$\qquad$
Instructions: Write complete solutions to the following problems in the space provided. Be sure to supply all the necessary steps that lead to your answers

1. Estimate the volume of the solid that lies below the surface $z=x y$ and above the following rectangle.

$$
\mathfrak{R}=\{(x, y) \mid 2 \leq x \leq 8,4 \leq y \leq 8\}
$$

a. Use a Riemann sum with $\mathrm{m}=3, \mathrm{n}=2$, and take the sample point to be the upper right corner of each square.

Ans. 1008
b. Use the Midpoint Rule to estimate the volume of the solid.

Ans. 720
2. A $20-\mathrm{ft}-\mathrm{by}-30-\mathrm{ft}$ swimming pool is filled with water. The depth is measured at $5-\mathrm{ft}$ intervals, starting at one corner of the pool, and the values are recorded in the table. Estimate the volume of water using the Midpoint Rule with $\mathrm{m}=2$ and $\mathrm{n}=3$.

| Ans. $3300 \mathrm{ft}^{3}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{x} \backslash \mathrm{y}$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
| 0 | 2 | 3 | 4 | 6 | 7 | 8 | 8 |
| 5 | 2 | 3 | 4 | 6 | 8 | 9 | 8 |
| 10 | 2 | 4 | 6 | 8 | 0 | 12 | 10 |
| 15 | 2 | 3 | 4 | 5 | 6 | 7 | 7 |
| 20 | 2 | 2 | 2 | 2 | 3 | 4 | 4 |

3. A contour map is shown for a function $f$ on the square $\mathfrak{R}=[0,2] \times[0,2]$.
a. Use the Midpoint Rule with $\mathrm{m}=\mathrm{n}=2$ to estimate the value of

$$
\iint_{\Re} f(x, y) d A
$$

Ans. 119

b. Estimate the average value of $f$.

Ans. 29.8
4. Estimate the volume of the solid that lies below the surface $z=3 x+2 y^{2}$ and above the rectangle $\mathfrak{R}=[0,2] \times[0,4]$
a. Use a Riemann sum with $\mathrm{m}=\mathrm{n}=2$ and choose the sample points to be lower right corners.

Ans. 68
b. Use the Midpoint Rule to estimate the volume in part a.

Ans. 104

