Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1.   Find the vector x determined by the given coordinate vector $[x]_B$ and the given basis B. Illustrate your answer with a figure, as in the solution of Practice Problem 2.

$$\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}, [x]_B = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

2.   The vector **x** is in a subspace H with a basis B $=\{b_1, b_2\}$. Find the B-coordinate vector of **x**.

$$b_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, b_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, x = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

3.   The vector **x** is in a subspace H with a basis B $=\{b_1, b_2\}$. Find the B-coordinate vector of **x**.

$$b_1 = \begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}, b_2 = \begin{bmatrix} -3 \\ -7 \\ 5 \end{bmatrix}, x = \begin{bmatrix} 4 \\ 10 \\ -7 \end{bmatrix}.$$

4. Display an echelon form of A, then find bases for Col A and Nul A, and then state the dimensions of each subspace.

$$A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 5 & 1 & 3 & 7 \\ 3 & 0 & 3 & 6 \end{bmatrix}$$

5. Find a basis for the subspace spanned by the given vectors. What is the dimension of the subspace?

$$v_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \\ -4 \end{bmatrix}, v_2 = \begin{bmatrix} -3 \\ 9 \\ -6 \\ 12 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ -1 \\ 4 \\ 2 \end{bmatrix}, v_4 = \begin{bmatrix} -4 \\ 5 \\ -3 \\ 7 \end{bmatrix}$$

6. Let H=Span $\{v_1,v_2,v_3\}$ and B=$\{v_1,v_2,v_3\}$. Show that B is a basis for H and **x** is in H, and find the B-coordinate vector of **x**, when

$$v_1 = \begin{bmatrix} -6 \\ 4 \\ -9 \\ 4 \end{bmatrix}, v_2 = \begin{bmatrix} 8 \\ -3 \\ 7 \\ -3 \end{bmatrix}, v_3 = \begin{bmatrix} -9 \\ 5 \\ -8 \\ 3 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 4 \\ 7 \\ -8 \\ 3 \end{bmatrix}$$