Practice Final Exam  
Math 10

You have two hours and fifteen minutes to complete this exam. It is closed book, but you may use one 8.5 by 11 inch piece of paper with notes and a calculator. You may not confer with any other person (other than the one administering the exam). You must show all work to receive full credit. If you write anything on the back of a page that you want to get graded, be sure to clearly state so on the front of the corresponding page.

1. The New York Times ran an article about a study in which Professor Denise Korniewicz and other Johns Hopkins researchers subjected laboratory gloves to stress. Among 200 vinyl gloves, 126 leaked. Among 200 latex gloves, 14 leaked. At the \( \alpha = 0.01 \) significance level test the claim that vinyl gloves have a larger leak rate than latex gloves. Write down the null and alternative hypotheses, compute the statistic, and find the critical value. What is your conclusion?

\[
\begin{align*}
H_0 & : \hat{p}_1 \leq \hat{p}_2 \\
H_a & : \hat{p}_1 > \hat{p}_2
\end{align*}
\]

\[
\hat{p}_1 = \frac{X_1}{n_1} = \frac{126}{200} = 0.63
\]

\[
\hat{p}_2 = \frac{X_2}{n_2} = \frac{14}{200} = 0.07
\]

\[
\hat{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{126 + 14}{200 + 200} = 0.35
\]

\[
z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}(1-\hat{p}) \frac{1}{n_1} + \hat{p}(1-\hat{p}) \frac{1}{n_2}}} = \frac{0.63 - 0.07}{\sqrt{0.35(1-0.35) \frac{1}{200} + 0.35(1-0.35) \frac{1}{200}}} = 11.74 \leftarrow \text{Test Statistic}
\]

Since the test statistic is inside the critical region, we will reject \( H_0 \) and conclude that vinyl gloves have a larger leak rate than latex gloves.
2. In a small pilot study of antibiotics, a standard dose of the broad-spectrum antibiotic, gentamicin was administered to five volunteers and in a later trial the same quantity of erythromycin was given to the same people. The results below show the concentration of each drug in the bloodstream one hour after administration. The measurements are in \( \mu g/ml \) and are assumed to be normally distributed. Is the bloodstream concentration of gentamicin significantly different from that of erythromycin? Perform the appropriate test using \( \alpha = 0.05 \). State the null and alternative hypotheses, compute the statistic, find the critical value(s) and provide your conclusions.

<table>
<thead>
<tr>
<th>Person</th>
<th>Gentamicin</th>
<th>Erythromycin</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28</td>
<td>31</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>58</td>
<td>67</td>
<td>-9</td>
</tr>
<tr>
<td>3</td>
<td>43</td>
<td>42</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>38</td>
<td>36</td>
<td>2</td>
</tr>
</tbody>
</table>

\[ H_0: \mu_d = 0 \]
\[ H_a: \mu_d \neq 0 \]

\[ \bar{X}_d = \frac{-3 - 9 + 1 + 2}{4} = \frac{-9}{4} = -2.25 \]

\[ S_d = \sqrt{\frac{\sum (X_d - \bar{X}_d)^2}{n-1}} = \sqrt{\frac{(-3 - (-2.25))^2 + (-9 - (-2.25))^2 + (-1 - (-2.25))^2 + (2 - (-2.25))^2}{4-1}} = 4.99 \]

\[ t = \frac{\bar{X}_d - \mu_d}{S_d / \sqrt{n}} = \frac{-2.25}{4.99 / \sqrt{4}} = -0.902 \]

\( \alpha = 0.05 \)

\( df = n-1 = 4-1 = 3 \)

Since our test statistic is outside the critical region, we will fail to reject \( H_0 \) and conclude that the bloodstream concentration of gentamicin and erythromycin are not significantly different.

Critical values are -3.182 and 3.182
3. Use the data in the table to solve the following parts.

\[
\begin{array}{ccccccc}
  x & 4 & 2 & 8 & 6 & 11 & \text{ } n = 5 \\
  y & 3 & 6 & 12 & 10 & 10 & \text{ } \\
\end{array}
\]

(a) Draw a scatterplot of the data. Is there a correlation between \(x\) and \(y\)? If so, is it positive or negative?

\[
\text{Looks like a positive correlation}
\]

(b) Calculate the correlation coefficient.

\[
r = \frac{n \sum xy - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \sqrt{n(\sum y^2) - (\sum y)^2}}
\]

\[
= \frac{5(290) - (31)(41)}{\sqrt{5(241) - 31^2} \sqrt{5(381) - 41^2}} = \frac{179}{253.8031} = 0.705
\]

(c) Conduct a hypothesis test to see if there is any correlation between \(x\) and \(y\). Assume \(\alpha = 0.05\)

\[
H_0: \rho = 0 \quad t = \frac{r}{\sqrt{1-r^2}} = \frac{0.705}{\sqrt{1-0.705^2}} = 1.72 \quad \text{Test Statistic}
\]

\[
\alpha = 0.05 \quad \text{df} = n-2 = 5.2 = 3
\]

Critical values are -3.182 and 3.182

Since our test statistic is outside the critical region, we will fail to reject \(H_0\) and conclude that \(x\) and \(y\) do not appear to be correlated.
4. For the following observed and expected frequencies, perform a goodness-of-fit test at the \( \alpha = 0.05 \) significance level. Write down the null and alternative hypotheses, compute the statistic, and find the critical value. What is your conclusion?

<table>
<thead>
<tr>
<th>Group</th>
<th>Observed</th>
<th>Expected</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>106</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>64</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>10</td>
</tr>
</tbody>
</table>

\( k = 4 \)

\( H_0: \) The observed frequency distribution is the same as the expected frequency distribution

\( H_a: \) The observed frequency distribution is not the same as the expected frequency distribution

\[
X^2 = \sum \frac{(O_i - E_i)^2}{E_i} = \frac{(106 - 100)^2}{100} + \frac{(64 - 60)^2}{60} + \frac{(24 - 30)^2}{30} + \frac{(6 - 10)^2}{10}
\]

\[
= 3.427 \quad \text{Test Statistic}
\]

\( \alpha = 0.05 \)

\( df = k - 1 = 4 - 1 = 3 \)

Critical value = 7.815

Since the test statistic is outside the critical region, we will fail to reject \( H_0 \) and conclude that the observed frequency distribution appears to be the same as the expected frequency distribution.
5. The General Social Survey asked 1373 men and 993 women in the United States whether they agreed that they were generally optimistic about the future. The results are presented in the following table.

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimistic</td>
<td>1148</td>
<td>815</td>
<td>1963</td>
</tr>
<tr>
<td>Pessimistic</td>
<td>225</td>
<td>178</td>
<td>403</td>
</tr>
<tr>
<td>Total</td>
<td>1373</td>
<td>993</td>
<td>2366</td>
</tr>
</tbody>
</table>

At an $\alpha = 0.05$ significance level, conduct a hypothesis test to test the claim that optimism is independent of gender. Write down the null and alternative hypotheses, compute the statistic, and find the critical value. What is your conclusion?

\[
E_1 = \frac{(1963)(1373)}{2366} = 1139.137 \\
E_2 = \frac{(1963)(993)}{2366} = 823.8626 \\
E_3 = \frac{(403)(1373)}{2366} = 233.8626 \\
E_4 = \frac{(403)(993)}{2366} = 169.1374
\]

\[
\chi^2 = \sum \frac{(O_i-E_i)^2}{E_i} = \frac{(1148-1139.137)^2}{1139.137} + \frac{(815-823.8626)^2}{823.8626} + \frac{(225-233.8626)^2}{233.8626} + \frac{(178-169.1374)^2}{169.1374} = 0.965 < \text{Test Statistic}
\]

$\alpha = 0.05$

\[
df = (r-1)(c-1) = (2-1)(2-1) = 1
\]

Critical value $= 3.481$

Since our test statistic is inside the critical region, we will fail to reject $H_0$ and conclude that optimism is independent of gender.