Answers to Practice Problems from Section 2.1

Class on Tues 10/24

① $f(x) = \sqrt[5]{4-x^2} = (4-x^2)^{1/5}$ Domain $[-2,2]$

$f'(x) = \frac{1}{5}(4-x^2)^{-4/5}(-2x)$

so $f'(x) = -\frac{2}{5}\frac{x}{(4-x^2)^{4/5}}$ or equivalently $-\frac{2}{5}\frac{x}{\sqrt[5]{(4-x^2)^4}}$ if simplified

$f'(x) = 0$ if numerator $= 0$ so $f'(x) = 0$ when $x = 0$

$f'(x)$ does not exist if denominator $= 0$

so $4 - x^2 = 0$ gives $x^2 = 4$

so $x = -2, 2$

$\therefore$ Critical Values are $x = -2, 0, 2$

② $f(x) = -x^4 + 4x^3 + 20x^2 - 60$

$f'(x) = -4x^3 + 12x^2 + 40x$ ← factor to solve OR use PLYSMLT2 POLYSMLT

$= -4(x^3 - 3x^2 - 10x)$

$= -4x(x^2 - 3x - 10)$

$= -4x(x-5)(x+2)$

$f'(x) = -4x(x-5)(x+2) = 0$

at $x = -2, 5, 0$ critical values

Interval	$(-\infty, -2)$ $x < -2$		$(-2, 0)$ $-2 < x < 0$		$(0, 5)$ $0 < x < 5$		$(5, \infty)$ $x > 5$
Critical Value		$\boxed{-2}$		$\boxed{0}$		$\boxed{5}$	
Test Point	$x = -3$		$x = -1$		$x = 1$		$x = 10$
	$f'(-3) > 0$		$f'(-1) < 0$		$f'(1) > 0$		$f'(10) < 0$
	$f' > 0$		$f' < 0$		$f' > 0$		$f' < 0$
	f increasing on $(-\infty, -2)$		f decreasing on $(-2, 0)$		f increasing on $(0, 5)$		f decreasing on $(5, \infty)$
		relative maximum at $x = -2$ $f(-2) = -28$ $(-2, -28)$		relative minimum at $x = 0$ $f(0) = -60$ $(0, -60)$		relative maximum at $x = 5$ $f(5) = 315$ $(5, 315)$	

Answers:

a) f is increasing on $(-\infty, -2)$ and $(0, 5)$

f is increasing for $x < -2$ and $0 < x < 5$

b) f is decreasing on $(-2, 0)$ and $(5, \infty)$

f is decreasing for $-2 < x < 0$ and $x > 5$

c) Relative minimum $(0, -60)$

d) Relative maxima $(-2, -28)$ and $(5, 315)$

③ $f(x) = \sqrt[3]{(2x-5)^2} = (2x-5)^{2/3}$

$f'(x) = \frac{2}{3}(2x-5)^{-1/3}(2) = \frac{4}{3}\,\frac{1}{\sqrt[3]{2x-5}}$

$f'(x)$ does not exist when $2x-5=0$

so $x = \frac{5}{2} = 2.5$ is the critical value

intervals	$x<2.5$		$x>2.5$
critical values		$\boxed{2.5}$	
test points	$x=0$		$x=3$
	$f'(0)<0$		$f'(3)>0$
	f decreasing for $x<2.5$		f increasing for $x>2.5$

relative MINIMUM
at $x=2.5$
$f(2.5)=0$
$(2.5, 0)$

Answers

a) f is increasing on $(2.5, \infty)$ (for $x>2.5$)

b) f is decreasing on $(-\infty, 2.5)$ (for $x<2.5$)

c) f has a relative MINIMUM at $(2.5, 0)$

d) there is no relative MAXIMUM

④ $f(x) = x^4 - 18x^3 + 10$

$f'(x) = 4x^3 - 48x^2$

$= 4x^2(x-16)$

$f'(x) = 4x^2(x-16) = 0$ at $x = 0,\ x = 16$

$x < 0$ | $0 < x < 16$ | $x > 16$

critical values $\boxed{0}$ $\boxed{16}$

test points $x = -1$ $\quad x = 1$ $\quad x = 20$

$f'(-1) < 0$ $\quad f'(1) < 0$ $\quad f'(20) > 0$

f decreasing on $(-\infty, 0)$ $\quad f$ decreasing on $(0, 16)$ $\quad f$ increasing on $(16, \infty)$

at $x = 0$
no relative min
no relative max

at $x = 16$
relative min
$f(16) = -8182$
$(16, -8182)$

Answers: a) f is decreasing on $(-\infty, 0)$ and $(0, 16)$

f is decreasing for $x < 0$ and $0 < x < 16$

OR also OK: $(-\infty, 16)$ or $x < 16$

b) f is increasing for $x > 16$, interval $(16, \infty)$

c) f has a relative minimum at $(16, -8182)$

d) f has no relative maxima