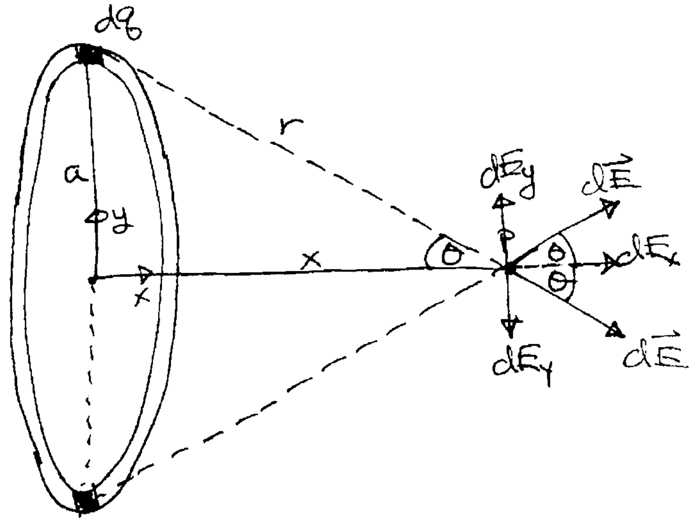
Ex. $\vec{E}$-field Due to a ring of charge $+Q$ distributed uniformly.



$r = (a^2+x^2)^{1/2}$

By symmetry $E_y = 0$

$dE_x = dE \cos\theta$

$$dE_x = \frac{k\,dq}{r^2}\left(\frac{x}{r}\right) = \frac{k\,dq\,x}{r^3}$$

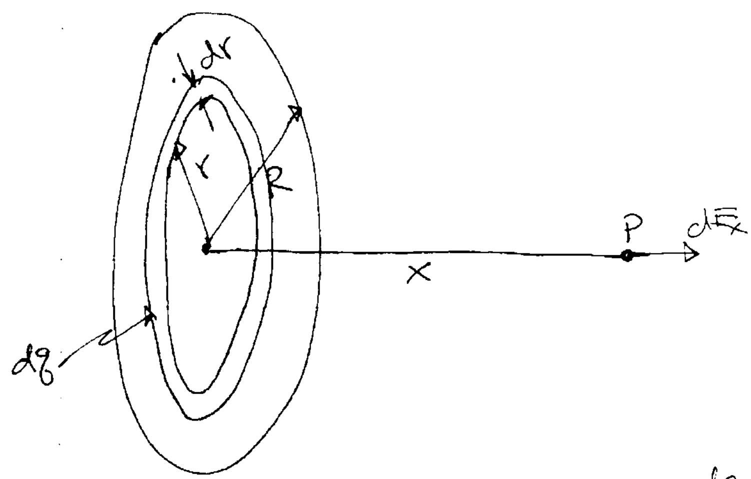
$$dE_x = \frac{kx(dq)}{(a^2+x^2)^{3/2}}$$

$$E_x = \frac{kx}{(a^2+x^2)^{3/2}}\int dq$$

$$E_x = \frac{kQx}{(a^2+x^2)^{3/2}}$$

If $x \gg a$, then $E_x \approx \dfrac{kQ}{x^2}$ ✓

Ex. $\vec{E}$-field Due to a disk of charge $+Q$ distributed uniformly.



$$\frac{Q}{\pi R^2} = \frac{dq}{dA}$$

$$dq = \left(\frac{Q}{\pi R^2}\right) dA$$

$$dq = \sigma\, dA$$

σ = surface charge density

$$dq = \sigma(2\pi r\, dr)$$

$$dE_x = \frac{Kx(dq)}{(r^2+x^2)^{3/2}} \quad (\vec{E}\text{-field due to ring})$$

$$dE_x = Kx\sigma\pi \frac{2r\,dr}{(r^2+x^2)^{3/2}}$$

let $u = r^2+x^2$

$du = 2r\,dr$

$u_i = x^2$

$u_f = R^2+x^2$

$$E_x = Kx\sigma\pi \int_0^R \frac{2r\,dr}{(r^2+x^2)^{3/2}}$$

$$= Kx\sigma\pi \int_{x^2}^{R^2+x^2} \frac{du}{u^{3/2}}$$

$$= Kx\sigma\pi \left.\frac{u^{-1/2}}{-1/2}\right|_{x^2}^{R^2+x^2}$$

$$= -2Kx\sigma\pi \left[\frac{1}{\sqrt{R^2+x^2}} - \frac{1}{x}\right]$$

$$\vec{E} = 2K\sigma\pi \left[1 - \frac{x}{\sqrt{R^2+x^2}}\right] \hat{i}$$

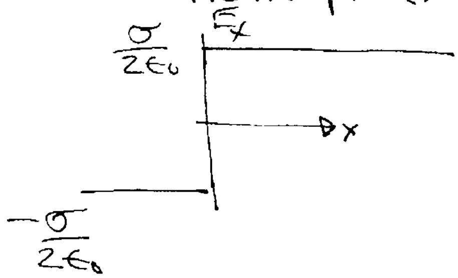
$$\lim_{R \to \infty} \vec{E} = 2K\sigma\pi \lim_{R \to \infty} \Big[1 - \underbrace{\frac{x}{\sqrt{R^2 + x^2}}}_{1} \Big] \hat{i}$$

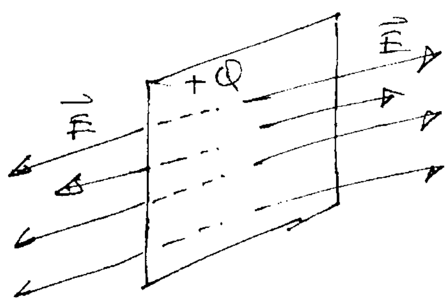
$$\vec{E} \approx 2K\sigma\pi \hat{i}$$

$$\vec{E} \approx 2 \frac{1}{4\pi\epsilon_0} \sigma\pi \hat{i}$$

$$\boxed{\vec{E} \approx \frac{\sigma}{2\epsilon_0} \hat{i}}$$ $\vec{E}$-field Due To an Infinite plane of charge

- field is uniform
- field is $\perp$ to plane
- field is independent of distance from plane.





In the limit as $x \gg R$, $\vec{E}$ should approach that of a point charge:

$$\vec{E} = 2K\sigma\pi \Big[1 - \frac{\cancel{x}}{\cancel{x}\sqrt{\frac{R^2}{x^2} + 1}} \Big] \hat{i} \qquad \left(\frac{R^2}{x^2} + 1 \right)^{-1/2} \approx 1 - \frac{1}{2}\frac{R^2}{x^2} + \dots$$

$$\vec{E} \approx 2K \frac{Q}{\cancel{\pi} R^2} \cancel{\pi} \Big[\cancel{1} - \big(\cancel{1} - \frac{1}{2} \frac{R^2}{x^2} \big) \Big] \hat{i}$$

$$\vec{E} \approx \frac{\cancel{2}KQ}{\cancel{R^2}} \left(\frac{1}{\cancel{2}} \frac{\cancel{R^2}}{x^2} \right) \hat{i}$$

$$\vec{E} \approx \frac{KQ}{x^2} \hat{i} \checkmark$$