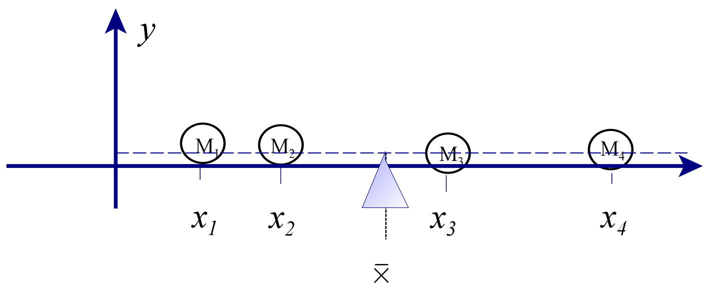
Center of Mass
Mass system placed along a straight line



Static equilibrium
A mass system is said to be in a static equilibrium if the sum of all moments of masses in the system about the fulcrum, $\bar{x}$ is zero. (no movement/rotation)

Suppose the masses in the figure are attached to a thin rod with negligible mass. The center of mass of the system is at a point called the fulcrum, the point at which the system is in static equilibrium

The moment of mass of mass mi, about the vertical line through the fulcrum is defined to be

$$M_i = m_i(x_i - \bar{x}) \quad \text{See the figure.}$$
$$M_i = \text{(mass) (arm)}$$

So, $$m_1(x_1 - \bar{x}) + m_2(x_2 - \bar{x}) + m_3(x_3 - \bar{x}) + m_4(x_4 - \bar{x}) = 0$$

If we isolate and solve for $\bar{x}$, we get

$$\bar{x} = \frac{\sum_{i=1}^{4} x_i m_i}{\sum_{i=1}^{4} m_i}$$

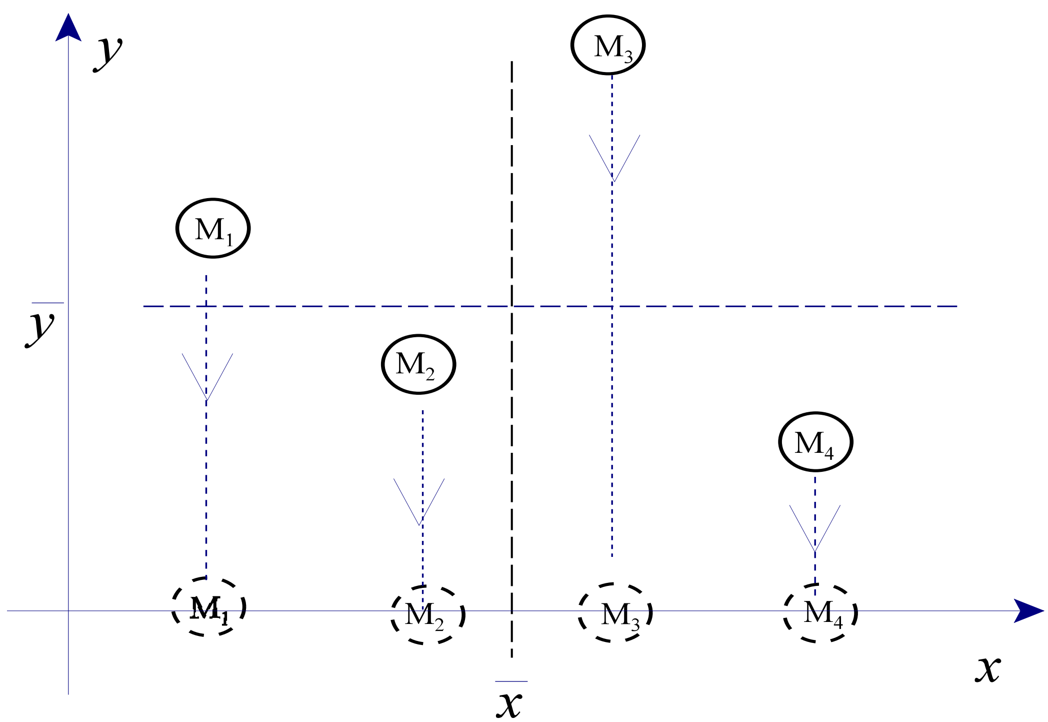
Mass System placed on a plane.

Suppose the masses in the figure are attached to a thin plate with negligible mass. The center of mass of the system is at a point called the fulcrum, the point at which the system is in static equilibrium

To find the fulcrum, the center of mass, of the system $(\bar{x}, \bar{y})$ find a vertical line though $\bar{x}$ so that the total moments of mass about this line is zero. Since the arm for each moment of mass about the line $x = \bar{x}$ in the system is independent of the y coordinate of the position of each mass, the same can be accomplished if the masses are translated downward and placed on the x axis.

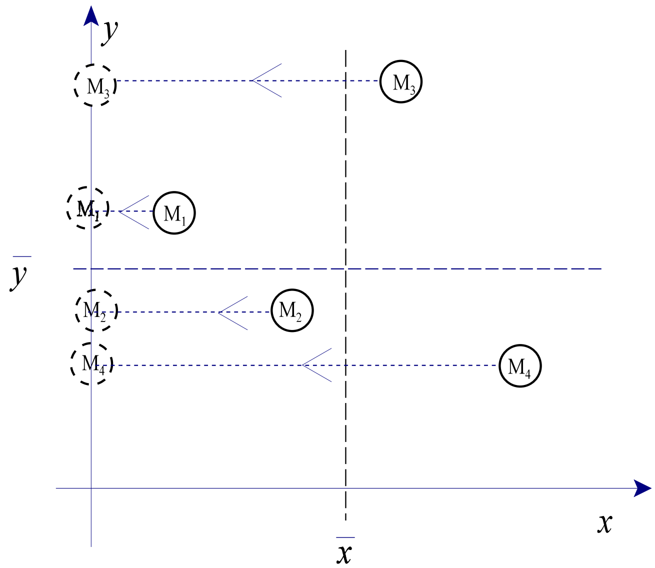
This, in tern, allows us to use the expression derived for $\bar{x}$, for masses placed along a straight line.

see the figure below



$$\bar{x} = \frac{\sum_{i=1}^{4} x_i m_i}{\sum_{i=1}^{4} m_i}$$

And similarly, translating the masses parallel to the x axis allows to compute $\bar{y}$



$$\bar{y} = \frac{\sum_{i=1}^{4} y_i m_i}{\sum_{i=1}^{4} m_i}$$

The numerator of the expression for both the x and y co-ordinates of the center of mass of the mass system is the sum of the moments of the masses about the x-axis and the y-axis respectively.

Summary:
To find the moment of mass of a mass system about the an axis, add up the moments of mass of all the masses about the axis. We say that the moment of mass about an axis is additive.

To find the x or y coordinate of a center of mass of a mass system use

$$\bar{x} = \frac{M_y}{M}$$

$$\bar{y} = \frac{M_x}{M}$$

Where M_x, M_y, M are the moment of mass about the x axis, the moment of mass about the y axis and the total mass respectively.