CHAPTER 8: CONFIDENCE INTERVAL ESTIMATES for Means and Proportions

Introduction: In this chapter we want to find out the value of a parameter for a population. We don't know the value of this parameter for the entire population. If we did already know it, we wouldn't have to do any statistical investigation or calculations. But usually we can't find out all information about the entire population, so the true value of the parameter is usually not known. We will use sample statistics to estimate population parameters

Recall from chapter 2: A parameter is ____

____

If we don't know the value of a population parameter, we can estimate it using a sample statistic.

Recall from chapter 2: A statistic is____

____

Using data from a sample to draw a conclusion about a population is called _________________ statistics.

Two Types of Estimates for population parameters:

1) POINT ESTIMATE: A population parameter can be estimated by one number: the sample statistic. This is called a **point estimate**.

(Statistical theory has identified desirable properties of point estimates, which are studied in more depth in upper level statistics classes. One property usually considered desirable is that a point estimate be "unbiased", meaning that the average of the point estimates from all possible samples would equal the true value of the population parameter.)

2) CONFIDENCE INTERVAL ESTIMATE:

- The population parameter is estimated by an **interval of numbers**, a range of numbers that we believe contains the true (unknown) value of the population parameter.
- Also, we are able to state how confident we are that the interval estimate contains the true value of the parameter.
- This **confidence interval estimate** is built using two items: a point estimate, and margins of error; the margins of error are also called error bounds.

We will use confidence interval estimates based on sample data to estimate

- a population average (mean)
- population proportion

Confidence intervals for means and proportions are symmetric; the point estimate is at the center of the interval. The endpoints of the interval are found as

- point estimate – error bound
- point estimate + error bound.

(For some other parameters, a confidence interval may not be symmetric about the point estimate, moving different distances above and below the point estimate to find the ends of the interval estimate.)

We'll learn by example to calculate the point estimates and the error bounds and what they mean. The last 3 pages these notes has a concise summary of formulas, procedures, and interpretations.

We'll start in class by examining a jar with beads to determine the proportion of beads in the jar that are blue; after we explore the concepts, then we'll move on to the mathematical calculations.

CHAPTER 8 EXAMPLE 1:	CONFIDENCE INTERVAL ESTIMATE for an unknown POPULATION PROPORTION p

a. Statistics and data in this example are based on information from :
http://sf.streetsblog.org/2014/08/15/car-free-households-are-booming-in-san-francisco/
http://en.wikipedia.org/wiki/List_of_U.S._cities_with_most_households_without_a_car

A trend in urban development is to reduce the need for residents to have a car; city neighborhoods are often ranked for "walkability'. In recent studies, the US city with the lowest car ownership rate is New York City; a majority (56%) of households are "car-free" with only 44% of households owning any vehicles. San Jose has the highest car ownership rate of large US cities; only about 6% of households "car-free". San Francisco's percent of "car-free" households has changed rapidly in recent years.

Suppose a recent study of 1200 households in San Francisco showed that 372 households were "car-free". Construct and interpret a 95% confidence interval for the true proportion of households in San Francisco that are "car-free". Use a 95% confidence level.

population parameter: p = ____

____

random variable p′ = ____

We are using sample data to estimate an unknown proportion for the whole population

HOW TO CALCULATE THE CONFIDENCE INTERVAL		
Point Estimate = **p ′**	Confidence Level **CL** is area in the middle	Standard Error
Error Bound = (Critical Value)(Standard Error) **EBP =** $Z_{\alpha/2}\sqrt{\frac{p'q'}{n}}$	Critical Value is $Z_{\alpha/2}$ is the Z value that creates area of CL in the middle; Z ~N(0,1) Use POSITIVE value of Z	$\sqrt{\frac{p'q'}{n}}$
Confidence Interval = Point Estimate + Error Bound Confidence Interval = p ′ ± EBP	invnorm(*area to left*, 0, 1)	

<u>Calculations and interpretation in context of the problem:</u>

CHAPTER 8 EXAMPLE 1 Continued:

b. Our sample proportion was p′=0.31. A city official had thought the percent would be 33%. Based on the confidence interval can we conclude that the true proportion of car-free households in San Francisco is different than 33%. Explain.

c. Can we conclude that more than 25% of San Francisco households are “car-free”. Explain.

d. What does it mean when we say the confidence level is 95% or when we say that we are "95% confident"?

CHAPTER 8 EXAMPLE 2:	CONFIDENCE INTERVAL ESTIMATE for unknown POPULATION MEAN μ when the POPULATION STANDARD DEVIATION σ is KNOWN

A soda bottling plant fills cans labeled to contain 12 ounces of soda. The filling machine varies and does not fill each can with exactly 12 ounces. To determine if the filling machine needs adjustment, each day the quality control manager measures the amount of soda per can for a random sample of 50 cans. Experience shows that its filling machines have a known population standard deviation of 0.35 ounces. In today's sample of 50 cans of soda, the sample average amount of soda per can is 12**.**1 ounces.

a. Construct and interpret a 90% confidence interval estimate for the true population average amount of soda contained in all cans filled today at this bottling plant. Use a 90% confidence level.

X = ____

population parameter: μ = ____

random variable $\overline{X}$ = ____

We are using sample data to estimate an unknown mean (average) for the whole population

HOW TO CALCULATE THE CONFIDENCE INTERVAL for μ		
When σ IS known, use the Standard normal distribution Z ~ N(0,1)		
Point Estimate = $\overline{X}$	Confidence Level **CL** is area in the middle	Standard Error
Error Bound = (Critical Value)(Standard Error) **EBM =** $Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$	Critical Value is $Z_{\alpha/2}$ is the Z value that creates area of CL in the middle; Z ~N(0,1)	
Confidence Interval = Point Estimate + Error Bound Confidence Interval = $\bar{x}$ ± EBM	Use POSITIVE value of Z invnorm$(area\ to\ left, 0, 1)$	$\frac{\sigma}{\sqrt{n}}$

Calculations and interpretation in context of the problem:

CHAPTER 8 EXAMPLE 3:	CONFIDENCE INTERVAL ESTIMATE for unknown POPULATION MEAN μ when the POPULATION STANDARD DEVIATION σ is NOT KNOWN

a. The traffic commissioner wants to know the average speed of all vehicles driving on River Rd. Police use radar to observe the speeds for a sample of 20 vehicles on River Rd. For the vehicles in the sample, the average speed is 31.3 miles per hour with standard deviation 7.0 mph. Construct and interpret a 98% confidence interval estimate of the true population average speed of all vehicles on River Rd. Use a 98% confidence level.

X = ____

population parameter: μ = ____

random variable $\bar{X}$ = ____

We are using sample data to estimate an unknown mean (average) for the whole population

HOW TO CALCULATE THE CONFIDENCE INTERVAL for μ		
When σ is NOT known, use the t distribution with degrees of freedom = sample size – 1 : t with df = n – 1)		
Point Estimate = $\bar{X}$	Confidence Level **CL** is area in the middle	Standard Error
Error Bound = (Critical Value)(Standard Error) **EBM =** $t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$	Critical Value $t_{\alpha/2}$ is the t value that creates an area of CL in the middle; Use t distribution with df = n – 1	$\frac{s}{\sqrt{n}}$
Confidence Interval = Point Estimate + Error Bound Confidence Interval = $\bar{x} \pm$ EBM	Use POSITIVE value of t TI-84: $t_{\alpha/2}$ = invT(*area to left, df*)	

Calculations and interpretation in context of the problem:

b. In Example 3, suppose that you were not given the sample mean and sample standard deviation and instead you were given a list of data for the speeds (in miles per hour) of the 20 vehicles.

19 19 22 24 25 27 28 37 35 30 37 36 39 40 43 30 31 36 33 35

How would you use the data to do this problem?

NOTE: Use of t-distribution requires the underlying population of individual values to be approximately normally distributed. It is OK if this assumption is violated a little, but if the underlying population of individual values has a distribution that differs too much from the normal distribution, then this confidence method is not appropriate, and statisticians would use other techniques that we do not study in Math 10.

CHAPTER 8 EXAMPLE 4: Working Backwards: Finding the Error Bound and Point Estimate if we know the confidence interval:

The average nightly cost of hotel rooms for two popular vacation areas are compared. Large random samples of hotel room costs are collected for each city. The resulting confidence interval estimates are reported in a hotel industry journal.

The 90% confidence interval estimate for the true population average nightly cost of all hotel rooms in Surf City is $134 to $159 per night.

The 90% confidence interval estimate for the true population average nightly cost of all hotel rooms in Ski Village is $123 to $141 per night.

a. Find the point estimate for the true average nightly cost of a hotel room in each city.
Which city has a higher point estimate?

b. Find the error bound for each city. Which city has a smaller margin of error?

c. Based on the confidence intervals only, would it be reasonable to conclude that the true average nightly cost of a hotel rooms are different in Surf City and in Ski Village?

d. Would it be true that 90% of hotel rooms cost between $134 and $159 per night in Surf City and that 90% of hotel rooms cost between $123 and $141 per night in Ski Village?

CHAPTER 8: Confidence Interval for a Proportion: Calculating the Sample Size needed in a Study

Given a desired confidence level and a desired margin of error, how large a sample is needed?

EBP = $Z_{\alpha/2} \sqrt{\frac{p'q'}{n}}$

We know the error bound EBP that we want.

We know the confidence level CL we want, so we can find $Z_{\alpha/2}$ corresponding to the desired CL.

We don't know p' or q' until we do the study, so we will assume for now that p' = q' = ½ = 0.5

Then we can substitute all these values into EBP = $Z_{\alpha/2} \sqrt{\frac{p'q'}{n}}$ and solve for n.

Solving EBP = $Z_{\alpha/2} \sqrt{\frac{p'q'}{n}}$ for n gives n = $\left(\frac{Z_{\alpha/2}}{EBP}\right)^2$ p' q'

Sample Size Formula to determine sample size n needed when estimating a population proportion p	n = $\left(\frac{Z_{\alpha/2}}{EBP}\right)^2 (.25)$	*The 0.25 appears in the formula because we are assuming that* p' = q' = ½ = 0.5 **ALWAYS ROUND UP to the next higher integer**

EXAMPLE 5: **Finding the Sample Size:**

a. Public opinion and political polls often do surveys with a 95% confidence level and 3% margin of error. Find the sample size needed.

n = $\left(\frac{Z_{\alpha/2}}{EBP}\right)^2 (.25) =$

b. Suppose a margin of error of 2% was wanted with a 95% confidence level. Find the sample size needed.

n = $\left(\frac{Z_{\alpha/2}}{EBP}\right)^2 (.25) =$

c. Suppose a margin of error of 3% was wanted with a 90% confidence level. Find the sample size needed.

n = $\left(\frac{Z_{\alpha/2}}{EBP}\right)^2 (.25) =$

d. Suppose a poll uses a sample size of n=100, and a confidence level of 95%.
Estimate the expected error bound using p' = q' = ½ = 0.5

EBP = $Z_{\alpha/2} \sqrt{\frac{p'q'}{n}}$

Note the actual error bound will differ after the study is done because we will know p' and q' and will no longer be estimating that p'=q'=0.5

e. Is the error bound in part d large or small compared to the examples in parts a, b, c?
Explain why this happened.

CHAPTER 8 EXTRA PRACTICE PROBLEMS : CALCULATING CONFIDENCE INTERVALS

Practice examples 6, 7, 8 are based on examples from Chapter 8 Practice and Chapter 9 homework in Introductory Statistics from OpenStax available for download for free at http://cnx.org/content 11562/latest/ *. Practice example 9 is based on information from Bureau of Labor Statistics 2012 cited on 8/31/2015 at* http://fivethirtyeight.com/datalab/how-many-women-earn-more-than-their-husbands/

PRACTICE EXAMPLE 6: A supermarket chain is deciding what produce providers to purchase from. A sample of 20 heads of lettuce is selected to estimate the average weight of the lettuce from this provider. The population standard deviation for the weight is known to be 0.2 pounds. The sample of 20 heads of lettuce had a mean weight of 2.2 pounds with a sample standard deviation of 0.1 pounds.

Calculate and interpret a confidence interval estimate for the true average weight of all heads of lettuce from this provider. Use a 90% confidence level.

PRACTICE EXAMPLE 7: Salaried employees (paid weekly, not hourly) generally do not need to report the number of hours they work per week. A start-up company wants to estimate the average number of hours its engineering employees work per week. For a sample of 10 engineering employees, the hours they report they worked in a typical week were 70, 45, 55, 60, 65, 55, 55, 60, 50, 55

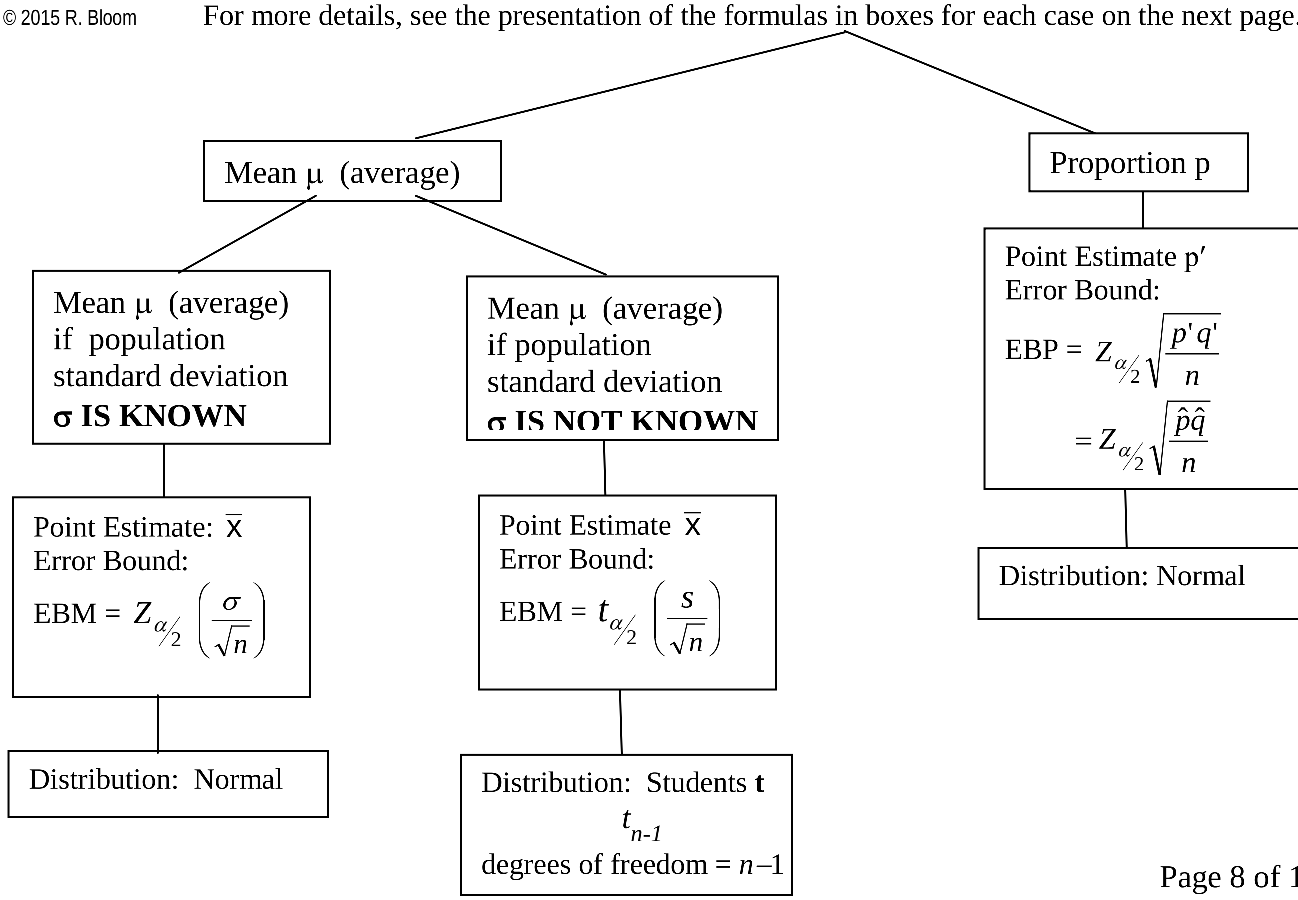
Calculate and interpret a 95% confidence interval estimate of the average number of hours worked per week by all engineering employees at this company. Use a 95% confidence level.

PRACTICE EXAMPLE 8: Suppose a company did a market research survey of 200 randomly selected households and found that in 120 of them the woman made the majority of purchasing decisions for their products. Calculate and interpret a confidence interval estimate for the true proportion of all households in which women make the majority of purchasing decisions for their products. Use a 95% confidence level.

PRACTICE EXAMPLE 9: Suppose a survey of 500 households of married couples where both partners have paid work found that only 29% of the women earn more than their husbands. Calculate and interpret a confidence interval estimate for the true proportion of all such households in which the women earn more than their husbands. Use a 92% confidence level.

CHAPTER 8: FLOW CHART VIEW OF FORMULAS FOR CONFIDENCE INTERVAL ESTIMATES

For more details, see the presentation of the formulas in boxes for each case on the next page.



Confidence Interval for a Proportion p

We want to estimate a population proportion p (binomial probability of success).

Point Estimate $\pm$ Margin of Error (Margin of Error is also called Error Bound)

Point Estimate: Sample Proportion: $p' = \frac{x}{n} = \frac{number\ \ of\ \ successes\ \ in\ \ sample}{total\ \ number\ \ in\ \ sample}$

Error Bound: EBP = (critical value)(standard error) = $Z_{\alpha/2}\sqrt{\frac{p'q'}{n}}$

The critical value depends on the confidence level. $Z_{\alpha/2}$ is the Z value that will put to an area equal to the confidence level (CL) in the middle of standard normal distribution N(0,1)

$Z_{\alpha/2}$ tells us how many "appropriate standard deviations" to enclose about the point estimate, where the "standard error" $\sqrt{\frac{p'q'}{n}}$ is the appropriate standard deviation for a proportion

Confidence Interval: $p' \pm EBP$ which is $p' \pm Z_{\alpha/2}\sqrt{\frac{p'q'}{n}}$

Confidence Interval for a Mean μ when σ is known

We want to estimate the population average μ and we already know the population standard deviation σ.

Point Estimate $\pm$ Margin of Error (Margin of Error is also called Error Bound)

Point Estimate: Sample Average (Sample Mean) $\bar{x}$

Error Bound: EBM = (critical value)(standard error) = $Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

The critical value depends on the confidence level. $Z_{\alpha/2}$ is the Z value that will put to an area equal to the confidence level (CL) in the middle of standard normal distribution N(0,1)

$Z_{\alpha/2}$ tells us how many "appropriate standard deviations" to enclose about the point estimate, where the "standard error" $\frac{\sigma}{\sqrt{n}}$ is the appropriate standard deviation for the sample mean

Confidence Interval: $\bar{x} \pm EBM$ which is $\bar{x} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

Confidence Interval for a Mean μ when σ is NOT known

We want to estimate the population average μ and we do not know the population standard deviation σ.
We use the sample standard deviation s to estimate the population standard deviation σ

Point Estimate $\pm$ Margin of Error (Margin of Error is also called Error Bound)

Point Estimate: Sample Average (Sample Mean) $\bar{x}$

Error Bound: EBM = (critical value)(standard error) = $t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$

The critical value depends on the confidence level. $t_{\alpha/2}$ is the t value that will put to an area equal to the confidence level (CL) in the middle of the student t-distribution with $n - 1$ degrees of freedom

$t_{\alpha/2}$ tells us how many "appropriate standard deviations" we need to move away from the point estimate, where $\frac{s}{\sqrt{n}}$ is an *estimat*e of the standard error ("appropriate standard deviation") for the sample mean

Confidence Interval: $\bar{x} \pm EBM$ which is $\bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$

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Interpreting the Confidence Interval for a PROPORTION (*2 ways to word it*)

We estimate with _____% confidence that the true <u>proportion</u> of the population that *describe the population parameter in the situation of this problem* is between ______ and ________

We estimate with _____% confidence that between _______% and _______% of the population *describe the population parameter in the situation of this problem*

Interpreting the Confidence Interval for a MEAN (average)

We estimate with _____% confidence that the true population <u>average</u> (or mean) *describe the population parameter in the situation of this problem* is between _______ and _________

What is the meaning of the Confidence Level? What does it mean to be CL% confident?

If we took repeated samples and calculated many confidence interval estimates (one for each sample), we expect that CL% of the confidence interval estimates would be "good estimates" that enclose (capture) the true value of the population parameter we are estimating.

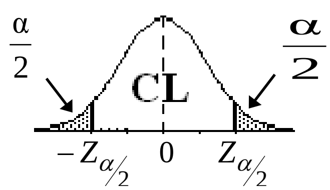
If we took repeated samples), we expect that 100% − CL% of the confidence interval estimates would be "bad estimates" that would NOT enclose (capture) the true value of the population parameter we are estimating.

Note that the confidence interval is about proportions or averages. It is not about individual data values. It does NOT imply that CL% of the data lies within the confidence interval.

Finding the Point Estimate and Error Bound (Margin of Error) if we know the Confidence Interval:
The interval is (lower bound, upper bound)
Point Estimate = (lower bound + upper bound)/2 Error Bound = (upper bound – lower bound)/2

To find Z that puts the area equal to the confidence level "in the middle"

CL tells use the area in the middle
$\alpha = 1 - CL$ is "outside", split equally between both tails
$\frac{\alpha}{2}$ is in one tail.

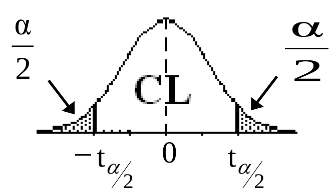


To find $Z_{\alpha/2}$: invnorm($1-\frac{\alpha}{2}$, 0, 1)
OR use invnorm($\frac{\alpha}{2}$, 0, 1) and take absolute value (drop the "−" sign)

Without calculator: Use a standard normal probability table to find Z.

To find t that puts the area equal to the confidence level "in the middle"

CL tells use the area in the middle
$\alpha = 1 - CL$ is "outside", split equally between both tails
$\frac{\alpha}{2}$ is in one tail. df = degrees of freedom = n−1



To find $t_{\alpha/2}$: TI-84+: invT($1-\frac{\alpha}{2}$, df)
OR use invT ($\frac{\alpha}{2}$, df) and take absolute value (drop the "−" sign)

TI-83,83+: Use INVT program; ask instructor to download it to your calculator: PRGM INVT
enter area to the left and df after prompts: area to left is $1-\frac{\alpha}{2}$; (if using $\frac{\alpha}{2}$ as area to left , drop the "−" sign)

Without calculator or if calculator does not have inverse t: Use a student's-t distribution probability table. Value of t is found at the intersection of the column for the confidence level and row for degrees of freedom

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