Hyperbolic Functions Problems

Assume two poles of equal height are spaced a certain distance apart from each other. If a heavy cable or wire is connected between two points at the same height on the poles, the resulting curve of the wire is in the form of a "catenary", with basic equation

$y = a \operatorname{Cosh}\left(\frac{x}{a}\right)$ (Graph this curve for different values of "a" : positive, negative, large, small)

Associated problems:

Given that $\operatorname{Sinh} x = \frac{e^x - e^{-x}}{2}$ $\operatorname{Cosh} x = \frac{e^x + e^{-x}}{2}$

1. Write the hyperbolic tangent function; Write the remaining three hyperbolic trig functions.

2. Establish the identities for:

   i) $\operatorname{Sinh}(2x)$ ii) $\operatorname{Cosh}(2x)$ (as in trigonometry, there should be 3 such identities)

   iii) $\operatorname{Sinh}(x + y)$ iv) $\operatorname{Cosh}(x + y)$ v) $\operatorname{Sinh}(-x)$ vi) $\operatorname{Cosh}(-x)$

3. Establish the expressions for $\operatorname{Cosh} x + \operatorname{Sinh} x = ??$ $\operatorname{Cosh} x - \operatorname{Sinh} x = ??$

4. Establish the "Pythagorean Identities" for the hyperbolic functions:

   Does $\operatorname{Sinh}^2 x + \operatorname{Cosh}^2 x = 1$? If not, what change(s) should be made to get a result of 1?

   Show that $1 - \operatorname{Tanh}^2 x = \operatorname{Sech}^2 x$ $1 - \operatorname{Coth}^2 x = -\operatorname{Csch}^2 x$

   How do these Hyperbolic Pythagorean Identities compare to the analogous trig identities?

5. Find $D_x \operatorname{Sinh} x$ Find $D_x \operatorname{Cosh} x$ Find $D_x \operatorname{Tanh} x$

   Find $D_x \operatorname{Coth} x$ Find $D_x \operatorname{Sech} x$ Find $D_x \operatorname{Csch} x$

6a. Show that $\operatorname{Sinh}^{-1} x = \ln(x + \sqrt{x^2 + 1})$. Find expressions for $\operatorname{Cosh}^{-1} x$ and $\operatorname{Tanh}^{-1} x$
What is the domain of each of these inverse hyperbolic functions?

6b. Derive a formula for the deriverative of the inverse hyperbolic sine function $y = \operatorname{Sinh}^{-1} x$ (hint: how is the deriverative of inverse sine derived: $D_x(\operatorname{Sin}^{-1} x) = ??$); Also, derive the deriverative of $y = \operatorname{Cosh}^{-1} x$ (i.e., $D_x(\operatorname{Cosh}^{-1} x) = ??$)

Find the derivative:

7. $y = \text{Tanh}\, x$

8. $f(x) = e^x \text{Cosh}\, x$

9. $y = \text{Sinh}\, e^{2x}$

10. $g(x) = \text{Cosh}^{-1} x^2$

11. $y = \text{Cosh}\, x^3$

12. $y = \text{Coth}(\ln x)$

13. $f(x) = \text{Sin}^{-1}(\text{Tanh}\, x^2)$

14. $f(x)\ \text{Tanh}^{-1}(\text{Cos}\, e^x)$

Find the antiderivative:

15. $\int \text{Sinh}^4 x\, \text{Cosh}\, x\, dx$

16. $\int x\, \text{Sech}^2 x^2\, dx$

17. $\int \frac{\text{Sinh}\sqrt{x}}{\sqrt{x}}\, dx$

18. $\int e^t\, \text{Cosh}\, e^t\, \text{Sinh}\, e^t\, dt$

19. $\int \frac{\text{Sinh}\, x}{1 + \text{Cosh}\, x}\, dx$

20. $\int \frac{\text{Cosh}\, t}{\sqrt{\text{Sinh}\, t}}\, dt$

21. At what point on the curve $y = \text{Cosh}\, x$ does the tangent have slope 1 ?

22. The *gudermannian* , named after the German mathematician Christoph Gudermann (1798 - 1852) is the function $\text{gd}\ x = \text{Tan}^{-1}(\text{Sinh}\, x)$ Show that $D_x(\text{gd}\, x) = \text{Sech}\, x$