

Answers to Practice Problems from Section 2.1
Class on Tues 10/24

$$\textcircled{1} f(x) = \sqrt[5]{4-x^2} = (4-x^2)^{1/5} \quad \text{Domain } [-2, 2]$$

$$f'(x) = \frac{1}{5} (4-x^2)^{-4/5} (-2x)$$

$$\text{so } f'(x) = -\frac{2}{5} \frac{x}{(4-x^2)^{4/5}} \quad \text{or equivalently } -\frac{2}{5} \frac{x}{\sqrt[5]{(4-x^2)^4}} \text{ if simplified}$$

$$f'(x) = 0 \text{ if numerator} = 0 \text{ so } f'(x) = 0 \text{ when } x = 0$$

$$f'(x) \text{ does not exist if denominator} = 0$$

$$\text{so } 4-x^2 = 0 \text{ gives } x^2 = 4$$

$$\text{so } x = -2, 2$$

$$\therefore \text{Critical Values are } x = -2, 0, 2$$

$$\textcircled{2} f(x) = -x^4 + 4x^3 + 20x^2 - 60$$

$$f'(x) = -4x^3 + 12x^2 + 40x \leftarrow \begin{array}{l} \text{factor to solve OR} \\ \text{USE PLYSMLT2} \\ \text{POLYSMLT} \end{array}$$

$$= -4(x^3 - 3x^2 - 10x)$$

$$= -4x(x^2 - 3x - 10)$$

$$= -4x(x-5)(x+2)$$

$$f'(x) = -4x(x-5)(x+2) = 0$$

at $x = -2, 5, 0$ critical values

Interval	$(-\infty, -2)$ $x < -2$	$(-2, 0)$ $-2 < x < 0$	$(0, 5)$ $0 < x < 5$	$(5, \infty)$ $x > 5$
Critical Value	-2	0	5	
Test Point	$x = -3$	$x = -1$	$x = 1$	$x = 10$
	$f'(-3) > 0$	$f'(-1) < 0$	$f'(1) > 0$	$f'(10) < 0$
	$f' > 0$	$f' < 0$	$f' > 0$	$f' < 0$
	f increasing on $(-\infty, -2)$	f decreasing on $(-2, 0)$	f increasing on $(0, 5)$	f decreasing on $(5, \infty)$
	relative Maximum at $x = -2$	relative Minimum at $x = 0$	relative Maximum at $x = 5$	
	$f(-2) = -28$	$f(0) = -60$	$f(5) = 315$	
	$(-2, -28)$	$(0, -60)$	$(5, 315)$	

- Answers:
- f is increasing on $(-\infty, -2)$ and $(0, 5)$
 f is increasing for $x < -2$ and $0 < x < 5$
 - f is decreasing on $(-2, 0)$ and $(5, \infty)$
 f is decreasing for $-2 < x < 0$ and $x > 5$
 - Relative minimum $(0, -60)$
 - Relative maxima $(-2, -28)$ and $(5, 315)$

$$\textcircled{3} f(x) = \sqrt[3]{(2x-5)^2} = (2x-5)^{2/3}$$

$$f'(x) = \frac{2}{3} (2x-5)^{-1/3} (2) = \frac{4}{3} \frac{1}{\sqrt[3]{2x-5}}$$

$f'(x)$ does not exist when $2x-5=0$

so $x = \frac{5}{2} = 2.5$ is the critical value

Intervals	$x < 2.5$	$x > 2.5$
Critical values	2.5	
Test points	$x=0$	$x=3$
	$f'(0) < 0$	$f'(3) > 0$
	f decreasing for $x < 2.5$	f increasing for $x > 2.5$
	$\uparrow$ relative minimum at $x=2.5$ $f(2.5) = 0$ $(2.5, 0)$	

Answers

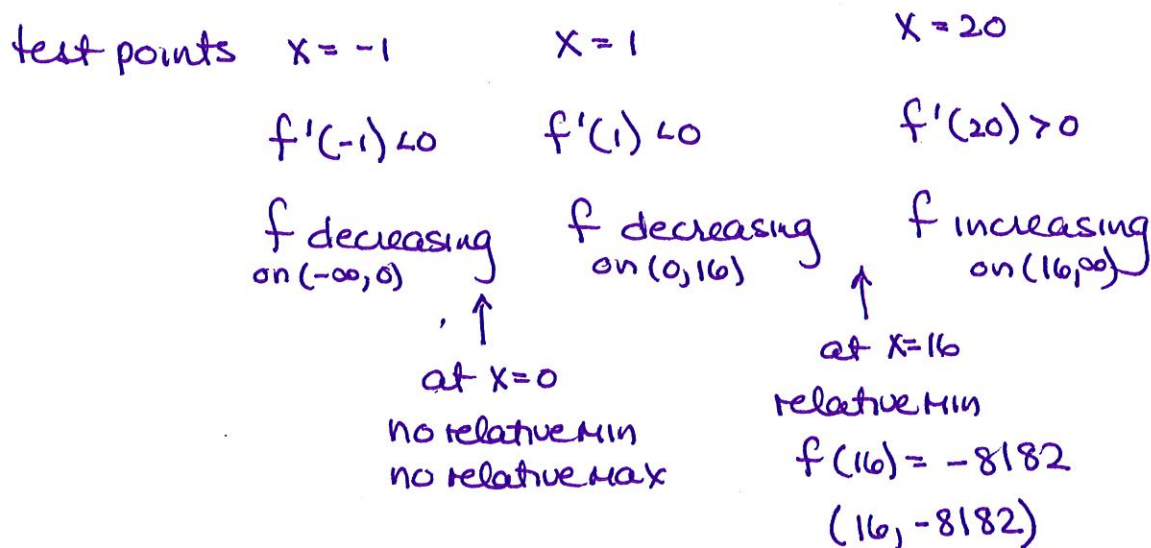
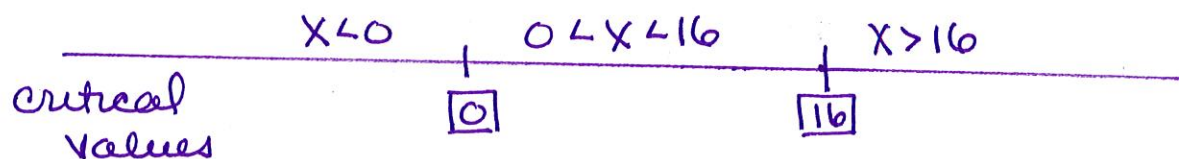
- f is increasing on $(2.5, \infty)$ (for $x > 2.5$)
- f is decreasing on $(-\infty, 2.5)$ (for $x < 2.5$)
- f has a relative minimum at $(2.5, 0)$
- There is no relative maximum

④ $f(x) = x^4 - 18x^3 + 10$

$$f'(x) = 4x^3 - 48x^2$$

$$= 4x^2(x-16)$$

$$f'(x) = 4x^2(x-16) = 0 \text{ at } x=0, x=16$$



Answers: a) f is decreasing on $(-\infty, 0)$ and $(0, 16)$

f is decreasing for $x < 0$ and $0 < x < 16$

OR also OK: $(-\infty, 16)$ or $x < 16$

b) f is increasing for $x > 16$, interval $(16, \infty)$

c) f has a relative minimum at $(16, -8182)$

d) f has no relative maxima