Inverse Functions I

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Definition:

Two functions $y = f(x)$ and $y = g(x)$ *are said to be inverses or inverse functions if applying one after the other to a value x in the domain of the first returns back x.*
Using function notation, it is written as $f(g(x)) = x$, or $g(f(x)) = x$.
In other words the second function reverses what the first function has done to x.

Notation

Use the notation $f^{-1}(x)$ to denote the inverse function of *f*.

Examples: The inverse of linear functions.

Note that addition can be undone or reversed by subtraction and vise versa. Also, multiplication can be reversed by division and vise vera.

1. Since addition can be reversed or undone using subtraction, the inverse of a linear function that adds is a linear function that subtract.

   Hence the inverse of $f(x) = x + 10$ is $f^{-1}(x) = x - 10$.
   $f(f^{-1}(x)) = f(x - 10) = (x - 10) + 10 = x$, and similarly $f^{-1}(f(x)) = x$
   Similarly, one can find the inverse function of a linear function that subtract.

2. Since multiplication can be revered or undone using division, the inverse of a linear function that multiplies is a linear function that divides.

   therefore the inverse of $f(x) = 5x$ is $f^{-1}(x) = \dfrac{x}{5}$

   $f^{-1}(f(x)) = f^{-1}(5x) = \dfrac{(5x)}{5} = x$, and similarly $f^{-1}(f(x)) = x$

3. For linear functions of the form $f(x) = ax + b$, this function first multiplies *x* by then adds b to a*x*. To reverse these two operation, the inverse has to undo the addition first by subtraction from *x* then undo the multiplication by division of *x*-b by a.

   $f(x)$: ax, $(ax) + b$, then $f(x) = ax + b$.

   $f^{-1}(x)$: $x - b$, $(x - b) \div a$, then $f^{-1}(x) = \dfrac{x - b}{a}$.

   The inverse of $f(x) = 3x + 4$, is $f^{-1}(x) = \dfrac{x - 4}{3}$,

   $f(f^{-1}(x)) = f(3x + 4) = \dfrac{(3x + 4) - 4}{3} = x$, and similarly $f^{-1}(f(x)) = x$

Problems: Find the inverse function.

a. $f(x) = 4x$ b. $f(x) = x + 4$ c. $f(x) = 2x - 3$