



The Geometric series

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$

Adds up to the real number two. The proof can be discerned by examining the figure above constructed as follow:

Start with a rectangle of width equal to two and height equal to one. This makes the total area of the rectangle two, the base times the height. Divide this rectangle into two equal parts by dropping a perpendicular that bisect the base. This will split the first rectangle into two rectangles each with area equal to one. Then continue by dividing one of the new rectangles into two rectangles each with area equal to one half. Continue the process by dividing one of the new rectangles by dividing the base or the height by horizontal or vertical lines as needed. See the figure. Note that as you continue dividing the new rectangles, we will produce rectangles of area equal to the terms of the series by doing this infinitely many times without ever leaving the original rectangle.

From the figure we conclude that the area of the rectangle is equal the sum of the areas of all the rectangles inside it and that is two. And since there is one to one correspondence between the rectangles and the term of the series, the sum of the terms of the series is also two.