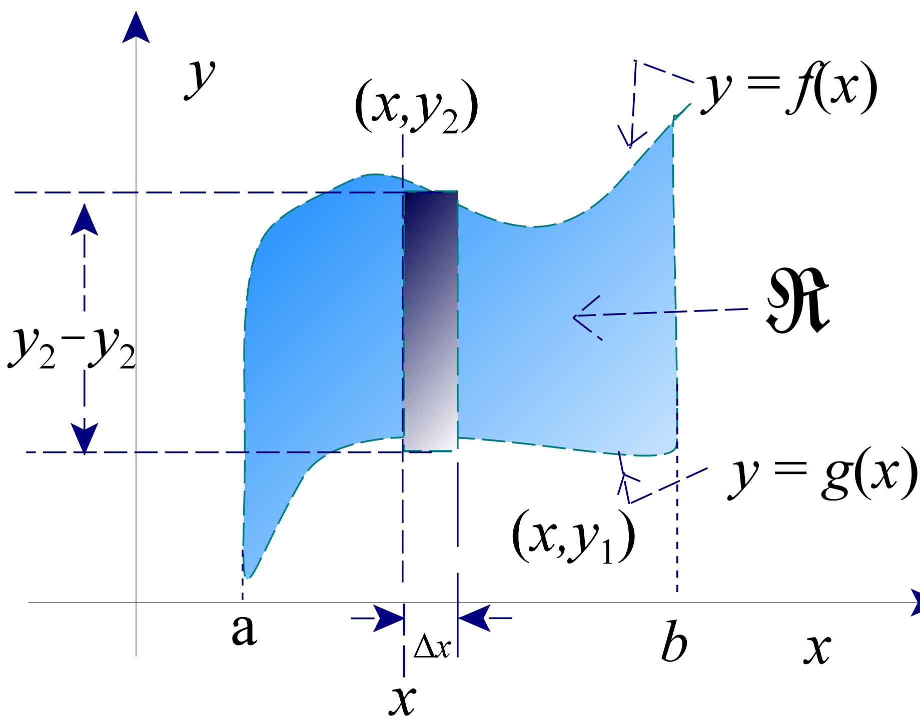
The mass of a lamina with a curvilinear boundary.

Find the mass of the Lamina defined by a region $\Re$ whose boundaries are shown in the figure below. Let ρ be the mass density of the Lamina per unit area.



Solution

Suppose the Mass of the Lamina from $t = a$ to $t = x$ given by $M(x)$ is known.
if t moves from x to $x + \Delta x$, then M changes by an amount ΔM

ΔM equal approximately the mass of the rectangular element of the lamina over the interval $[x, x + \Delta x]$.

$$\Delta M = \rho(y_2 - y_1)\Delta x, \quad a \le x \le b$$

If If $\Delta x \to 0$, then the equation for ΔM becomes,

$$dM = \rho(y_2 - y_1)dx, \quad a \le x \le b.$$

The equation above is integrated for M.