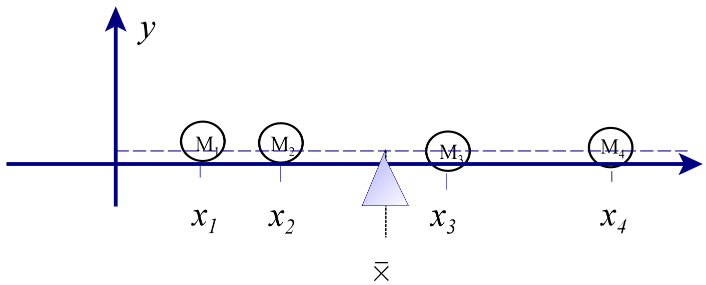
Center of Mass

I Mass system placed along a straight line



Static equilibrium

A mass system is said to be in a static equilibrium if the sum of all moments of mass in the system about the fulcrum, $\bar{x}$ is zero. (no movement/rotation)

Suppose the masses in the figure are attached to a thin rod with negligible mass. The center of mass of the system is at a point called the fulcrum, the point at which the system is in static equilibrium

The moment of mass of mass m_i about the vertical line through the fulcrum is defined to be

$$M_i = m_i(x_i - \bar{x}) \qquad \text{See the figure above.}$$
$$M_i = \text{(mass) (arm)}$$

So, $$m_1(x_1 - \bar{x}) + m_2(x_2 - \bar{x}) + m_3(x_3 - \bar{x}) + m_4(x_4 - \bar{x}) = 0$$

If we isolate and solve for $\bar{x}$, we get

$$\bar{x} = \frac{\sum_{i=1}^{4} x_i m_i}{\sum_{i=1}^{4} m_i}.$$

Problem

Find the center of mass of the mass system: 10g at x=5, 15g at x=9, 25 g at x=6.

Mass System placed on a plane.

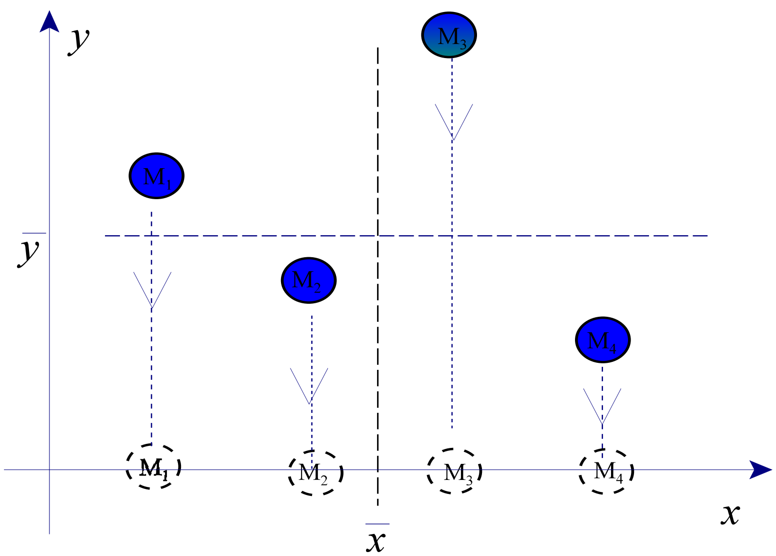
Suppose the masses in the figure are attached to a thin plate with negligible mass. The center of mass of the system is at a point called the fulcrum, the point at which the system is in static equilibrium

To find the fulcrum, the center of mass, of the system $(\bar{x}, \bar{y})$ find a vertical line through $\bar{x}$ so that the total moments of mass about this line is zero. Since the arm for each moment of mass about the line $x = \bar{x}$ in the system is independent of the y coordinate of the position of each mass, the same can be accomplished if the masses are translated downward or upward depending on the position and placed on the x axis.

This, in tern, allows us to use the expression derived for $\bar{x}$, for masses placed along a straight line.

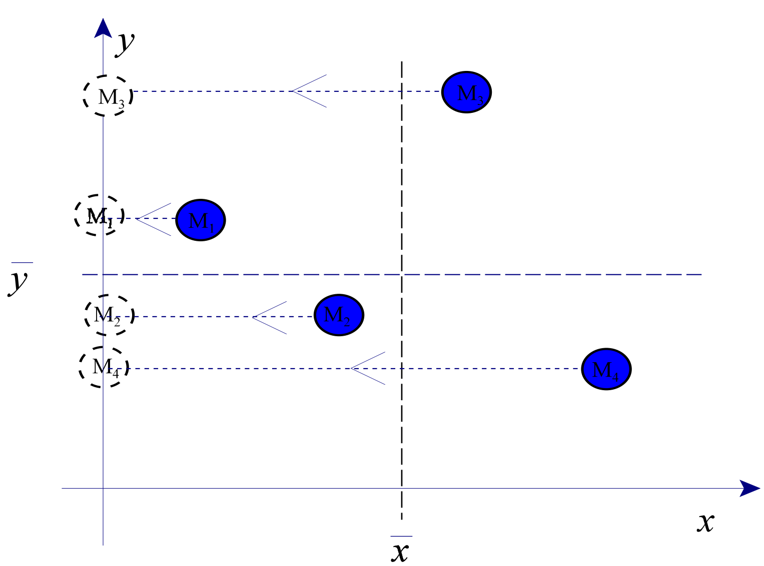
see the figure below

$$\bar{x} = \frac{\sum_{i=1}^{4} x_i m_i}{\sum_{i=1}^{4} m_i}$$



And similarly, translating the masses parallel to the x axis allows to compute $\bar{y}$

$$\bar{y} = \frac{\sum_{i=1}^{4} y_i m_i}{\sum_{i=1}^{4} m_i}$$



The numerator of the expression for both the x and y coordinates of the center of mass of the mass system is the sum of the moments of masses about the y-axis and the x-axis respectively.

Summary:
To find the moment of mass of a mass system about an axis, add up the moments of mass of all masses in the system about that axis. We say that the moment of mass about an axis is additive.

To find the x or y coordinate of a center of mass of a mass system use

$$\bar{x} = \frac{M_y}{M}$$

$$\bar{y} = \frac{M_x}{M}$$

Where M_x, M_y, M are the moment of mass about the x axis, the moment of mass about the y axis and the total mass respectively.

Problems.

1. Find the center of mass of the mass system:
10g at (3,4), 8g at (-2,-1), 15g at (5,1), and 5g at (-3,2).

2. Find the center of mass of the lamina shown in the figure. Assume mass density equal 1. The laminas are rectangular plates two each 2x2 and one 4x2.