$$x^2 y'' + xy' + (x^2 - v^2)y = 0 \qquad (1)$$

Bessel Equation

A Bessel Equation is an ordinary differential equation that arises in many applications in physics and engineering in areas of mechanical vibrations, heat conduction and wave phenomena.

This equation can solved by the method of Frobenius , and it is a routine exercise to substitute a power series definition of the solution into the equation and produce the coefficients for the solution.

For the case when n = r = v. The solution is given by

$$J_n(x) = x^n \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+n} m!(n+m)!} \qquad (2)$$

This is a convergent power series for all x and the solutions described by (2) are called Bessel Functions of the first kind and order n.

The graphs of , J_1, J_2, J_2, J_4 are shown in the figure above.

We will study the Method of Frobenius to solve Ordinary differential equations, (ODE) with polynomial coefficients such as Bessel Equations.