

Definition

The composition of two functions f and g is a function defined by the equation

$$(f \circ g)(x) = f[g(x)]$$

The action of the composition of two functions f and g is obtained by first applying g to x then applying f to the image $g(x)$.

The Domain of $f \circ g$

All x in the domain of g such that $g(x)$ is in the domain of f .

Example: Find the domain and range of $F(x) = \frac{1}{x^2 - 1}$

If we consider f as a composition of two functions

$$g(x) = x^2 - 1, f(x) = \frac{1}{x}, \text{ and } F(x) = f(g(x)) = \frac{1}{x^2 - 1}$$

then the domain of g , the inside function, is all real numbers and the domain of f , the outside function, is all real numbers except zero. So in the domain of the composition F , we include all the elements in the domain set of g except those which are mapped to zero. Those x coordinates that corresponds to zero namely $x = \pm 1$.

Hence the domain of F is all real numbers except $x = \pm 1$.

Problems: Find the domain and range of $f \circ g$

1. $f = \{(0,1), (1,2), (2,3), (4,4)\}, g = \{(0,0), (1,2), (3,3), (4,4)\}$

Domain:

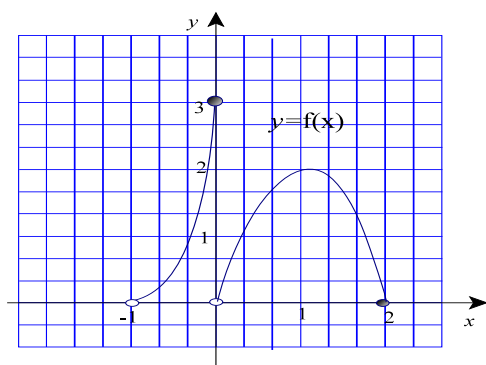
Range:

2. $f(x) = \sqrt{x}, g(x) = x^2 - x$

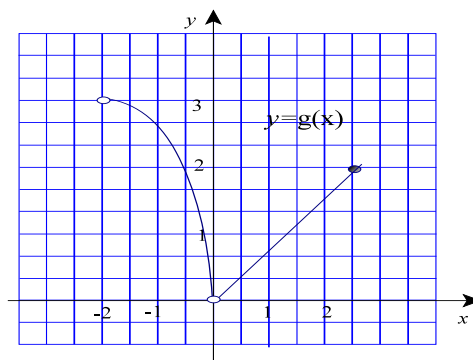
Domain

Range:

3. The graphs of f and g are shown in the figures. (Hint: Plot the domain of on the range of g)



Domain



Range: