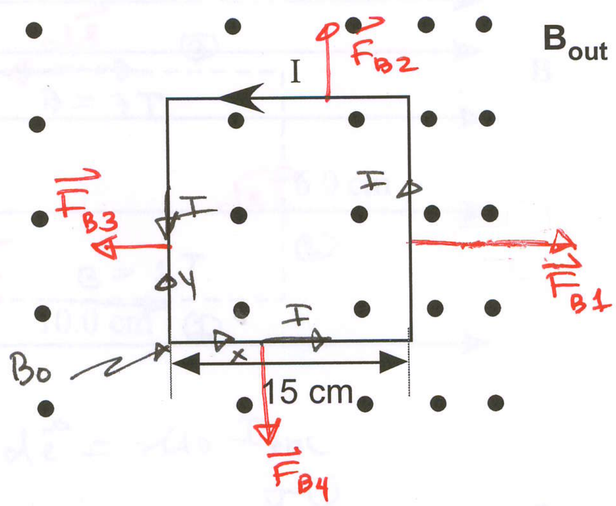
DO NOT TURN THIS PAGE!!!!!

Name: KEY

Physics 4B
Winter 2011
EXAM 3

Partial credit will be given, so do what you can, and show all your work in complete detail with appropriate units. NO CREDIT WILL BE GIVEN IF NO WORK IS SHOWN!

1. A non-uniform magnetic field points out of the page as shown below in the figure. The field increases at a constant rate of 2.0 mT as you move to the right. A square wire loop of 15 cm on a side lies on a plane perpendicular to the field, and a 2.5 A current circles the loop in the counter-clockwise direction. Calculate the magnitude and direction of the net magnetic force on the loop. (10 pts)



$\vec{F}_B = I\vec{\ell} \times \vec{B}$

$d\vec{F}_B = I d\vec{\ell} \times \vec{B}$

$\frac{dB}{dx} = 2.0 \frac{mT}{cm} \Rightarrow B = B_0 + 2x$

By symmetry $|\vec{F}_{B2}| = |\vec{F}_{B4}|$ (cancel out)

$\therefore F_{net} = F_{B1} - F_{B3}$

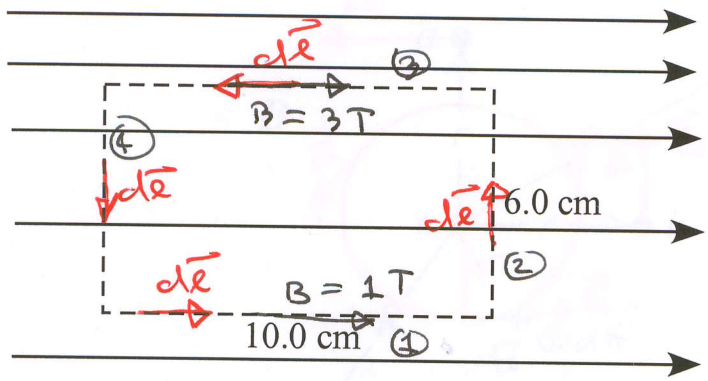
$= I\ell(B_0 + 2(15)) - I\ell B_0$

$= I\ell(2\,mT)(15\,cm)$

$F_{net} = (2.5A)(0.15m)(30 \times 10^{-3}\,T)$

$F_{net} = 1.125 \times 10^{-2} N$

2. The figure below shows a non-uniform magnetic field that varies along the y-axis. At the top and bottom of the rectangular loop shown the field strengths are 3.0 T and 1.0 T respectively. (10 pts)
   a) Calculate the amount of current that flows through the area bounded by the loop.
   b) In what direction does the current flow?
   c) What is the source of the **B**-field shown.



a) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

$\int_{(1)} \bar{B} \cdot d\bar{l} + \int_{(2)} \bar{B} \cdot d\bar{l} + \int_{(3)} \bar{B} \cdot d\bar{l} + \int_{(4)} \bar{B} \cdot d\bar{l} = \mu_0 I_{enc}$

$(1T)(0.10m) - (3T)(0.10m) = \mu_0 I_{enc}$

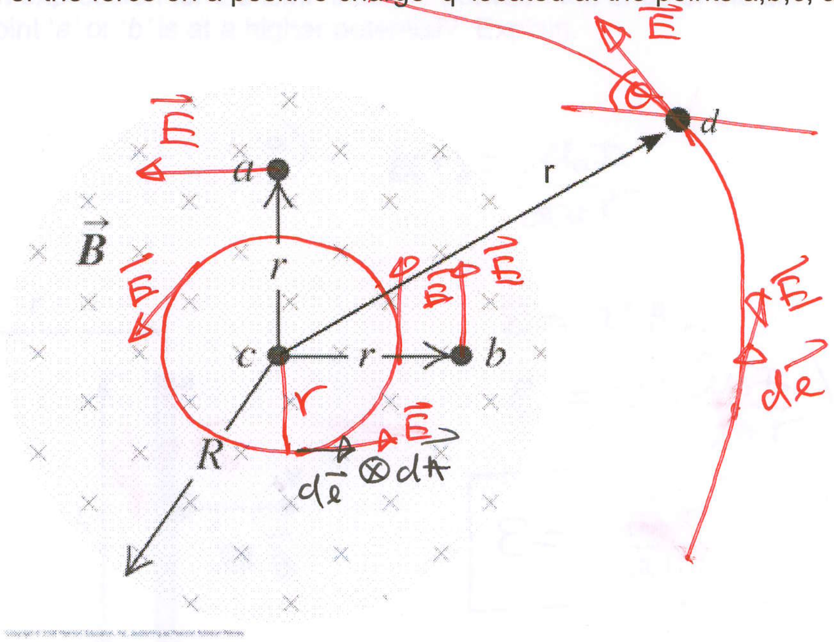
$-0.2 T\cdot m = (4\pi \times 10^{-7} \frac{T\cdot m}{A}) I_{enc}$

$I_{enc} = -1.59 \times 10^{5} A$

b) Into page!

c) I_{enc} + all other surrounding current!

3. A magnetic field **B** is confined to a circular region in space as shown below. The **B**-field is directed into the plane of page and increasing at a rate of dB/dt. Find the magnitude and direction of the force on a positive charge 'q' located at the points a,b,c, and d. (10 pts)



$r<R$

$\oint \vec{E}\cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

[Evaluate over path shown!]

$\oint E\,d\ell\cos 0° = -\pi r^2 \frac{dB}{dt}$

$E\oint d\ell = -\pi r^2 \frac{dB}{dt}$

$E\,2\pi r = -\pi r^2 \frac{dB}{dt}$

(3) $E = -\frac{r}{2}\frac{dB}{dt}$ $(r<R)$

$\Phi_B = \int \vec{B}\cdot d\vec{A} = BA = B\pi r^2$

$\frac{d\Phi_B}{dt} = \pi r^2 \frac{dB}{dt}$

$r>R$

$E\,2\pi r = -\pi R^2 \frac{dB}{dt}$

(3) $E = -\frac{R^2}{2r}\frac{dB}{dt}$ $(r>R)$

(1) $E(c) = 0 \Rightarrow F(c) = 0$

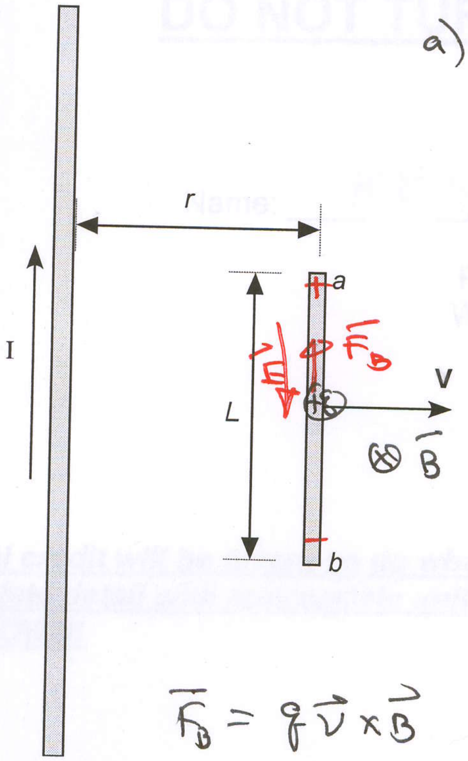
$E(a) = -\frac{r_a}{2}\frac{dB}{dt}$ (left)

(2) $F_a = qE = q\frac{r_a}{2}\frac{dB}{dt}$ (left)

$F_b = q\frac{r_b}{2}\frac{dB}{dt}$ upward.

(1) $F_d = \frac{qR^2}{2r_d}\frac{dB}{dt}$ (at θ w/ horiz)

4. A long, straight wire as shown below carries a constant current I. A metal bar with length L is moving at a constant velocity **V** as shown. (10 pts)
   a) Derive an expression for the EMF induced between the ends of the bar.
   b) Which point *'a'* or *'b'* is at a higher potential? Explain.



a) $B(r) = \frac{\mu_0 I}{2\pi r}$

$\mathcal{E} = vBL$

$\mathcal{E} = v\left(\frac{\mu_0 I}{2\pi r}\right)L$

$\boxed{\mathcal{E} = \frac{\mu_0 I v L}{2\pi r}}$ ✓

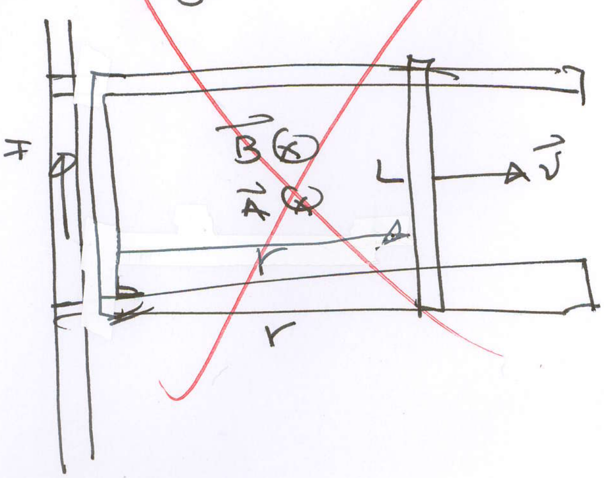
$\vec{F}_B = q\vec{v}\times\vec{B}$

b) $V_a > V_b$

$\vec{E}$ directed ↓

+charges move up!

Assuming part of circuit loop:



$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BrL)$

$\mathcal{E} = -LB\frac{dr}{dt}$

$\boxed{\mathcal{E} = -L\frac{\mu_0 I}{2\pi r}v}$ ✓