Math001D Review Fall 2102

1. Find the limit if it exists, or show it does not exist

$$\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$$

2. Given $f(x,y) = x^2 + y^2,\ x = \sin t,\ y = \cos t$

   Find $\frac{dz}{dt} =$

3. Given $f(x,y) = x^2 + 4y^2$

   Find $\nabla f(1, \sqrt{3}/2)$

4. Find the equation of the normal line to the surface $z = xy^2$, at the point $(1,2,4)$

5. Find the equation of the tangent plane to the surface $z = xy^2$, at the point $(1,2,4)$

6. Find the x coordinate of the center of mass of the triangular lamina in the first quadrant bounded by the x an y axes and the line $y = 3 - x$, if the density function is $\rho(x,y) = 3x + y$

7. Suppose that the a business model has profit function $P(x,y,z) = 3x + 6y + 6z$, and a manufacturing constraint $2x^2 + y^2 + 4z^2 \le 8800$. Maximize the profit.

8. Find the minimum value of the function $f(x,y) = 2x^2 + 3y^2$

9. Find the shortest distance from the origin to the surface $z^2 = 2xy + 2$

10. Find the minimum value of $f(x,y) = 2x^2 + y^2$, subject to the constraint $xy = 2$.

11. Find the work done by the force field $\mathbf{F(x,y)} = \langle -x^2, xy \rangle$ on a particle that moves once around the circle $x^2 + y^2 = 4$ counterclockwise.

12. Show that $\int_C 2xy^2dx + 2x^2ydy$ is independent of the path, then evaluate the integral. Where C is the path from $(0,0)$ to $(2,-1)$.

13. Use Green's theorem to evaluate the line integral along the given path.

$\int_C x^2y^2dx + 4xy^3dy$ C is the vertices of the triangle (0,0), (1,3), (0,3).

14. Determine if the vector field is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

a) $\mathbf{F(x,y)} = \langle yz, xz, xy \rangle$

b) $\mathbf{F(x,y,z)} = \langle 3xy, x^2 + 2y, y^2 \rangle$

15. Evaluate

$\int_C ydx + 2xydy$, where C is the curve is the line from (1,1) to (5,3).

16. Evaluate $\int_C \vec{F} \bullet d\vec{r}$ where $\vec{\mathbf{F}}(x,y) = \langle xe^{x^2} - 2, \sin y \rangle$ and C is the portion of the parabola $y = x^2$, from (-2,4) to (2,4) .

17. Evaluate the surface integral $\iint_S zdS$, where S is the part of the cylinder $z = \sqrt{1-x^2}$ that lies above the square with verices $(-1,0),(-1,1),(1,1),(1,0)$.

18. Let $\mathbf{F}(x,y,z) = x\mathbf{i} - y\mathbf{j} + z\mathbf{k}$. and let ∂Q be th boundary surface of the solid $Q = \{(x,y,z) | x^2 + y^2 + z^2 \le 1\}$, evaluate the surface integral $\iint_{\partial Q} \mathbf{F} \bullet \mathbf{n} ds$

19. Evaluate the flux integral $\iint_S (x\mathbf{i} - y\mathbf{j} + 3z\mathbf{k}) \bullet \mathbf{n} dS$ over the boundary of the ball $x^2 + y^2 + z^2 \le 4$

20. Use the change of variable $u = 2x - y, v = x + y$ to evaluate $\iint_R (x - 3y)dA$, where R is the region bounded $2x - y = 1, 2x - y = 3, x + y = 1, x + y = 2$.