Motion In 1-D with Constant Acceleration

There are several practical applications in which an object moves with constant or nearly constant acceleration. We need to be able to analyze the motion of such objects. Our objective is to derive 4 equations called the kinematic equations of motion that can be used to describe any object moving with constant (uniform) acceleration in 1D.

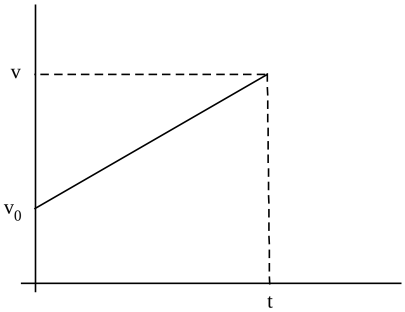
$$a_{ave} = a = \frac{\Delta v}{\Delta t} \text{ (because } a = \text{ constant)}$$

$$a = \frac{v_f - v_i}{t_f - t_i}$$

For convenience we choose that at:

$$t_i = 0,\ v_i = v_o$$
$$t_f = t,\ v_f = v$$
Thus,
$$a = \frac{v - v_o}{t}$$

1. $\boxed{v = v_o + at}$



Recall that,

$$v_{ave} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$
$$x_f - x_i = v_{ave}(t_f - t_i)$$

For convenience we choose that at:

$$t_i = 0,\ x_i = x_o$$
$$t_f = t,\ x_f = x$$
Thus,
$$x - x_o = v_{ave}t$$

Since the velocity changes at a constant rate with respect to time ($v = v_o + at$), the average velocity in any time interval is equal to the arithmetic mean of the initial and final velocity. Thus, for the time interval (0,t), the average velocity if equal to

$$\boxed{v_{ave} = \frac{v_o + v}{2}}$$ Valid only when a = constant

Substituting this equation into the equation $x - x_o = v_{ave}t$ we get

2. $$\boxed{x - x_o = \left(\frac{v_o + v}{2}\right)t}$$

Substituting Eq. 1 for 'v' into Eq.2 gives

$$x - x_o = \left(\frac{v_o + v_o + at}{2}\right)t$$

$$x - x_o = \left(\frac{2v_o + at}{2}\right)t = v_o t + \frac{1}{2}at^2$$

3. $$\boxed{x = x_o + v_o t + \frac{1}{2}at^2}$$

Solving for 't' in Eq. 1 gives

$$t = \frac{v - v_o}{a}$$

Substituting into Eq. 2

$$x - x_o = \left(\frac{v + v_o}{2}\right)\left(\frac{v - v_o}{a}\right) = \frac{v^2 - v_o^2}{2a}$$

4. $$\boxed{v^2 = v_o^2 + 2a(x - x_o)}$$

Kinematic Equations of Motion For Constant Acceleration

1. $v = v_o + at$

2. $x - x_o = \left(\dfrac{v_o + v}{2}\right)t$

3. $x = x_o + v_o t + \dfrac{1}{2}at^2$

4. $v^2 = v_o^2 + 2a(x - x_o)$

Graphs of Equations of Motion