Relations and Functions

The amount of electricity produced in recent years in the U.S. can be shown using ordered pairs. The first number is the year, and the second number is the amount in billions of kilowatt-hours.

{(1974, 1870), (1976, 2040), (1978, 2200),

(1980, 2290), (1982, 2240), (1984, 2300)}

Year	Kilowatt Hours (billions)
1974	1870
1976	2040
1978	2200
1980	2290
1982	2240
1984	2300

A set of ordered pairs is called a **relation.** The set of first coordinates, in this case years, is called the **domain** of the relation. The set of second coordinates, kilowatt hours (in billions), is called the **range** of the relation.

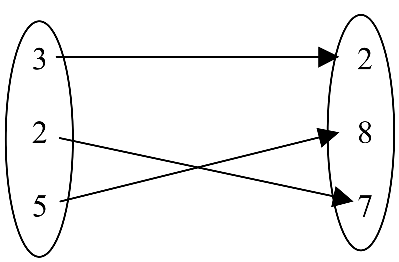
Definition of Relation, Domain, and Range

A relation is a set of ordered pairs. The domain is the set of all first coordinates of the ordered pairs. The range is the set of all second coordinates of the ordered pairs.

A mapping illustrates how each element in the domain of a relation is paired with an element in the range. The following diagrams are mappings of the given relations.

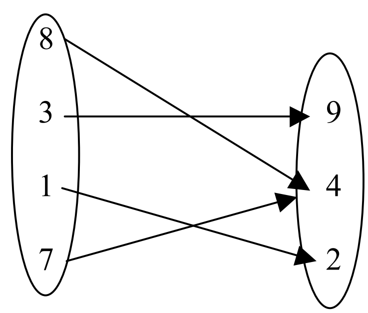
{(3, 2), (2, 7), (5, 8)}

domain **range**



{(8, 4), (3, 9), (1, 2), (7, 4)}

domain **range**

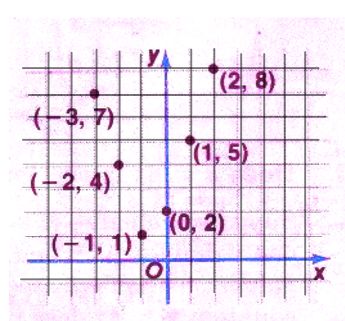


State a relation shown by the graph. Then state the domain and range of the relation.

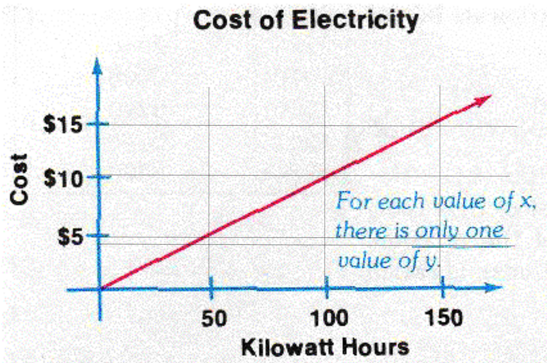
The relation is
{(-3, 7), (-2, 4), (-1, 1), (0, 2), (1, 5), (2, 8)}.

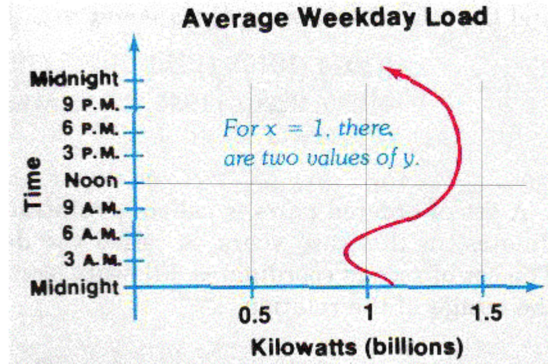
The domain is {-3, -2, -1, 0, 1, 2}.

The range is {7, 4, 1, 2, 5, 8}.



The graph on the left shows how the amount of electricity consumed compares with the cost of the electricity. The graph on the right shows the amount of electricity used in a certain area at different hours in an average weekday.





The first graph shows a special type of relation called a **function.**

A function is a relation in which each element of the domain is paired with exactly one element of the range. ***Definition of Function***

Examples:

Is {(2, 3), (3, -4), (4, 1), (1, 3)} a function?

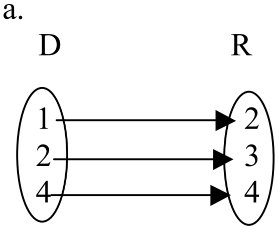
This relation is a function since each element of the domain is paired with exactly one element of the range. Is {(4, 4), (-2, 3), (4, 2), (3, 4), (1, 1)} a function? This relation is not a function. The element 4 of the domain is paired with two different elements of the range, 4 and 2. Which of the following mappings represent functions?

Is {(4, 4), (-2, 3), (4, 2), (3, 4), (1, 1)}

This relation is not a function. The element 4 of the domain is paired with two different elements of the range, 4, and 2.

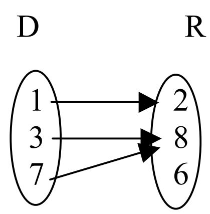
Which of the following mappings represent functions?

a.



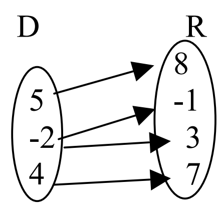
yes

b.



yes

c.

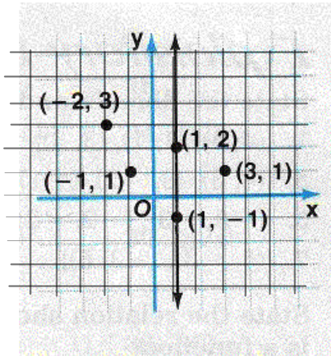


no

Does y = 4x represent a function?

Suppose the value of x is 3. What is the corresponding value of y? Is there more than one value for y? When x is 3, y is 12. There is only one value for y. If you try other values of x you will see that they are always paired with exactly one value of y. The equation y = 4x does represent a function.

The graph on the right represents the following relation.



{(-2, 3), (- 1, 1), (1, 2), (1, - 1), (3, 1)}

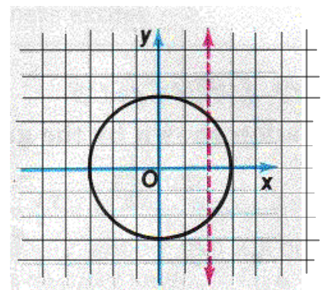
Suppose you drew a vertical line through each point on the graph. The vertical line through (1, 2) would also pass through(1, - 1). This shows that the relation is not a function. There are two elements of the range, 2 and - 1, that pair with one element of the domain, 1.

If **any vertical line drawn on the graph of a relation passes through no more than one point of that graph, then the relation is a function.**

Vertical Line Test for a Function

Use the vertical line test to determine if the relation graphed is a function.

The vertical line whose equation is x = 2 intersects the graph at (2, 2) and (2, -2). Therefore, the relation is not a function.



Equations that represent functions often are written in a special way. The equation y = 2x + 1 can be written f(x) = 2x + 1. The symbol f(x) is read "f of x." Likewise, the symbol f(3) is read "f of 3."If 3 is an element of the domain of the function, then f(3) is the corresponding element of the range. To show that the value of f(3) is 7, you write f(3) = 7.

Letters other than f can be used to represent a function. for example, the equation, y = 4x + 3 can be written g(x) = 4x + 3

Example:

Find f(15) if f(x) = $100x - 5x^2$.

f(x) = $100x - 5x^2$
f(15) = $100(15) - 5(15)^2$
= 1500 - 5(225)
= 375
Therefore, f(15) = 375

Find g(a + 2) if g(x) = $x^2 - 7$

g(x) = $x^2 - 7$
g(a + 2) = $(a+2)^2 - 7$
= $a^2 + 4a + 4 - 7$
= $a^2 + 4a - 3$
Therefore, g(a + 2) = $a^2 + 4a - 3$

In this book, sometimes the equation for a function is given without a specified domain. In such cases, the domain is understood to be all real numbers for which the function is defined. The corresponding range values are also real numbers.