Objectives

By the end of this set of slides, you should be able to:

1. Classify hypothesis tests by type
2. Conduct and interpret hypothesis tests for two population means
3. Conduct and interpret hypothesis tests for two population proportions
4. Conduct and interpret hypothesis tests for matched pairs or samples
Hypothesis Testing with Two Samples

- We are interested now in conducting hypothesis testing when we have two groups.

- Characteristics of this type of test:
  - Test of two means compares the population means for 2 populations for the same variable:
  - **Example**: Comparing the average age of male and female community college students.
  - Test of two proportions compares population proportions for 2 populations for the same variable:
  - **Example**: Comparing proportions of high school students and college students who take math classes.
Hypothesis Testing with Two Samples

- There are going to be 4 different tests (or test statistics) to consider:
  1. Comparing the population average of two groups when the population standard deviation is **KNOWN**
  2. Comparing the population average of two groups when the population standard deviation is **UNKNOWN**
  3. Comparing the population proportions of two groups
  4. Comparing the average difference between two dependent groups

- **Requirements:** We now need to consider whether or not the two groups are independent or dependent.
- The test in 1-3 are for when the two groups are independent.
- The test in 4 is when the two groups are dependent.
- But first we need to be able to identify which test we are dealing with, let’s look at Handout #8.
Two Population Means with Known Standard Deviation

- We want to compare the population means $\mu_1$ and $\mu_2$
- There are 3 possible null and alternative hypothesis:
  - $H_0: \mu_1 = \mu_2$ versus $H_a: \mu_1 \neq \mu_2$
  - $H_0: \mu_1 \leq \mu_2$ versus $H_a: \mu_1 > \mu_2$
  - $H_0: \mu_1 \geq \mu_2$ versus $H_a: \mu_1 < \mu_2$
- When the two population standard deviations are **KNOWN** the test statistic is
  \[
  z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}}}
  \]
  - $\bar{x}_1$ and $\bar{x}_2$ are the sample averages of the two groups
  - $\sigma_1$ and $\sigma_2$ are the population standard deviation of the two groups
  - $n_1$ and $n_2$ are the samples sizes of group 1 and group 2
Two Population Means with Unknown Standard Deviation

- We want to compare the population means $\mu_1$ and $\mu_2$
- There are 3 possible null and alternative hypothesis:
  - $H_0: \mu_1 = \mu_2$ versus $H_a: \mu_1 \neq \mu_2$
  - $H_0: \mu_1 \leq \mu_2$ versus $H_a: \mu_1 > \mu_2$
  - $H_0: \mu_1 \geq \mu_2$ versus $H_a: \mu_1 < \mu_2$

- When the two population standard deviations are **UNKNOWN** the test statistic is

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}}$$

- $\bar{x}_1$ and $\bar{x}_2$ are the sample averages of the two groups
- $s_1$ and $s_2$ are the sample standard deviation of the two groups
- $n_1$ and $n_2$ are the samples sizes of group 1 and group 2
The Degrees of Freedom

- The number of the degrees of freedom (df) requires a somewhat complicated calculation
- The degrees of freedom are not always a whole number
- The degrees of freedom are the following

\[
df = \frac{\left( \frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2} \right)^2}{\left( \frac{1}{n_1-1} \right) \left( \frac{(s_1)^2}{n_1} \right)^2 + \left( \frac{1}{n_2-1} \right) \left( \frac{(s_2)^2}{n_2} \right)^2}
\]

- The value of df is rounded down to the nearest integer
- To simplify calculations, we will calculate the degree of freedom as the minimum between \( n_1 - 1 \) and \( n_2 - 1 \)
Two Population Proportions

- We want to compare the population proportions $p_1$ and $p_2$
- Their are 3 possible null and alternative hypothesis:
  - $H_0: p_1 = p_2$ versus $H_a: p_1 \neq p_2$
  - $H_0: p_1 \leq p_2$ versus $H_a: p_1 > p_2$
  - $H_0: p_1 \geq p_2$ versus $H_a: p_1 < p_2$

- The test statistic is
  \[
  z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1} + \frac{\bar{p}(1-\bar{p})}{n_2}}}
  \]

- $\hat{p}_1$ and $\hat{p}_2$ are the sample averages of the two groups
- $n_1$ and $n_2$ are the samples sizes of group 1 and group 2
- $\bar{p}$ is the pooled proportion
  \[
  \bar{p} = \frac{x_1 + x_2}{n_1 + n_2}
  \]

- $x_1$ and $x_2$ are the number of successes in group 1 and group 2
Matched or Paired Samples

• We want to compare the population means $\mu_1$ and $\mu_2$ however the samples are not independent.

• There are 3 possible null and alternative hypothesis:
  - $H_0: \mu_d = 0$ versus $H_a: \mu_d \neq 0$
  - $H_0: \mu_d \leq 0$ versus $H_a: \mu_d > 0$
  - $H_0: \mu_d \geq 0$ versus $H_a: \mu_d < 0$

• The test statistic is

$$t = \frac{\bar{x}_d - \mu_d}{\left( \frac{s_d}{\sqrt{n}} \right)}$$

• $\bar{x}_d$ is the average difference between the pairs.
• $\sigma_d$ is the standard deviation of the difference between the pairs.
• $n$ is the number of pairs.
• $df = n - 1$
Matched or Paired Samples

- So what is $\bar{x}_d$ and $s_d$?

- Example: Let’s imagine we want to compare the effects of a new diet on weight loss. We have 4 individuals using the diet and their before and after weights are recorded.

<table>
<thead>
<tr>
<th>Weight Before Diet</th>
<th>130</th>
<th>190</th>
<th>170</th>
<th>220</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight After Diet</td>
<td>120</td>
<td>185</td>
<td>175</td>
<td>200</td>
</tr>
<tr>
<td>Difference</td>
<td>10</td>
<td>5</td>
<td>-5</td>
<td>20</td>
</tr>
</tbody>
</table>

- $\bar{x}_d$ is the average of the differences between pairs

$$\bar{x}_d = \frac{10 + 5 + (-5) + 20}{4} = \frac{30}{4} = 7.5$$

- $s_d$ is the standard deviation of the differences between pairs

$$s_d = \sqrt{\frac{(10 - 7.5)^2 + (5 - 7.5)^2 + (-5 - 7.5)^2 + (20 - 7.5)^2}{3}} = 10.41$$
Matched or Paired Samples

- Let’s take a look at examples of all 4 tests on Handout #8