0756	0.2646	0.4116	0.2401

Probabilities are found as: binompdf(4, 0.70, x) or can be calculated using probability rules.

A statistics student decides to sample many groups of 4 students to see if the binomial distribution is a good fit for this situation. She and her friends collect data for 150 groups of 4 students. The table below shows the number of groups of students in which 0,1,2,3 or 4 students receive financial aid.

X = Number of Students getting financial aid	0	1	2	3	4
Frequency	3	15	36	55	41

Perform a goodness of fit test to determine if this situation fits the binomial distribution B(4, 0.70)

χ^2 Hypothesis Test for Independence

Review: Understanding Independence in a Contingency Table:

EXAMPLE 7: Lina is a statistics student doing a project to compare ice cream flavor preferences at 3 ice cream stores in different cities. She wants to determine if customer preferences are related to store location or if they are independent. She will select a sample of customers, and categorize each customer by store location and flavor preference.

Assume that there are only 3 flavors

A.

	Fremont(F)	Gilroy(G)	Hayward (H)	Total
Chocolate (C)	25	25	50	100
Vanilla (V)	15	15	30	60
Mint Chip (M)	10	10	20	40
Total	50	50	100	200

Are flavor preference and store location INDEPENDENT?

B:

	Fremont (F)	Gilroy(G)	Hayward (H)	Total
Chocolate (C)	20	10	70	100
Vanilla (V)	20	10	20	50
Mint Chip (M)	10	30	10	50
Total	50	50	100	200

Are flavor preference and store location INDEPENDENT?

C:

	Fremont (F)	Gilroy(G)	Hayward (H)	Total
Chocolate (C)	27	24	49	100
Vanilla (V)	13	15	32	60
Mint Chip (M)	10	11	19	40
Total	50	50	100	200

Are flavor preference and store location INDEPENDENT?

D.

	Fremont (F)	Gilroy(G)	Hayward (H)	Total
Chocolate (C)	22	30	48	100
Vanilla (V)	10	12	38	60
Mint Chip (M)	18	8	14	40
Total	50	50	100	200

Are flavor preference and store location INDEPENDENT?

How far can our data vary from the independent data in part A and still be considered "similar"?
How far must our data vary from the independent data in part A in order to be considered "different"?

χ^2 Hypothesis Test for Independence

We use a hypothesis Test for Independence to help us decide whether two qualitative variables are independent or not.

We calculate what the data is expected to look like if the variables are independent.

We compare the observed data to the expected data
Is the "observed data" sufficiently different from "expected" to decide that Ho is not true.
OR is the "observed data" reasonably close to "expected"

Writing the Hypothesis Test for Independence

This is an overview of the process.
Calculator instructions are on the last page of the notes for this chapter.

- Write the hypotheses
  Null Hypothesis: Ho: The variables (*DESCRIBE THEM*) are independent
  Alternate Hypothesis: H_A: The variables (*DESCRIBE THEM*) are NOT independent
- Decide upon significance level α
- Collect observed data. Calculate expected data.
- Find the test statistic and pvalue: Test statistic $\sum_{\substack{all\ cells \\ in\ table}} \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$

  Find the pvalue using the χ^2 distribution , right tailed test
  df = (# of Rows – 1) (# of Columns -1)
  χ^2cdf (test statistic, 10^99, df)
  Draw the graph ; shade the pvalue; label axis, test statistic & pvalue
- Compare pvalue to the significance level α and make a decision.
- Write a conclusion in the context of the problem.

Calculator Instructions for Test of Independence:

Entering the data:

2nd MATRIX EDIT; select matrix [A] and press enter.
Enter matrix size: number of rows x number of columns, pressing enter after each is entered
NOTE: Look only at data when figuring out size; do NOT include row or column totals in the matrix. Matrix on calculator will change shape on the screen to match the stated size.
Enter observed data into matrix A, press enter after each number is entered.

Performing the hypothesis test to find the pvalue and test statistic

STAT TESTS: χ^2 Test
Observed: 2nd Matrix 1: [A] Enter *(This is the input to the test)*

Expected: 2nd Matrix 2: [B] Enter *(This is output the calculator creates when doing the test)*

OUTPUT SCREEN will show pvalue and test statistic and degrees of freedom

ON HOME SCREEN: To see matrices containing observed and expected data
Observed: 2nd Matrix 1: [A] Enter
Expected: 2nd Matrix 2: [B] Enter
You may need to scroll to the right to see part of the matrix if it is too wide to fit on y our calculator screen

EXAMPLE 8: χ^2 Hypothesis Test for Independence

Use the following sample of data from 3 ice cream store locations to determine whether flavor preference and store location are independent, or whether they are dependent (related to teach other).

OBSERVED DATA	Fremont (F)	Gilroy(G)	Hayward (H)	Total
Chocolate (C)	22	30	48	100
Vanilla (V)	10	12	38	60
Mint Chip (M)	18	8	14	40
Total	50	50	100	200

Ho: _____

Ha: _____

Calculating the EXPECTED DATA (assumes independence)

EXPECTED DATA	Fremont (F)	Gilroy(G)	Hayward (H)	Total
Chocolate (C)				100
Vanilla (V)				60
Mint Chip (M)				40
Total	50	50	100	200

Compare OBSERVED Data to how the data is EXPECTED to look **IF** it were INDEPENDENT.

How close is actual "observed data (Obs)" to "expected data (E)"?

Measure of the difference for each cell inside the table: $\frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$

Add them all up to get: **Test Statistic** = $\sum_{\substack{all\ cells \\ in\ table}} \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$

Test Statistic = $(22-25)^2/25 + (30-25)^2/25 + (48-50)^2/50$

$+ (10-15)^2/15 + (12-15)^2/15 + (38-30)^2/30$

$+ (18-10)^2/10 + (8-10)^2/10 + (14-20)^2/20$ **Test Statistic = 14.44**

Degrees of freedom = (number of rows – 1)(number of columns – 1); df =(___ – 1)(____ – 1) = _____

Probability Distribution to use to calculate pvalue: _____ with df = _____

pvalue : Draw, shade and label the graph

P($\chi^2 \geq$ _____) = _____ (_____, _____, ___)

pvalue = _____

Decision: _____Reason for Decision: _____

Is the result Statistically Significant (circle one): YES NO

Conclusion:

EXAMPLE 9: χ^2 Hypothesis Test for Independence

An economics professor believes that whether a student uses the textbook in print form or as an ebook won't affect their exam grade. The exam grade and the format of textbook the student used are summarized in the table below for a sample of 300 randomly selected students over the academic year. At a 5% level of significance, can we conclude that textbook format and exam grade are related?

OBSERVED DATA		Textbook Format Used		
		Print textbook	**ebook**	TOTAL
Grade Earned On Exam	**A**	23	17	40
	B	42	48	90
	C	55	75	130
	D	12	8	20
	F	8	12	20
	TOTAL	140	160	300

Ho: _____

Ha: _____

Use the _____ test on your calculator to find the following:

Test Statistic = _____ pvalue = _____

Show calculation for Degrees of freedom = _____

What probability distribution is used to calculate the pvalue:_____

Draw, shade and label the graph

Decision: _____Reason for Decision: _____

Is the result Statistically Significant (circle one): YES NO

Conclusion:

Find the output matrix with the EXPECTED DATA and fill it in. Then show the calculation for the expected data for "A and ebook" and the expected value for "D and print". Round to 2 decimal places).

OBSERVED DATA		Textbook Format Used		
		Print textbook	**ebook**	TOTAL
Grade Earned On Exam	**A**			40
	B			90
	C			130
	D			20
	F			20
	TOTAL	140	160	300

Practice Examples for Test of Independence
Reminder: If alpha is not stated in the problem, use 5%.

Example 11. A random sample of 1343 US adults age 25 & over were surveyed and asked about their educational status and whether they were employed or unemployed. Are employment status and educational status independent?

SOURCE: *Based on information in the Current Population Survey Feb 2013*
http://www.bls.gov/webapps/legacy/cpsatab4.htm http://data.bls.gov/pdq/SurveyOutputServlet

OBSERVED DATA	Not a high school Graduate	High School Graduate	Some College or Associate Degree	Bachelors Degree or Higher	TOTAL
Employed	100	333	348	476	1257
Unemployed	13	29	25	19	86
TOTAL	113	362	373	495	1343

Example 12. A personal trainer at a gym wants to compare three exercise plans (Plans A, B, C) to see if they are equally effective in improving the fitness of the participants. A sample of 260 clients yields the following data about which exercise plan the client uses and the fitness improvement results.

	A	B	C
Moderate or large improvement	64	75	53
No or little improvement	18	24	26

A sample of people who train at the gym are randomly assigned to one of the exercise plans and the trainer evaluates whether fitness improvement occurs or not.
Perform a hypothesis test to determine if the improvement in fitness depends on the fitness plan or if improvement is independent of the particular fitness plan the person participates in.

Example 13. *http://www.people-press.org/files/2015/06/6-4-15-Immigration-release.pdf*
The Pew Research Center conducted a poll of US adults' opinions on immigration. One of the questions asked was whether undocumented immigrants should be permitted a way to stay in the US legally, if requirements are met. The responses are summarized below.

		Political Party Affiliation			Total
		Democrat	Republican	Unaffiliated or Other	
Response to Immigration Question	Yes	509	283	576	1368
	No	121	218	174	513
	Don't Know	6	5	8	19
Total		636	506	758	1900

At a 5% level of significance, are resident's opinions about illegal immigration independent of their political party affiliation or is there a relationship between their opinions and their political party affiliation?