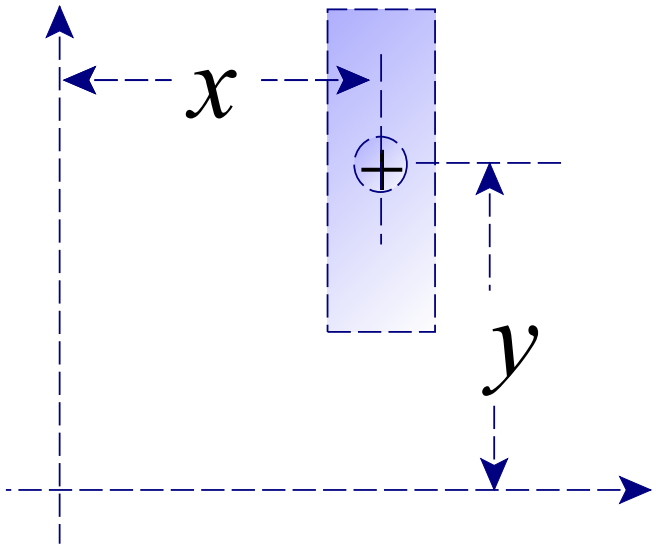
Moment of Mass



The moment of mass of a point mass about an axis is the product of the mass and the signed distance between the center of mass and the axis. The signed distance is called the arm.

$$M_y = (Mass)(Arm)$$
$$M_y = (M)(x)$$
$$M_x = (M)(y)$$

Lamina

A lamina is a thin sheet of uniform density. An example of a lamina is a flat piece of sheet metal, or a thin solid piece of cardboard.

The density of a lamina is given in terms of mass units per area units.

For Example

The mass density of a piece of solid cardboard is 0.8 grams per square centimeter.

The mass of a lamina whose area is 20 cm^2, and its mass density $\rho = 0.8 g/cm^2$ is

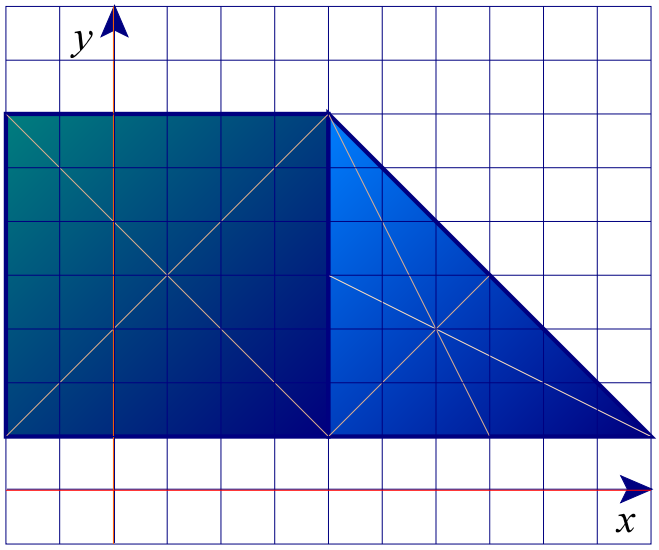
$$Mass = (Mass\ \text{Density})(Area)$$
$$Mass = \left(20 cm^2\right)\left(0.8\, g/cm^2\right) = 16 g$$

Geometric center

The center of mass of a lamina is its geometric center, we use the term centroid, instead of the center of mass, when applying the methods of computing center of mass to find geometrical centers of regions in the xy-plane.

For example the center of mass of a rectangular lamina is the intersection of the diagonals, the center of mass of a circular lamina is located at the center of the disk, and for a triangular lamina, the center of mass is located at the point of intersection of the medians.

Problem
Find the centroid of the Lamina shown in the figure.
Grid size 1x1, and $\rho = 1 g/cm^2$



1. Divide the Lamina into two parts, as shown in the figure then find the geometric center and the mass of each part.

2. Collapse each part into a point mass at the location of the corresponding centroid, then compute the centroid $(\overline{x}, \overline{y})$ using

$$\overline{x} = \frac{\sum_{i=1}^{2} x_i m_i}{\sum_{i=1}^{2} m_i}, \quad \overline{y} = \frac{\sum_{i=1}^{2} y_i m_i}{\sum_{i=1}^{2} m_i}$$

3. If we are to spin the piece of lamina above about an axis perpendicular to the lamina, where would this axis intersect the lamina?

Answer (8/3,11/3)