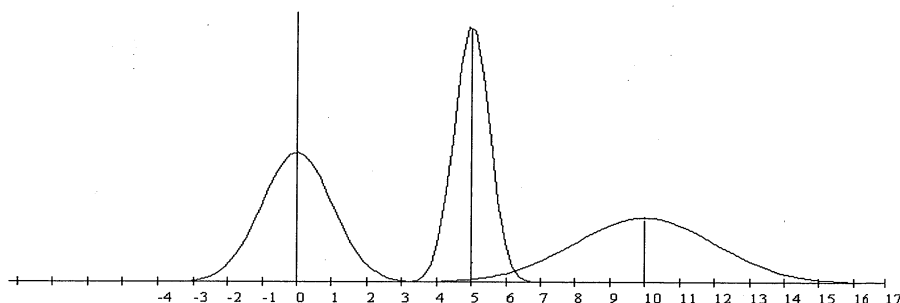
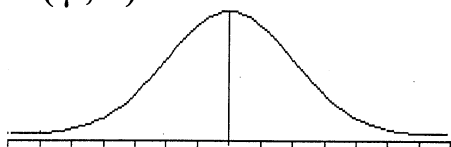


CHAPTER 6: THE NORMAL PROBABILITY DISTRIBUTION

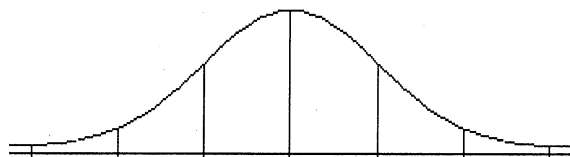
Exploring the Normal Probability Distribution

$$X \sim N(\mu, \sigma)$$



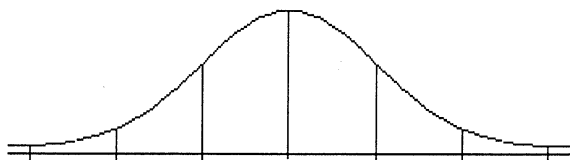
- a. The normal distributions above have means $\mu = 5$, $\mu = 10$ and $\mu = 0$.
Identify which graph has which mean.
- b. The normal distributions above have standard deviations of $\sigma = 0.5$, $\sigma = 1$ and $\sigma = 2$.
Identify graph has which standard deviation.

The following are true for ANY normal probability distribution:



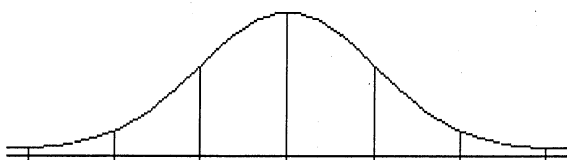
$$P(\mu - \sigma < X < \mu + \sigma) = \underline{\hspace{2cm}}$$

The probability for the interval within ± 1 standard deviations from the mean ($\mu \pm 1\sigma$) is $\underline{\hspace{2cm}}$



$$P(\mu - 2\sigma < X < \mu + 2\sigma) = \underline{\hspace{2cm}}$$

The probability for the interval within ± 2 standard deviations from the mean ($\mu \pm 2\sigma$) is $\underline{\hspace{2cm}}$



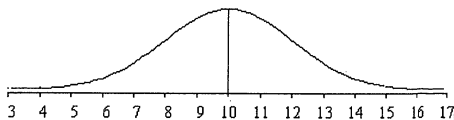
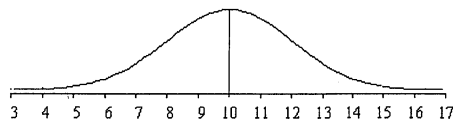
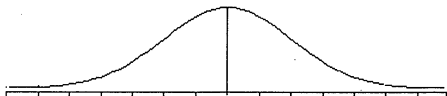
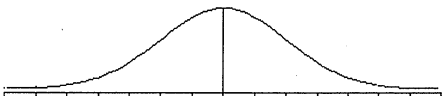
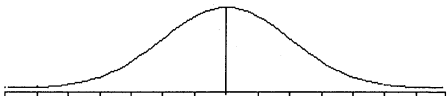
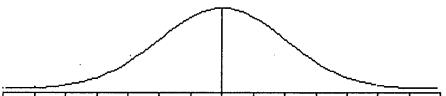
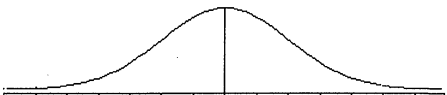
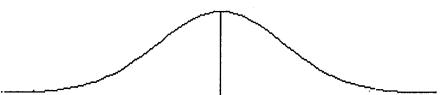
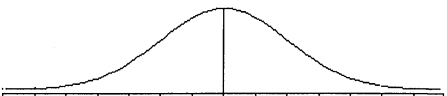
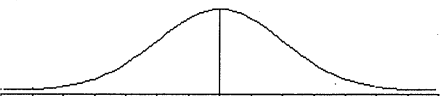
$$P(\mu - 3\sigma < X < \mu + 3\sigma) = \underline{\hspace{2cm}}$$

The probability for the interval within ± 3 standard deviations from the mean ($\mu \pm 3\sigma$) is $\underline{\hspace{2cm}}$

CHAPTER 6: THE NORMAL PROBABILITY DISTRIBUTION: Calculating Probabilities

- Shade the area representing the probability.
- Label the value of the mean μ and the X values along the horizontal axis
- Label area above or inside the graph.
- Use your calculator: 2nd Distr normalcdf to find the probabilities if given X
or use 2nd Distr invnorm to find X if given the probability (invnorm requires input of area to the left)
- Write the answer as a mathematical probability statement using correct notation.

A botanist is studying a certain type of plant. Suppose that it takes an average of 10 days for seeds to germinate (sprout) with standard deviation 2 days. X = germination time for one seed

FINDING AREA IN BETWEEN $X \sim N(10, 2)$ normalcdf(lower, upper, μ , σ)	 $P(7 < X < 9)$	 $P(11 < X < 14)$
FINDING AREA TO THE RIGHT $X \sim N(10, 2)$ normalcdf(lower, 1E99, μ , σ) normalcdf(lower, 10^99, μ , σ)	 $P(X > 9) =$	 $P(X \geq 13) =$
FINDING AREA TO THE LEFT $X \sim N(10, 2)$ normalcdf((-) 1E99, upper, μ , σ) normalcdf((-) 10^99, upper, μ , σ) use (-) key for negative number	 $P(X \leq 8) =$	 $P(X < 11.5) =$
FINDING PERCENTILES: FINDING X when we know the AREA TO THE LEFT $X \sim N(10, 2)$ invnorm (area to the left, μ , σ)	 25 th percentile	 80 th percentile
FINDING X when we know the AREA TO THE RIGHT $X \sim N(10, 2)$ First find the area to the left Then use the calculator invnorm (area to the left, μ , σ)	 $P(X \geq k) = 0.60$ Find k	 $P(X > k) = 0.30$ Find k

CHAPTER 6: THE NORMAL PROBABILITY DISTRIBUTION: Calculating Probabilities

FINDING TWO X VALUES WHEN WE KNOW THE AREA IN THE MIDDLE

$$X \sim N(10, 2)$$

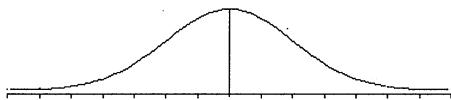


The middle 90% of values are between _____ and _____
 $P(c \leq X \leq d) = 0.90$ Find c and d:

1. First figure out the area to the left of the lower bound.



2. Then figure out area to the left of the upper bound



3. Then use the calculator with `invnorm` TWICE to find the lower and upper bounds:

lower bound =

$$\text{invnorm} \left(\begin{array}{l} \text{area to left of} \\ \text{lower bound} \end{array} , \mu , \sigma \right)$$

upper bound =

$$\text{invnorm} \left(\begin{array}{l} \text{area to left of} \\ \text{upper bound} \end{array} , \mu , \sigma \right)$$

CHAPTER 6: THE NORMAL PROBABILITY DISTRIBUTION

Exploring Connections between Percentiles and Z-Scores

Two students, John and Ali, from different high schools, wanted to determine who had the higher GPA when compared relative to his classmates at his school.

Suppose we also know that GPAs at each high school follow a normal distribution.

Dean, S., & Illowsky, B. Collaborative Statistics, Ch. 2(10/27/2008) Connexions <http://cnx.org/content/m10522/>

Student	Student's GPA	School Mean GPA	School Standard Deviation
John	2.85	3.0	0.7
Ali	77	80	10

Which student's GPA would be considered better (John or Ali) when each is compared relative to the other students at his own school?

X = GPA of a student at John's school: $X \sim$ _____

Y = GPA of a student at Ali's school: $Y \sim$ _____

Calculate and compare Z-scores (we did this in chapter 2; chapter 6 repeats this concept)

John	Ali
------	-----

Compare percentiles for John and Ali by calculating probabilities using their actual GPAs

John: Find $P(X \leq 2.85)$ using $X \sim N(3.0, 0.7)$	Ali: Find $P(Y \leq 77)$ using $Y \sim N(80, 10)$
--	---

Compare percentiles for John and Ali by calculating probabilities using their z-scores

John: Find $P(Z < -0.2143)$ using $Z \sim N(0, 1)$	Ali: Find $P(Z < -0.30)$ using $Z \sim N(0, 1)$
--	---

CONCLUSION: _____'s GPA of _____ would be considered better than _____'s GPA of _____ when each is compared relative to other students at his own school.

NOTE

- Using z-scores and percentiles lead to the equivalent results and the same conclusion..
- Using the specified normal distribution $X \sim N(3.0, 0.7)$ and $Y \sim N(80, 10)$ give the same probabilities and percentiles as are obtained by using the Z score with the standard normal distribution, $Z \sim N(0, 1)$

CHAPTER 6: THE NORMAL PROBABILITY DISTRIBUTION: PRACTICE

- Draw the graph and label all pertinent values.
- Label the value of the mean μ and the X values along the horizontal axis
- Label areas above or inside the graph.
- Use your calculator to find the requested values. Write your calculator commands.
- Write the answer as a mathematical probability statement using correct notation.

PRACTICE EXAMPLE N: In California, the wine and fruit industries invest significant resources in the study of growing grapes in order to improve crop yield and quality.

Suppose that a certain type of grape, when ripe, has diameter that is normally distributed with a mean of 1.8 cm with a standard deviation of 0.3 cm.

- Find the probability that a randomly selected grape has diameter more than 2.2 cm
- Find the probability that a randomly selected grape has diameter less than 1.6 cm.
- The middle 80% of this type of grapes have diameters between _____ and _____ cm.
- Find the IQR (Interquartile Range) of diameters for this type of grape.
- The largest 15% of grapes have diameters of at least _____ cm.
- Find the probability that a randomly selected grape has diameter between 1.5 and 2.0 cm using each method as instructed below:
 - Use $X \sim N(1.8, 0.3)$ when using normalcdf on your calculator
 - Find z-scores for $x = 1.5$ and $x = 2.0$ and use $Z \sim N(0,1)$ standard normal distribution when using normalcdf on your calculator