Equations in two variables

The expression $G(x,y)=c$ *defines an equation in two variables x and y where c is a real number.*

Examples

$$x^2-y=0,\ \text{here } G(x,y)=x^2-y \text{ and } c=0.$$

$$x^2+y^2=1$$

$$\sqrt{(x-1)^2+y}=2$$

Any expression written in terms of x and y and an equal sign and a constant can be represented in the form

$$G(x,y)=c \qquad (1)$$

by moving everything except the constant to the left side of the equation, the expression on the left is G(x,y)

Example

$$y=1+x^2$$

$$y-x^2=1$$

$$G(x,y)=y-x^2,\ \text{and}$$

$$G(x,y)=1$$

Implicit Definition

In the event equation (1) Can be solved for one of the variables with a single expression on the other side, the new equation is said to be defined implicitly by the original equation.

The equations for the upper and lower semicircles

$$y=\sqrt{1-x^2} \text{ and } y=-\sqrt{1-x^2} \text{ respectively}$$

are defined implicitly by the equation of the circle

$$x^2+y^2=1$$

Exercises

Solve the given equations for the variable y, then graph the equation where possible.

1. $3x - 2y + 5 = 0$

2. $x^2 + 4y^2 - 4 = 0$

3. $\ln(xy) = e$

4. $\sqrt{x+y} = x$

Find an equation in two variables that defines each of the given equations implicitly, then graph the equation.

1. $x = 4 - y^2$

2. $x = ye^y$

3. $y = \sqrt{\left(4 - x^{\frac{2}{3}}\right)^3}$

4. $x = |1 - y|$