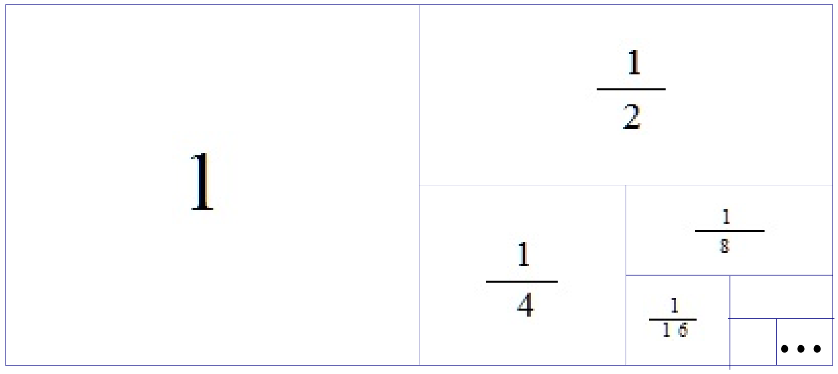

The Geometric series

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\ldots\ldots$$

Adds up to the real number two. The proof can be discerned by examining the figure above constructed as follow:

Start with a rectangle of width equal to two and height equal to one. This makes the total area of the rectangle two, the base times the height. Divide this rectangle into two equal parts by dropping a perpendicular that bisect the base. This will split the first rectangle into two rectangles each with area equal to one. Then continue by dividing one of the new rectangles into two rectangles each with area equal to one half. Continue the process by dividing one of the new rectangles by dividing the base or the height by horizontal or vertical lines as needed. See the figure. Note that as you continue dividing the new rectangles , you will produce rectangles of area equal to the terms of the series by doing this infinitely many times without ever leaving the original rectangle.

From the figure we conclude that the area of the rectangle is equal the sum of the areas of all the rectangles inside it and that is two. And since there is one two one correspondence between the rectangles and the term of the series, the sum of the terms of the series is also two.

Note that addition of infinite sums is not always possible. Some infinite series have sums that can be arrived at by deriving general formulas and arguments like the one presented above, some others continue to grow and become infinite or oscillate between different values and never fix on any given number. In the case the sum is infinite, such as the sum

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\ldots\ldots\ldots$$

there are methods studied in Calculus and advanced mathematics that can be used to approximate the sum of a large number of terms without actually performing the addition.