Application of the Definite Integral H. Bourgoub

Problem

Set up integrals for the area of the region bounded by the given curves

$x = y^2 - 1,\ \text{and}\ y = x - 1$.

The curves intersect at $(0,-1),\ \text{and}\ (3,2)$, and the parabola can be expressed as $y = \pm\sqrt{x+1}$

Solution 1. Using two vertical strips.

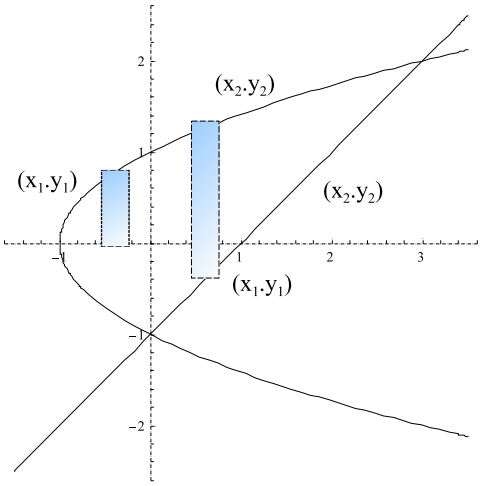
$$\Delta A = \quad \Delta A_1 \quad + \quad \Delta A_2$$

$$\Delta A = 2(y_2 - 0)\Delta x + (y_2 - y_1)\Delta x$$

$$\Delta A = 2\left(\sqrt{x+1}) - 0\right)\Delta x + \left(\sqrt{x+1} - (x-1)\right)\Delta x$$

$$-1 \le x \le 0 \qquad\qquad 0 \le x \le 3$$

$$\int_0^1 \left(2(\sqrt{x+1}) - 0)\right)dx + \int_0^3 \left(\sqrt{x+1} - (x-1)\right)dx$$



Solution 2. Using one horizontal strip.

$$\Delta A = (x_2 - x_1)\Delta y$$

$$\Delta A = \left((y^2 - 1) - (y+1)\right)\Delta y$$

$$-1 \le y \le 2$$

$$\int_{-1}^{2} \left((y^2 - 1) - (y+1)\right)dy =$$