Math02B Assignment 5.7 Last Name_____1st_____

Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1. A particle moving in a planar force field has position vector x that satisfies $\mathbf{x}' = \mathbf{A}\mathbf{x}$ The 2x2 matrix A has eigenvalues 2 and 3, with corresponding eigenvectors $\mathbf{v}_1 = (-2, 1)$, and $\mathbf{v}_2 = (2, 1)$. Find the position of the particle at time t assuming $x(0) = (-3, 3)$

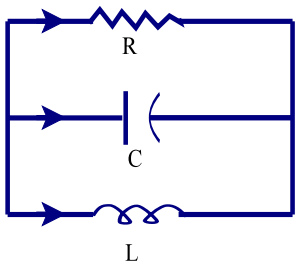
2. Solve the initial value problem $\mathbf{x}' = \mathbf{A}\mathbf{x}$ for $t >= 0$ with $x(0) = (4, 3)$. Classify the nature of the origin as an attractor, repeller, or a saddle point of the dynamical system described by $\mathbf{x}' = \mathbf{A}\mathbf{x}$. Find the direction of the greatest attraction and or/ repulsion. When the origin is a saddle point, sketch a typical trajectory.

$$\mathbf{A} = \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$$

3. The circuit in the figure is described by the equation

$$\begin{bmatrix} i_L' \\ v_C' \end{bmatrix} = \begin{bmatrix} 0 & 1/L \\ -1/C & -1/(RC) \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

Where $i_L$ is the current through the inductor L and $v_C$ is the voltage drop across the capacitor C. Find a formulas for $i_L$ and $v_C$ when R = 0.5 ohms and C = 2.5 farads, L = 0.5 henry, the initial current is 0.0 amp and the initia voltage is 12.0 volts.




4. Solve the given system of linear equations initial value problem.

$$\begin{cases} \dfrac{dy_1}{dt} = -0.2y_1 \\ \dfrac{dy_2}{dt} = 0.2y_1 - 0.1y_2, \quad y_1(0) = 1,\ y_2(0) = 1,\ y_3(0) = 1, \\ \dfrac{dy_3}{dt} = 0.1y_3 \end{cases}$$