CONCEPTUAL UNDERSTANDING VERSUS PLUG 'N' CHUG

"Plug 'n' chug" is a term, often used in a mocking way, to describe the mechanics of doing algebraic (or computational) work. The sound of the term suggests a machine, and refers to the possibility that the person may be doing algebra simply by rote, without really understanding what they're doing, and especially why they're doing what they're doing. Basically, they've become algebra machines, instead of learning human beings.

Plug 'n' chug is common among students who have any type of math phobia, because the (vaguely compulsive) ritual is a useful tool to help to ease their anxiety, but it should not be the final goal. It also occurs among some higher achieving students, who simply memorize solutions and are adept at algebraic manipulation.

Signs that the tutee doesn't understand the concept and is relying only on plug 'n' chug:

1. The tutee cannot tell when the plug 'n' chug technique applies, and when it doesn't, or when it might not be the most efficient way. (See Math 212 example.)
2. The tutee confuses similar situations and mixes up corresponding techniques. (See Math 114 example 1.)
3. The tutee insists on solving problems which are similar, yet subtly different, in the exact same way every time. (See Math 212 example.)
4. The tutee cannot solve a similar problem when the same relevant information is presented in a different way (eg. by graph, versus by formula, versus by table of values, versus a verbal description). (See Math 1A example.)
5. The tutee confuses the different variables within a single formula. (See Math 212 example 2 and Math 10 example.)
6. After seeming to understand the concept, the tutee reverts to the original techniques, mistakes and misunderstandings.
7. The tutee cannot remember a significant amount of foundation material from a recently taken prerequisite course, or from earlier in the class.
8. When asked to explain the reasoning behind their work, the tutee simply echoes back each algebra step that was executed, without being able to explain why they did it, or why it worked ("add 2, then divide by 5").
9. The tutee does not check their answer, or even know how to.
10. At calculus level and above, the tutee does not know when an algebraic technique based on a theorem can or cannot be used.

Signs that the tutor is teaching the tutee plug 'n' chug without concepts:

1. The tutor feels impatient, and uses phrases like "When you see that, just do this".
2. The tutor teaches shortcuts, but doesn't explain or make sure the tutee understands why the shortcuts work.
3. The tutor writes down an example, and asks the tutee to imitate it, usually in silence or just stating the algebraic manipulations, without any reasoning.

Getting tutees beyond plug 'n' chug to real conceptual understanding is a difficult task, and one that many education specialists have tried to tackle. (Reform Calculus was a major move in that direction.) Math anxiety is an obstacle, as is earlier educational experience. The tutee's attitude toward math and learning in general can also be a huge hurdle.

Ideas on how to get beyond plug 'n' chug as a tutor:

1. Identify the underlying concept the tutee is struggling with (eg. order of operations, inverse operations, definitions of mathematical terms) and review it carefully and slowly. It may be far below the level of their current class.
2. Use examples which involve specifics (eg. numbers instead of variables), and generalize from them to algebra.
3. Present the concept non-algebraically first (eg. using graphs, pictures, physical objects, motion, words).
4. Write down the meaning and units of all variables. Write down all formulae before substituting in values. Use physical motion (pointing) and spoken words to connect a substituted value to the original variable in the formula, and the meaning and units of that variable.
5. Ask the tutee to explain why each step is important, and how it helps to solve the problem.
6. Present the tutee with similar problems that should be handled differently, and ask them to compare and contrast which techniques work best and why, and how to recognize when one technique is preferable to another.
7. Ask the tutee to explain to a peer who is having difficulty with the same material. The peer should ask your tutee for clarifications if they don't understand.
8. ***? Your suggestions ?***

MATH SCENARIOS

In each scenario, identify the concept(s) the tutee needs to grasp, and find a way to present that concept to help the tutee overcome their difficulty.

Math 210:

The tutee knows that division involving 0 is tricky, but he can never remember whether $1/0$, $0/1$ and $0/0$ are 0 or undefined. The tutee also doesn't understand why $0/0 \neq 0$ and $0/0 \neq 1$.

Math 212:

The tutee can solve equations like $12x - 19 = 53$ by addition and division, and equations like $3(3x-1) = 14$ using distribution, addition and division, but is unsure how to start solving $18(12x-19) = 90$, because she is afraid of large numbers (anything bigger than 50). How do you explain to her an alternate method that doesn't require large numbers, and how do you explain to her when to use that method versus her traditional method ?

Math 212:

To find the *x*-intercept of $f(x) = 2x + 7$, the tutee sets x to 0 and gets 7.

Math 114:

In trying to solve inequalities like $|x-3| < 5$, the tutee expands the inequality into

$x - 3 < 5 \quad and \quad x - 3 < -5$

OR $\quad x - 3 < 5 \quad or \quad x - 3 > -5$

In trying to solve inequalities like $|x-3| > 5$, he expands the inequality into

$x - 3 > 5 \quad and \quad x - 3 > -5$

OR $\quad x - 3 > 5 \quad and \quad x - 3 < -5$

Math 114:

In trying to simplify $\dfrac{x^2 - 3x + 2}{x^2 - 4x + 4}$, the tutee cancels out the x^2's, and the x's, and the 2 against the 4, and gets $\dfrac{1-3+1}{1-4+2} = \dfrac{-1}{-1} = 1$. The tutee doesn't understand why this answer is obviously wrong (regardless of what technique was used), and why this technique is wrong.

Math 10:

The tutee is asked to find the *z*-score of a weight of 25 pounds, when the mean is 15 pounds and the standard deviation is 5 pounds, and gives an answer of $\dfrac{15-25}{5} = -2$. The tutee doesn't understand why this answer is obviously wrong, and how to correct it. Alternately, the tutee gives an answer of $\dfrac{25-5}{15} = 1.333$.

Math 51:

The tutee repeatedly confuses the period of $y = \sin x$ as π, instead of 2π. Also, he calculates the period of $y = \sin 3x$ as $3 \times \pi = 3\pi$.

Math 1A:

The tutee can take the derivatives of functions like $x^3 \tan^{-1} x \ln x$ using the product rule very quickly. When asked to find $g'(3)$ given that $g(x) = x^2 f(x)$, and that $f(3) = 5$ and $f'(3) = 7$, she has no idea how to begin.

Math 1A:

The tutee is asked to find the equation of the tangent line to $y = x^4$ at $x = 3$, and gives an answer of $y - 81 = 4x^3(x-3)$. He fails to understand why this answer is clearly wrong, and how to correct it.

DISCUSSION OF MATH SCENARIOS

Math 212:

> The tutee can solve equations like $12x-19=53$ by addition and division, and equations like $3(3x-1)=14$ using distribution, addition and division, but is unsure how to start solving $18(12x-19)=90$, because she is afraid of large numbers (anything bigger than 50). How do you explain to her an alternate method that doesn't require large numbers, and how do you explain to her when to use that method versus her traditional method ?

This scenario is a typical result of the tutee being told "when you see a linear equation, expand it all out, collect all terms with *x* on one side, collect all constants on the other, and divide" – an algorithm without an explanation.

The key concept here is that **when trying to solve for x (when x only appears once in the entire equation), you need to undo what was done to x in the first place, but in the reverse order**. If they learn to think about it this way, they will find solving non-linear equations (when x only appears once in the entire equation) much easier, because they will invoke the same concept. Otherwise, they will view each type of equation as distinct and requiring a separate method.

So in this case, starting with x,	x
we multiplied it by 12,	$12x$
then subtracted 19,	$12x-19$
then multiplied by 18.	$18(12x-19)$

So to isolate x again,	$18(12x-19)=90$
we need to divide both sides by 18,	$12x-19=5$
then add 19 to both sides,	$12x=24$
then divide both sides by 12.	$x=2$

The key to deciding when to use this method is whether it introduces ugly fractions. Because of the division in the first step, it is most efficient to use this method when the division gives an integer.

You can compare the order of undoing to taking a walk. **[INSERT MAP LEAVING HOME]**
If you start at home,
then you walk north 2 blocks,
then you walk east 3 blocks,
then you walk south 1 block,
then the safest way to get back home is, **[INSERT MAP GOING BACK HOME]**
you walk north 1 block,
then you walk west 3 blocks,
then you walk south 2 blocks.

[If they ask why you don't just walk 1 block south and 3 blocks west to go home, you can say that the blocks you walked originally were safe, and all the other blocks had killer bunny rabbits on them.]

Math 212:

> To find the *x*-intercept of $f(x)=2x+7$, the tutee sets x to 0 and gets 7.

This scenario is a typical result of the tutee being told "to find the *x*-intercept, set *y* to 0; to find the *y*-intercept, set *x* to 0" and then they confuse the variables because of the extremely similar structure of the two instructions.

The key concepts here are that **the *x*-intercept is where the graph crosses the *x*-axis**, and that **every point on the *x*-axis has a *y*-coordinate of 0**. The first concept is a ***definition***, so it has to be memorized. The second concept can be demonstrated by asking the tutee to identify the coordinates of various points along the *x*-axis, and asking them what the coordinates of all the points have in common.

To find the *x*-intercept then, we want to know what the value of *x* is when the value of *y* is 0. Since $f(x)$ is the stand-in for y, we replace $f(x)$ with 0 in the function definition above, and solve for x.

Math 114:

In trying to simplify $\frac{x^2-3x+2}{x^2-4x+4}$, the tutee cancels out the x^2's, and the x's, and the 2 against the 4, and gets $\frac{1-3+1}{1-4+2}=\frac{-1}{-1}=1$. The tutee doesn't understand why this answer is obviously wrong (regardless of what technique was used), and why this technique is wrong.

This scenario is a typical result of the tutee being told "if you see the same expression in the numerator and denominator, cancel it out" without emphasizing that the expression must be a factor of the entire numerator. Another variation of this error is $\frac{x+2}{x+4}+\frac{x-3}{x+1}=\frac{(x+2)(x+1)+(x-3)(x+4)}{(x+4)(x+1)}=\frac{(x+2)+(x-3)}{1}=2x-1$, where the $(x+4)$ and $(x+1)$ were cancelled.

The key concept here is that **cancelling is writing a fraction as a product of 2 fractions, where one of the fractions actually equals 1**.

$$\frac{x^2-3x+2}{x^2-4x+4} = \frac{(x-1)(x-2)}{(x-2)(x-2)} = \frac{x-1}{x-2}\times\frac{x-2}{x-2} = \frac{x-1}{x-2}\times 1 = \frac{x-1}{x-2}$$

It's impossible to do the same thing with the "cancelling" the tutee is trying to do. Ask her to try, and point out the algebraic errors that arise, like $\frac{x^2-3x+2}{x^2-4x+4} = \frac{(x^2)(-3x)(2)}{(x^2)(-4x)(4)}$. (Also, her answer is obviously wrong because the numerator and denominator are not equal.)

The tutee should do several problems fully written out as above, before they start just cancelling out factors in the simplified way.

Math 10:

The tutee is asked to find the *z*-score of a weight of 25 pounds, when the mean is 15 pounds and the standard deviation is 5 pounds, and gives an answer of $\frac{15-25}{5}=-2$. The tutee doesn't understand why this answer is obviously wrong, and how to correct it.

Alternately, the tutee gives an answer of $\frac{25-5}{15}=1.333$.

This scenario is a typical result of the tutee being told "just memorize the formula", when the reasoning and operations behind the formula are actually fairly straightforward.

The key concept here is that **the *z*-score is a way to measure how far a piece of numerical data is from the mean, using a scale (standard deviations) that allows us to compare the spread of data regardless of meaning, value and unit of measurement.**

```
                              15 lb
-----|------|------+------|------|-----
  5 lb    5 lb    5 lb    5 lb    5 lb    5 lb
z =  -2μ     -1μ      0μ      1μ      2μ
```

Since the data of 25 pounds is more than 15 pounds, it is to the right of the mean on the scale, so its *z*-score must be positive. The data is actually $25\ lb - 15\ lb = 10\ lb$ to the right of the mean, and since each notch is $5\ lb$, the data is $\frac{10\ lb}{5\ lb}=2$ notches or 2 standard deviations from the mean, so its z-score is 2.

If the tutee can fully understand the concept, and they use the above method for several different problems correctly, you can begin to ask them what the pattern of operations is, "data minus mean first, then divide by standard deviation".

NOTE: It is not customary to write μ as part of the *z*-score, so you should only use it in diagrams, but not in formal work.

Math 1A:

The tutee can take the derivatives of functions like $x^3 \tan^{-1} x \ln x$ using the product rule very quickly. When asked to find $g'(3)$ given that $g(x) = x^2 f(x)$, and that $f(3) = 5$ and $f'(3) = 7$, she has no idea how to begin.

This scenario is a typical result of the tutee not memorizing the exact formula and the conditions under which it is to be used, instead shortcutting it to "the derivative of the first times the second, plus the first times the derivative of the second".

The key concept here is that **the derivative function is a (shortcut) way to find the derivative** *at any point* **using its coordinates without using limits**, and that **the product rule is a way to find the derivative of a product of functions** *at any point* **using the functions and their derivatives** *at that point*.

The product rule says that if $w(x) = u(x)v(x)$, then $w'(x) = u'(x)v(x) + u(x)v'(x)$

So, since $g(x) = x^2 f(x)$

So, $g'(x) = (x^2)' f(x) + x^2 f'(x) = 2xf(x) + x^2 f'(x)$

So, $g'(3) = 2(3)f(3) + (3)^2 f'(3) = 6(5) + 9(7) = 93$

The tutee must memorize the product rule as written, not just have an idea of how to use it. (The rule is too complicated for the tutee to derive whenever they need it, so complete memorization is required in this situation.). To reinforce the concept that the derivative applies at every point where all functions are differentiable, highlight the step $g'(3) = 2(3)f(3) + (3)^2 f'(3)$.

Now that you've seen some examples, identify the key concept(s) for the remaining scenarios, and find a way to present them. At the very least, try the Math 210 division-by-zero scenario.

NOTE: None of the situations below require that the tutee memorize any formulae or values. They can all be explained conceptually.

Math 210:

The tutee knows that division involving 0 is tricky, but he can never remember whether $1/0$, $0/1$ and $0/0$ are 0 or undefined. The tutee also doesn't understand why $0/0 \neq 0$ and $0/0 \neq 1$.

Math 114:

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CHEMISTRY SCENARIOS

In each scenario, identify the concept(s) the tutee needs to grasp, and find a way to present that concept to help the tutee overcome their difficulty.

NOTE: All mass/mole conversions are correct. All arithmetic is numerically correct, but may not be the correct arithmetic required.

1:

Question: Determine the number of nitrogen atoms in 43.5 g of $Mg(NO_3)_2$.

Solution: 43.5 g • (1 mol/148.305 g) = 0.2933 moles $Mg(NO_3)_2$
0.2933 moles $Mg(NO_3)_2$ • (6.022 x 10^{23} N atoms/1 mol N atoms)= 1.77 x 10^{23} N atoms

2:

Question: Given the reaction $Ca_3(PO_4)_2 + 4\ HNO_3 \rightarrow 2\ Ca(NO_3)_2 + Ca(H_2PO_4)_2$,
If 95.0 kg of $Ca_3(PO_4)_2$ is reacted with 75 kg of HNO_3, what is the theoretical yield of $Ca(NO_3)_2$?

Solution: 95.0 x 10^3 g • (1 mol/310.18 g) = 306.27 mol $Ca_3(PO_4)_2$
75 x 10^3 g • (1 mol/63.108 g) = 1.188 x 10^3 mol HNO_3
Fewer moles of $Ca_3(PO_4)_2$ than HNO_3
306.27 mol $Ca_3(PO_4)_2$ • (2 mol $Ca(NO_3)_2$/1 mol $Ca_3(PO_4)_2$) • (164.10 g/1mol) = 1.005 x 10^5 g or 100.5 kg $Ca(NO_3)_2$

DISCUSSION OF CHEMISTRY SCENARIOS

Since I haven't taken chemistry since 19✷✷, the terminology below probably has errors.
Please help by identifying incorrect use of language. Possible errors are highlighted in red.

1:

The key concept here is that **the number of atoms of an element in a molecule is determined by all functional groups in which it appears, and the multiplicity of those functional groups**.

Even though N appears without a subscript, suggesting that there is 1 atom of N per molecule, its functional group appears twice, so the total number of atoms of N per molecule is actually (1 N atom/functional group) × (2 functional groups/molecule) = 2 N atoms/molecule.

2:

The key concept here is that **a reaction is limited by the number of moles present of each reagent**, *and* **the number of moles required of each reagent**.

Even though there are more moles of $Ca_3(PO_4)_2$ than of HNO_3, the reaction requires 4 times as many moles of HNO_3 as of $Ca_3(PO_4)_2$, which is not the case here. So the yield of $Ca(H_2PO_4)_2$ is actually limited by the number of moles of HNO_3, and not of $Ca_3(PO_4)_2$.

Either divide the number of moles present of each reagent by the number of molecules required of that reagent in the equation, and then select the lowest ratio to determine which reagent limits the yield, or calculate the yield based on each individual reagent (assuming an infinite amount of all other reagents), and then select the lowest yield, since the corresponding reagent would prevent the yield from being any higher.