Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1.   Compute the orthogonal projection of the vector **u** onto the col(**A**) using a projection Matrix.

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad u = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

2.   consider the system of equations

$$\begin{cases} x + y = 7 \\ -x + y = 0 \\ -x + 2y = -7 \end{cases}$$

a.   Find the projection Matrix onto Col(**A**), call it **P**.

b.   Find the projection of **b** onto the col(**A**)

c.   Solve the system **Ax = Pb** using an rref of an augmented matrix.

d.   Find the associated normal system, $Ax = b$.

e.   Give the least squares solution of the system.

f.   Find the least squares vector and computes the least squares error.

3.   Factor the Matrix A into QR in problem 2 then us the factorization to find the least squares solution.