QUIZ #2 - SOLUTIONS

1) Is the following series absolutely convergent, conditionally convergent or divergent? Explain.

\[ \sum_{k=1}^{\infty} \frac{\sin k\pi}{k} \]

\[ \sin (k\pi) = 0 \quad \forall k \Rightarrow \sum_{k=1}^{\infty} \frac{\sin k\pi}{k} = \sum_{k=1}^{\infty} 0 = 0 \Rightarrow \]

the series converges absolutely.

2) Use the **Integral Test** to determine convergence or divergence of

\[ \sum_{k=2}^{\infty} \frac{\ln k}{k} \]

Let \( f(x) = \frac{\ln x}{x} \) which is continuous and positive on \([2, \infty)\).

\[ f'(x) = \frac{1-\ln x}{x^2} < 0 \Rightarrow f \text{ is decreasing.} \]

Now use the Integral Test.

\[ \int_{2}^{\infty} \frac{\ln x}{x} \, dx = \lim_{t \to \infty} \int_{2}^{t} \frac{\ln x}{x} \, dx = \lim_{t \to \infty} \left[ \frac{(\ln x)^2}{2} \right]_{2}^{t} = \]

\[ \lim_{t \to \infty} \left[ \frac{(\ln t)^2}{2} - \frac{(\ln 2)^2}{2} \right] = \infty \]

Hence, our series diverges.