2.4 Proving a Limit

The precise definition can be used to prove that a limit is as stated.

Recall: \( \lim_{x \to a} f(x) = L \)

if for every number \( \varepsilon > 0 \) we can find a number \( \delta > 0 \) such that

\[
\text{if } 0 < |x - a| < \delta \text{ then } |f(x) - L| < \varepsilon
\]

To use this definition for a problem, we need to do 2 things:

1. For any number \( \varepsilon > 0 \), find a suitable \( \delta > 0 \)
2. Show that this \( \delta \) “works? – it satisfied statement (1)
   above
Example. Prove that \( \lim_{x \to 3}(4x-5) = 7 \)

1. Guess a value for \( \delta \)

2. Show that this \( \delta \) “works”
Do: Prove \( \lim_{x \to -2} (3x+5) = -1 \)
Example 2: Prove \( \lim_{x \to -3} \frac{x^2 - 9}{x + 3} = -6 \)