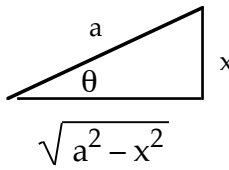
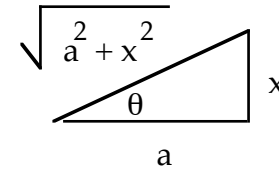
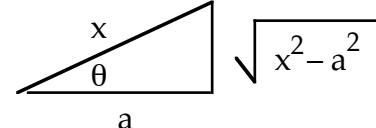


## Trigonometric Substitutions

This handout covers how to work with the following forms:

	$\sqrt{a^2 - x^2}$	$\sqrt{a^2 + x^2}$	$\sqrt{x^2 - a^2}$
Choose the expression	Make the substitution	dx and dθ	relation between x and θ
$x = a \sin \theta$ in $\sqrt{a^2 - x^2}$	$\sqrt{a^2 - x^2}$ $\sqrt{a^2 - (a \sin \theta)^2} =$ $\sqrt{a^2 - a^2 \sin^2 \theta} =$	$dx = d(a \sin \theta) =$ (take the derivative)	
finish the simplification			

$x = a \tan \theta$ in $\sqrt{a^2 + x^2}$	$\sqrt{a^2 + x^2}$ $\sqrt{a^2 + (a \tan \theta)^2} =$	$dx = d(a \tan \theta) =$ (take the derivative)	
finish the simplification			

$x = a \sec \theta$ in $\sqrt{x^2 - a^2}$	$\sqrt{x^2 - a^2}$ $\sqrt{(a \sec \theta)^2 - a^2} =$	$dx = d(a \sec \theta) =$ (take the derivative)	
finish the simplification			

## Trigonometric Substitution Problems

1.  $\int_1^2 \frac{dx}{x^2 \sqrt{9-x^2}}$

2.  $\int \frac{dx}{(x^2+1)^{3/2}}$

3.  $\int_{\sqrt{3}}^2 \frac{\sqrt{x^2-3}}{x} dx$  or  $\int_{\sqrt{3}}^2 \frac{\sqrt{x^2-3}}{x} dx$  hint: remember, when you switch variables, what happens to the limits?

4.  $\int \frac{x dx}{\sqrt{x^2+9}}$

5.  $\int \frac{\sqrt{1-x^2}}{x^4} dx$

6.  $\int e^{2x} \sqrt{e^{2x}-1} dx$

7.  $\int_0^{3/2} \frac{t^2 dt}{(1-t^2)^{3/2}}$  hint: when you switch variables, what happens to the limits?

8.  $\int_0^3 \frac{t^3}{\sqrt{t^2+9}} dt$

9.  $\int_1^4 \frac{\sqrt{u-1}}{\sqrt{u}} du$