Difference Quotients H. Bourgoub

Definition

Given a function $y = f(x)$, *then the Difference Quotient of* f *near* $x = a$

is defined to be $\dfrac{f(x) - f(a)}{x - a}$

There are many interpretation of the difference quotient of a function near a point on its graph, one of which is the value of the slope of the secant line that passes through the points $(x, f(x)), (a, f(a))$

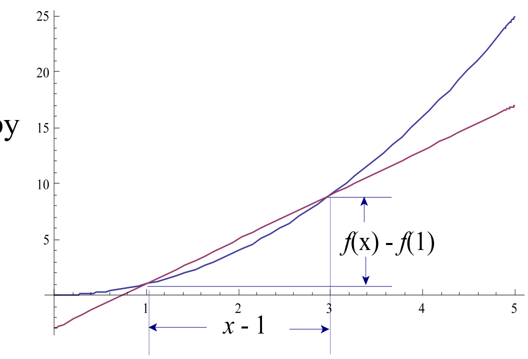
Example

The difference quotient for $f(x) = x^2$, near $x = 1$ is given by

$$\frac{f(x) - f(1)}{x - 1} = \frac{x^2 - 1}{x - 1} = x + 1$$

So the value of this difference quotient at x = 3 is

$$x + 1 = 3 + 1 = 4.$$



The graph of f and the secant line through (1,1) and (3,9)

which is the slope of the line that intersects the graph of f at the points

$$(1, f(1)) = (1,1), \text{ and } (3, f(3)) = (3,9)$$

Note that the slope of the secant line $m_{sec} = \dfrac{9-1}{3-1} = 4$, which the value of the difference quotient at x 3.

Problems

Let $f(x) = x^2 + x$, Find the difference quotients of f near the points with the given x coordinates, then simplify the expression.

a. $x = 1$

b. $x = 2$

c. Use the difference quotient near x =1 to find the slope of the secant lines that passes through the point $(1, f(1))$ and the points in two different ways.

i $(2, f(2))$

ii $(3, f(3))$