Math 2A final Exam Review Name:________________________

1. Determine a region in the xy plane in which the given differential equation has a unique solution whose graph passes through the point (x_0, y_0).

$$\frac{dy}{dx} = \sqrt{xy}.$$

2. A tank initially has 300 gallons of pure water in which 50 pounds of salts has been dissolved. Pure water is pumped into the tank at a rate of 3 gal/min, and the well-stirred mixture leaves the tank at the same rate.

a) Give the initial value problem for the amount of salt y.

b) Find the amount of salt in the tank at any time t, and the limiting value of the amount of salt in the tank as t approaches infinity.

3. Find the critical points and the phase portrait of the given autonomous 1^{st} order differential equation, then classify each point as asymptotically stable , unstable or semi-stable. By hand sketch typical solution curves in the in the xy plane determined by the equilibrium solutions.

$$\frac{dy}{dx} = y(y^2 - 4)$$

4. Solve the equation $y' = x\sqrt{1-y^2}$. by separation of variables.

5. Solve the initial value problem. Give the largest interval over which the solution is defined. $xy' + y = e^x, \quad y(1) = 2.$

6. Solve the differential equation $6xydx + (4y + 9x^2)dy = 0.$ by using an integrating factor.

7. Use Euler's method with step h = 0.1 to find four decimal approximation of y(0.5) where $y' = e^{-y}, \quad y(0) = 0.$

8. The radio active isotope of Lead Pb-209, decays at rate proportional to the amount present at time t, and has a half life of 3.3 years. If one gram of lead is present initially, how long will it take for 80% of the lead to decay.

9. Verify that the functions $y = e^{-3x}$, and $y = e^{4x}$, form a fundamental se of the differential equation $y'' - y' - 12y = 0, \quad -\infty < x < \infty$.

10. Use the method of reduction of order to find a 2^{nd} solution of the differential equation $y'' + 16y = 0$, given one solution $y_1(x) = \cos(4x)$.

11. Find the general solution of differential equation $y'' - y' - 6y = 0.$

12. Use the method of undetermined coefficients to solve the differential equation

$$y'' + 3y = -48x^2 e^{3x}.$$

13. Use the method of variation of parameters to solve the differential equation

$$y'' - 4y = e^{2x}/x$$

14. Solve the system of equations $\begin{cases} x' = -y + t \\ y' = -t + x \end{cases}$.

15. Use the Laplace transform to solve the differential equation $y' + 6y = e^{4t}, \; y(0) = 2.$

16. Use the Laplace transform to solve the differential equation $y' + 2y = f(t), \; y(0) = 0.$
where

$$f(t) = \begin{cases} t & 0 \le t < 1 \\ 1 & t \ge 1 \end{cases}$$

17. Use the Laplace transform to solve the differential equation

$$y'' + y = f(t), \; y(0) = 1, \; y'(0) = 0$$

where

$$f(t) = \begin{cases} 1 & 0 \le t < \pi/2 \\ \sin t & t \ge \pi/2 \end{cases}$$

18. Solve the equation $(e^x + y)dx + (2 + x + ye^y)dy = 0, \quad y(0) = 1.$

19. Solve the equation $ydx = 2(x + y)dy.$