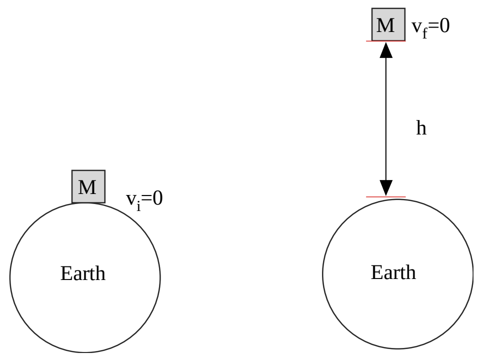
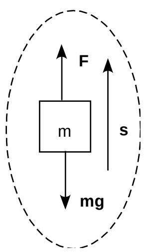
POTENTIAL ENERGY

Often the work done on a system of two or more objects does not change the kinetic energy of the system but instead it is stored as a new type of energy called POTENTIAL ENERGY. To demonstrate this new type of energy let's consider the following situation.

Ex. Consider lifting a block of mass 'm' through a vertical height 'h' by a force **F**.

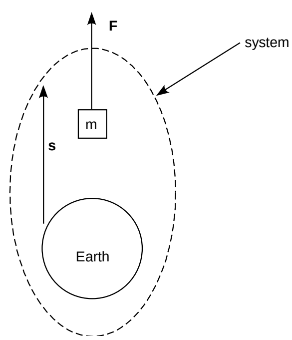


System = Block



$$W_{net} = \Delta K$$
$$\Delta K = 0 \text{ (since } v_i = v_f = 0)$$
$$W_{net} = W_F + W_g = 0$$
$$W_F = -W_g = -mgh\cos 180^o$$
$$\boxed{W_F = mgh}$$

System = block + earth

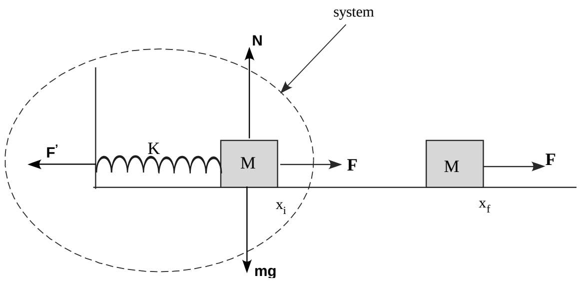


$$Wnet = \Delta K$$
$$\Delta K = 0$$
$$W_{net} = W_F = mgh$$
$$\boxed{W_F = mgh}$$

Clearly the work done by **F** is not zero and there is no change in KE of the system. Where has the work gone into? Because recall that positive work means energy transfer into the system. Where did the energy go into? The work done by F_{ext} must show up as an increase in the energy of the system.

The work done by **F** ends up stored as *POTENTIAL ENERGY* (gravitational) in the Earth-Block System. This potential energy has the "potential" to be recovered in the form of kinetic energy if the block is released.

Ex. Spring-Mass System



$$w_{net} = \Delta K$$
$$\Delta K = 0 \text{ (Since } v_{\mathrm{i}} = v_f = 0)$$
$$w_{net} = w_{F'} + w_N + w_{mg} + w_F$$
$$w_{net} = w_F = -w_s$$
$$w_s = \frac{1}{2}k_i^2 - \frac{1}{2}k_f^2$$
$$w_F = -w_s = \frac{1}{2}k_f^2 - \frac{1}{2}k_i^2$$

The work done by **F** ends up stored as *POTENTIAL ENERGY* (elastic) in the Spring-Mass System.

Potential Energy - Energy associated with the configuration of a system involving two or more objects. It is a shared property of the objects making up the system.

Before we define the potential energy function mathematically, we need to define a conservative force.

Conservative Forces

Conservative forces are very important because as we will see they imply conservation of energy for an isolated system. With every conservative force there is a potential energy function associated with that force.

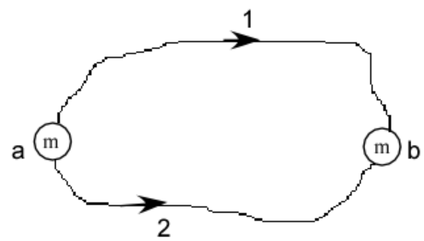
A conservative force has the following two equivalent definitions:

Def 1: A force is conservative if the work done by that force is path independent.

Def 2: The work done by a conservative force around any closed path is zero.

$$\boxed{w_c = \oint \vec{F}_c \bullet d\vec{s} = 0}$$

Consider a particle acted on by a conservative force between two points a and b along two different paths.



$$W_{ab(1)} = W_{ab(2)}$$

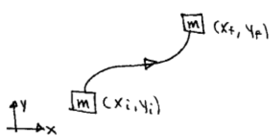
$$\int_{a\ \text{(path1)}}^{b} \vec{F}_c \bullet d\vec{s} = \int_{a\ \text{(path 2)}}^{b} \vec{F}_c \bullet d\vec{s}$$

$$W_{ab(1)} + W_{ba(2)} = 0 \Rightarrow W_{ab(1)} = -W_{ba(2)}$$

$$\int_{a\ \text{(path1)}}^{b} \vec{F}_c \bullet d\vec{s} + \int_{\text{b (path 2)}}^{a} \vec{F}_c \bullet d\vec{s} = 0$$

As examples of conservative forces let's see if the gravitational force and spring force are conservative.

Ex. Gravitational Force



$$W_g = \int_1^2 \vec{F}_g \bullet d\vec{\ell} = \int_1^2 -mg\vec{j} \bullet dx\vec{i} + dy\vec{j}$$

$$W_g = -mg \int_{y_i}^{y_f} dy$$

$$W_g = -mg\, y \Big|_{y_i}^{y_f} = -mg(y_f - y_i)$$

$$\boxed{W_g = mg y_i - mg y_f}$$

Since W_g is path independent, then $\vec{F_g} = m\vec{g}$ is a conservative force.

Ex Spring Force

$$\boxed{W_s = \tfrac{1}{2} k x_i^2 - \tfrac{1}{2} k x_f^2}$$

Since W_s is path independent, then $F_s = -kx$ is a conservative fore

Potential Energy Function

$$W_g = mgy_i - mgy_f$$

$\boxed{U_g = mgy}$ Gravitational Potential Energy Function

$$W_g = U_i - U_f = -(U_f - U_i)$$

(1) $\boxed{W_g = -\Delta U_g}$

Elastic Potential Energy Function

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

$\boxed{U_s = \frac{1}{2}kx^2}$ Elastic Potential Energy Function

$$W_s = U_i - U_f = -(U_f - U_i)$$

(2) $\boxed{W_s = -\Delta U_s}$

Note from equation (1) and (2) that only changes in potential energy have a physical significance!

a) For the gravitational PE function we may choose the reference value of $U_g = mgy = 0$ at any convenient reference point.
b) For the elastic PE function, $U_s = 0$ only when $x = 0$.

- With every conservative force there is a potential energy function associated with that force.

Equation (1) and (2) above provide a formal definition for the potential energy function.

Def: The change in PE function associated with a conservative force is equal to the negative of the work done by that conservative force.

$\boxed{\Delta U = -W_c = -\int_a^b \vec{F}_c \bullet d\vec{s}}$ Definition of Potential Energy

Ex. Gravitational PE Function

$$U_f - U_i = -\int_a^b \vec{F}_g \cdot d\vec{s} = -\int_a^b mg - \vec{j} \cdot (dx\vec{i} + dy\vec{j})$$

$$U_f - \cancel{U_i}^{\,0} = mg \int_{y_i}^{y_f} dy = mgy_f - mg\cancel{y_i}^{\,0}$$

$$\boxed{U_g = mgy}\ \checkmark$$

Ex. Elastic PE Function

$$U_f - U_i = -\int_a^b \vec{F}_s \cdot d\vec{s} = -\int_{x_i}^{x_f} -kx\vec{i} \cdot dx\vec{i}$$

$$U_f - \cancel{U_i}^{\,0} = k\int_{x_i}^{x_f} x\,dx = \frac{1}{2}kx_f^2 - \frac{1}{2}k\cancel{x_i^2}^{\,0}$$

$$\boxed{U_s = \frac{1}{2}kx^2}$$