CHAPTER 4 : DISCRETE PROBABILITY DISTRIBUTIONS

Probability distributions can be represented by tables or by formulas.
The simplest type of probability distribution can be displayed in a table.

Discrete Probability Distributions using PDF Tables

EXAMPLE D1: Students who live in the dormitories at Hudson University must buy a meal plan. They must select from four available meal plans: 10 meals, 14 meals, 18 meals, or 21 meals per week. The Food and Housing Office has determined that the 15% of students purchase 10 meal plan, 45% purchase the 14 meal plan of students, 30% purchase the 18 meal plan ,10% purchase the 21 meal plan.

a. What is the random variable? X = ____

____

Notation: P(Event) = probability value

P(X = 10) is the probability that a student purchases a meal plan with 10 meals per week
P(X >14) is the probability that a student purchases a meal plan with more than 14 meals per week

b. Make a table that shows the probability distribution

This table is called the PDF
Probability Distribution Function

x =Number of Meals	Probability P(x)
10	
14	
18	
21	

We can create an extra column next to the PDF table to help calculate the mean

xP(x)

c. Find the probability that a student purchases more than 14 meals:

d. Find the probability that a student does not purchase 21 meals.

e. On average, how many meals does a student purchase per week in their meal plan?

Calculate the mean. **Mean = Expected Value: $\mu = \Sigma xP(x)$** μ = _________

f. Write a sentence that interprets the mean in the context of the problem.

NOTE that it is acceptable that the mean is not whole number; it can have a fraction or a decimal.

CHAPTER 4 : DISCRETE PROBABILITY DISTRIBUTIONS
Discrete Probability Distributions using PDF Tables

- **PDF: Probability Distribution Function**
  All probabilities are between 0 and 1, inclusive AND All probabilities must sum to 1.

- **Mean = Expected Value = $\mu = \Sigma xP(x)$**
  Interpreted as a long term average over many observations
  Formula is a "weighted" average where each value is "weighted" according to how likely is its to occur

- **Standard Deviation = $\sigma = \sqrt{\sum (x-\mu)^2 P(x)}$**
  Measures variation in the probability distribution
  Formula is a "weighted" average of the squared distances between each data value and the mean

- **Variance = (standard deviation)^2 = $\sigma^2 = \sum (x-\mu)^2 P(x)$**
  Measures variation in the probability distribution
  Before widespread technology, variance was easier to calculate than standard deviation
  Variance is used in some types of "statistical tests" instead of standard deviation

---

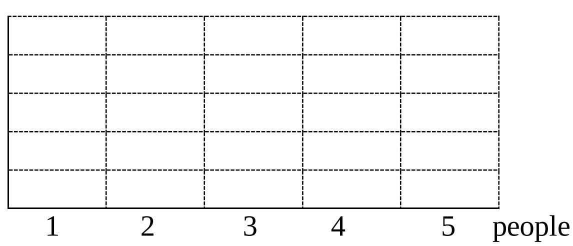
EXAMPLE D2: The Highway Commissioner wants to know how many people are in vehicles that use the carpool lanes on Ocean Expressway.

The best way to estimate occupancy of a car accurately is with human observers; electronic methods often are not accurate. A team of "observers" on highway overpasses counts the number of occupants in a sample of vehicles passing below the overpass. X = the number of people in a vehicle in the carpool lane.

The probability distribution for the number of occupants in vehicles in the carpool lane on Ocean Expressway is:

X	P(X)
1	0.1
2	0.4
3	0.1
4	0.2
5	0.1
6	0.1

a. Draw a relative frequency histogram; label the vertical axis scale to show the relative frequency values.



b. Find the expected value and write a sentence that interprets its meaning in the context of the problem

c. Use your calculator 1-VarStats to find the standard deviation

d. Find the variance σ^2 :

EXAMPLE D3: *At the county fair, a booth has a coin flipping game.*
You pay $1 to flip **three fair coins**.

If the result contains three heads, you win $4.

If the result is three tails, you win $2.

Otherwise there is no prize.

We are interested in the net amount of money gained or lost in one game.
a. Define the random variable and the values it can have.
b. Write the probability distribution function (PDF) for the amount gained or lost in one game.
c. Find the expected value for this game (Expected NET GAIN OR LOSS)
d. Find the expected total net gain or loss if you play this game 100 times.

***EXAMPLE D4 PRACTICE*:** Suppose you play a different game. In this game, you flip a **biased coin** twice.
A biased or unfair coin has different probabilities for landing on heads and tails.
Suppose that for this coin, P(HEAD) = 2/3 and P(TAIL) = 1/3.
In this game you do not pay in order to play.
You toss the coin twice, and then win or lose according to the following:

win $3 if you toss two tails

win $1 if you toss two heads

pay (lose) $2 if you toss one head and one tail.

We are interested in the net amount of money gained or lost in one game.
a. Define the random variable and the values it can have.
b. Write the PDF for the amount gained or lost in one game.
c. Find the expected value for this game (Expected NET GAIN OR LOSS)

EXAMPLE D5: In this game we roll ONE fair EIGHT SIDED DIE once.
(The eight sides of the die are numbered 1, 2, 3, 4, 5, 6, 7, 8 ; it has an equal chance of landing on each side.)

Suppose that you
- win \$6 if you roll an 8,
- win \$2.50 if you roll a 2,
- lose \$2 if you roll an odd number,
- and if you roll a 4 or 6 you neither win anything nor lose anything.

We are interested in the monetary outcome for one game.

a. Define the random variable and the values it can have.
b. Write the PDF for the amount gained or lost in one game.
c. Find the expected value for this game (Expected NET GAIN OR LOSS)
d. Find the expected total net gain or loss if you play this game 40 times.

EXAMPLE D6 PRACTICE: A real estate developer is presenting plans to the Planning Commission for a proposed housing development. He needs to estimate the impact on the local schools so he must estimate the number of children expected to attend the schools. He hires a statistician who studies the demographics of the neighborhood and of similar housing developments; she provides the estimates below.

Let X = the number of school age children per household.

a. Assuming no families have more than 5 school age children, complete the pdf table.
b. Find the probability that a family has at least 3 school age children
c. Find the expected number of school age children per household
d. Find the expected number of school age children in the housing development if it has 120 housing units.

X	P(X)
0	0.30
1	0.18
2	0.24
3	
4	0.06
5	0.02
6 ore more	0

CHAPTER 4 : BINOMIAL PROBABILITY DISTRIBUTION

The Binomial Distribution is a special discrete probability distribution that arises often.

A BINOMIAL probability experiment has the following properties

- There are a fixed number **n** of repeated trials
- Each trial has outcomes that we can classify as "success or "failure"
- Outcome of trials are independent (*Outcome of a trial does not influence outcome of future*
- The probability of success on a single trial, **p**, is constant (the same) for all trials.
- **We are interested in the number of successes, x, in n trials** **Notation: X~B(n,p)**

EXAMPLE B1: A college claims that 70% of students receive financial aid. Suppose that 4 students at the college are randomly selected. We are interested in the number of students in the sample who receive financial aid.

X = ____

p = the probability that a student receives financial aid: p = _____ q = 1−p = _____

X ~ B(4, 0.7) : Binomial with n = 4 and p = 0.7

X	P(x)
0	
1	
2	
3	
4	

Ways to get x successes in n trials

n = 4, x = 1	n = 4, x = 2	n = 4, x = 3
Abcd	**AB**cd	a**BCD**
a**B**cd	**A**b**C**d	**A**b**CD**
ab**C**d	**A**bc**D**	**AB**c**D**
abc**D**	a**BC**d	**ABC**d
	a**B**c**D**	
	ab**CD**	

a. Find the probability that AT MOST 2 of the students in the sample receive financial aid:

b. Find the probability that AT LEAST 3 of the students in the sample receive financial aid:

c. Find the **mean and the standard deviation** using the **shortcut formulas for the binomial distribution**:

$\boldsymbol{\mu = np}$; $\boldsymbol{\sigma = \sqrt{npq}}$ where $q = 1 - p$ **<u>only</u> for Binomial distribution.**

These shortcut formulas for μ and σ give the same results as the definitions $\mu=\Sigma xP(x)$, $\sigma = \sqrt{\sum (x-\mu)^2 P(x)}$

Formulas for Binomial Distribution: **X~B(*n*,*p*)** $P(X = x) = {}_nC_x\, p^x\, (1-p)^{n-x}$

$P(X = x)$ is the probability of obtaining x successes in n independent trial

$\mu = np$; $\sigma = \sqrt{npq}$ **where $q = 1 - p$ <u>only</u> for binomial distribution.**

${}_nC_x$ represents the number of ways (patterns) in which it is possible to get x successes in n trials

$${}_nC_x = \frac{n!}{x!(n-x)!}$$ *where n! = n(n−1)(n−2)(n−3). . .(3)(2)(1) for integers n > 0 and 0! = 1 by definition*

Examples: 4! = (4)(3)(2)(1) = 24 3! = (3)(2)(1) = 6 2! = (2)(1) = 6

$${}_4C_2 = \frac{4!}{2!(4-2)!} = \frac{4!}{(2!)(2!)} = \frac{(4)(3)(2)(1)}{(2)(1)(2)(1)} = 6$$

${}_nC_x$ using calculator MATH PROB nCr :
Example: 4 MATH PROB nCr 2 ENTER

Binomial Distribution on the TI 83, 84

2nd Distr binompdf or 2nd Distr binompdf

binomPdf : P(X = value) probability distribution function

binomCdf : P(X ≤ value) cumulative distribution function

P(X= x)	**binompdf (n,p,x)**
P(X ≤ x)	**binomcdf (n,p,x)**
P(X < x)	**binomcdf (n,p,x − 1)**
P(X > x)	**1 − binomcdf (n,p,x)**
P(X ≥ x)	**1 − binomcdf (n,p,x − 1)**

EXAMPLE B2: *http://www.pewresearch.org/fact-tank/2016/01/05/pew-research-center-will-call-75-cellphones-for-surveys-in-2016/?utm_source=Pew+Research+Center&utm_campaign=4a62041804-Methods_Newsletter_for_June6_24_2015*

Many major survey organizations that conduct "public opinion polls" gather their data through telephone surveys. These include political polls, polls about current events, and other subjects about demographics, lifestyle, economic issues, and more. In recent years, these organizations have had to change their data gathering techniques, as sampling from only landline phones no longer yields a representative sample of the population.

Kyley McGeeney, a research methodologist at Pew Research Center, wrote
"All major survey organizations that conduct telephone surveys include cellphones in their samples. They have to, because the kinds of people who rely only on a cellphone are different from those reachable on a landline, even though being cellphone-only is becoming more mainstream. Cellphone-only individuals are considerably younger than people with a landline. They tend to have less education and lower incomes than people with a landline. They are also more likely to be Hispanic and to live in urban areas. For this reason, excluding cellphones from a poll – or not including enough of them – would provide a sample that is not representative of all U.S. adults."

The Pew Research Center cites that:

65.7% of 25- to 29-year-olds live in wireless-only households and do not have landlines.

Suppose we took a sample consisting of a group of 100 people age 25-29.
We are interested in how many people in the group have only cell phone service.

X = (*description*) ____

____

p = (*description*) ____

____

p = (*value*) ________________ X ~ ________________

a. Find the probability that 60 have only cell phone service

b. Find the probability that at most (≤)60 have only cell phone service

c. Find the probability that less than 60 have only cell phone service

d. Find the probability that the number who have only cell phone service exceeds (is more than) 60

e. Find the probability that at least (≥) 60 have only cell phone service

CHAPTER 4: BINOMIAL DISTRIBUTION

Binomial Distribution on the TI 83, 84

2nd Distr binompdf or 2nd Distr binompdf

binomPdf : P(X = value) probability distribution function

binomCdf : P(X ≤ value) cumulative distribution function

TI-89 APPS; 1: FlashApps; highlight Stats/List Editor ENTER F5: Distr

P(X= x)	**binompdf (n,p,x)**
P(X ≤ x)	**binomcdf (n,p,x)**
P(X < x)	**binomcdf (n,p,x − 1)**
P(X > x)	**1 − binomcdf (n,p,x)**
P(X ≥ x)	**1 − binomcdf (n,p,x − 1)**

EXAMPLE B3: *PRACTICE*

http://statusofwomendata.org/explore-the-data/

http://www.theatlantic.com/business/archive/2015/04/women-are-owning-more-and-more-small-businesses/390642/

A report from the Institute for Women's Policy Research states that 71% of small business owners in the US are men. In a group of 24 small business owners, we are interested in the number that are women.

Hint: Make sure that the probability of "success" matches the definition of a "success" for the problem

X = (*description*) ____

____

p = (*description*) ____

____

p = (*value*) ________________ X ~ __________________

a. Find the probability that exactly half the small business owners in this group are women.

b. Find the probability that none are women.

c. Find the probability that less than 10 are women.

d. Find the probability that at least 15 are women.

e. Find the probability more than 6 are women.

f. If we examined many groups of 24 small business owners, on average how many women would we expect to find per group?

g. Find the standard deviation.

NOTE: Recognizing Scientific Notation on your Calculator:

Sometimes probabilities are very small. If the number is very close to 0, your calculator automatically uses scientific notation: 0.000068 = 6.8 x 10-5 appears on the calculator as 6.80000000E-5

4.26710E-6 = 4.26713x 10-6 = 0.00000426713