

Definition: $z = f(x, y)$

A function of two variable is a correspondence that assign to every ordered pair in a set $D \subseteq \mathbb{R}^2$ a unique real number value z in \mathbb{R} .

Notation

$$z = f(x, y), \quad f : D \subseteq \mathbb{R}^2 \rightarrow \mathbb{R}$$

Examples: 1. $f(x, y) = \frac{x}{x - y^2}$

Domain All (x, y) in \mathbb{R}^2 , Except points on the parabola $y = x^2$.

$$2. \quad g(x, y) = \sqrt{4 - x^2 - y^2}$$

Domain All (x, y) in \mathbb{R}^2 , Except points on and insdide the circle $y^2 + x^2 \leq 4$.

Graphing functions of two variables..

Traces

The intersection of the surface $z = f(x, y)$ and a standard plane. A trace is a space curve.

Traces parallel to the xy planes

Set $z = \text{constant}$, and examine $c = f(x, y)$.

Traces parallel to the yz plane

Set $x = \text{constant}$, and examine $z = f(c, y)$.

Traces parallel to the xz plane

Set $y = \text{constant}$, and examine $z = f(x, c)$.

Level Curves.

Level curves are traces parallel to the xy plane

$$z = \text{constant}.$$

Contour Graphs.

Level curves $f(x, y) = c$ for choices of the constant c , all graphed in the xy plane.

Examples

For $f(x, y) = \sqrt{9 - x^2 - y^2}$, describe all traces parallel to the standard planes, find level curves for five choices of the constant c , then produce a contour graph.