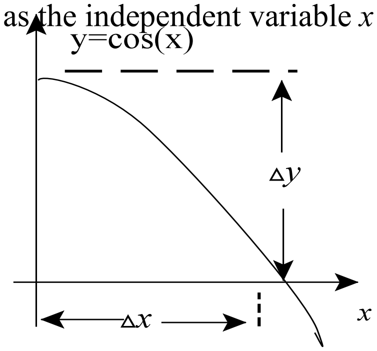
Average Rate of Change of Values of Functions

The average rate of change of the values of a function $y = f(x)$, as the independent variable x varies over the interval $[x_1, x_2]$ is denoted by the quotient



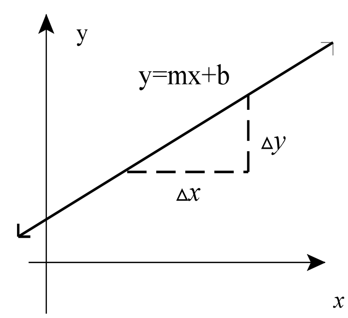
$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

For example the average rate of change of $f(x) = \cos x$ ov$[0, \pi/2]$rval is given b

$$\frac{\Delta y}{\Delta y} = \frac{\cos(\frac{\pi}{2}) - \cos(0)}{\frac{\pi}{2} - 0} = -\frac{1}{\pi}$$

Where the change in x is $\Delta x = \frac{\pi}{2} - 0$

The Rate of Change of Linear Functions.



The rate of change of a linear function $f(x) = mx + b$ equals the slope of the line.

Average rate of change of f = m.

Since m is constant, it is referred to as constant rate of change, which implies the slope of a line is independent of the points used to compute it. We express the rate of change of y with respect to x by

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

the familiar slope formula.

Example: **Application of constant rate of change**.

A Ferris Wheel with 100 ft diameter completes one revolution in 12 minutes. Assuming the wheel rotates at constant speed, and that a rider boards at ground level.

a. Find an expression for the angle in standard position as a function of time.
b. Use the sine function to find an expression for the height of a rider above the ground as a function of time.

Solution

Note that the angle of rotation is formed by the positive t axis and a line that passes through a point of at the position of the rider on the Ferris Wheel. See figure below.
Since the wheel rotates at a constant angular speed ω, the rate of change of the angle of rotation θ with respect to time t is constant, and that the relation between θ and t is linear.

This gives, $\theta(t) = \dfrac{\Delta\theta}{\Delta t} t + t_0$, where θ_0 is the value of θ when t = 0.

Using the constant angular speed $\omega = \dfrac{\Delta\theta}{\Delta t}$ the equation becomes $\theta(t) = \omega t + t_0$, where θ is measured in radians.

Now, $(0, -\pi/2)$ and $(6, \pi/2)$ are two points that satisfy the linear relation above. The rate of change of θ with respect to time is given by

$$\frac{\Delta\theta}{\Delta t} = \frac{\pi/2 - (-\pi/2)}{6-0} = \frac{\pi}{6}$$

This is the slope of our linear relation,

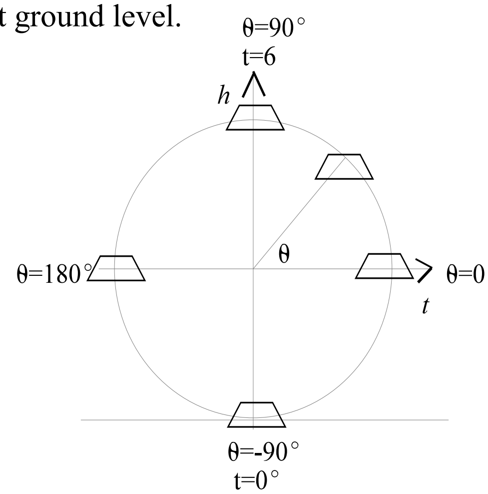
and the initial value is $\theta_0 = -\pi/2$ where the rider boards at ground level.

Hence the relation is $\theta = (\pi/6)t - \pi/2$

Using the definition of $y = \sin x$ as a circular function and a vertical shift equal the radius of the wheel, we get

$$h(\theta) = 50 + 50\sin(\theta)$$

$$h(t) = 50 + 50\sin\left(\frac{\pi}{6}t - \frac{\pi}{2}\right) \qquad \text{(b)}$$



Wheel radius is 100 ft