Identifying the Type of Test

Let's try to identify the type of test in following scenarios.

a) A hypothesis test is performed to determine if the proportion of patients undergoing a particular surgery who are cured is greater than 60%.

One sample proportion test

\[ H_0: \ p \leq 0.60 \]
\[ H_a: \ p > 0.60 \]

b) A hypothesis test is performed to determine if the proportion of patients undergoing a particular surgery who are cured is greater than the proportion who are cured by the current non-surgical standard treatment.

Two sample proportion test

\[ H_0: \ p_1 = p_2 \]
\[ H_a: \ p_1 > p_2 \]

c) A hypothesis test is performed to determine if the average time that a medication lasts is more than 3 hours.

One sample test of the mean

\[ H_0: \ \mu \leq 3 \]
\[ H_a: \ \mu > 3 \]

d) A hypothesis test is performed to determine if the average times that medication A and medication B last are the same.

Two sample test of the mean

\[ H_0: \ \mu_1 = \mu_2 \]
\[ H_a: \ \mu_1 \neq \mu_2 \]

or

\[ H_0: \ \mu_d = 0 \]
\[ H_a: \ \mu_d \neq 0 \]

e) A hypothesis test is performed to determine if the average time to graduation is at Hudson University is 4 years.

One sample test of the mean

\[ H_0: \ \mu = 4 \]
\[ H_a: \ \mu \neq 4 \]

f) A hypothesis test is performed to determine if the average time to graduation is longer at a public university than at a private college.

Two sample test of the mean

\[ H_0: \ \mu_1 \leq \mu_2 \]
\[ H_a: \ \mu_1 > \mu_2 \]
g) A hypothesis test is performed to determine if taking an SAT training class increases SAT scores of those who take the training, on average.

Two sample test of the mean

\[ H_0: \mu_1 \leq \mu_2 \quad \text{or} \quad H_0: \mu_d \leq 0 \quad \text{if difference is after-before} \]

\[ H_0: \mu_1 > \mu_2 \quad H_0: \mu_d > 0 \]

h) A hypothesis test is performed to determine if recent female college graduates are subject to pay discrimination, earning less on average for similar work than recent male college graduates with similar qualifications.

Two sample test of the mean

\[ H_0: \mu_1 = \mu_2 \]

\[ H_0: \mu_1 < \mu_2 \]

Testing Hypotheses

In the following examples, identify the null hypothesis, alternative hypothesis, test statistic, critical value(s) and state the conclusion of the test.

Example #1

Mean entry-level salaries for college graduates with mechanical engineering degrees and electrical engineering degrees are believed to be approximately the same. A recruiting office thinks that the mean mechanical engineering salary is actually lower than the mean electrical engineering salary. The recruiting office randomly surveys 50 entry level mechanical engineers and 60 entry level electrical engineers. Their salaries were $46,100 and $46,700, respectively. Their sample standard deviations were $3,450 and $4,210, respectively. Conduct a hypothesis test to determine if you agree that the mean entry-level mechanical engineering salary is lower than the mean entry level electrical engineering salary.

\[ \mu_1 = \text{mean salary for mechanical engineers} \]
\[ \mu_2 = \text{mean salary for electrical engineers} \]

Two independent samples

Population SD is unknown

\[ \begin{align*}
\mu_0: \mu_1 & > \mu_2 \\
\mu_0: \mu_1 & < \mu_2 \\
\end{align*} \]

\[ z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{46100 - 46700}{\sqrt{\frac{(3450)^2}{50} + \frac{(4210)^2}{60}}} = \frac{-600}{730.3778} = -0.82 \]

\[ \alpha = 0.05 \quad df = \min\left\{n_1 - 1, n_2 - 1\right\} = \min\{49, 59\} = 49 \]

Critical value = -1.645

Fail to reject \( H_0 \) since our test statistic is not in the critical region. There does not appear to be sufficient evidence to conclude that the mean entry-level salary for mechanical engineers is lower than that of electrical engineers.
Example #2

Some manufacturers claim that non-hybrid sedans cars have a lower mean miles-per-gallon (mpg) than hybrid ones. Suppose that consumers test 21 hybrid sedans and get a mean of 31 mpg with a standard deviation of 7 mpg. 31 non-hybrid sedans get a mean of 22 mpg with a standard deviation of 4 mpg. Suppose that the population standard deviations are known to be 6 and 3, respectively. Conduct a hypothesis test to evaluate the manufacturers claim.

\[ \mu_1 = \text{mean mpg of non-hybrid sedans} \]  
\[ \mu_2 = \text{mean mpg of hybrid sedans} \]  
\[ H_0: \mu_1 = \mu_2 \]  
\[ H_a: \mu_1 < \mu_2 \]

Two independent samples
Population SD is known

\[ Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(22-31)}{\sqrt{\frac{6^2}{21} + \frac{3^2}{21}}} = -9 \]

\[ = -6.36 \]

Critical value: -1.645

Reject Ho since the test statistic is inside the critical region. Conclude that the mean mpg of non-hybrid sedans is less than that of hybrid sedans

Example #3

Researchers conduct a study to find out if there is a difference in the use of eReaders by different age groups. Randomly selected participants were divided into two age groups. In the 16-29 year old group, 7% of the 628 surveyed used eReaders, while 11% of the 2,309 participants 30 years and older used eReaders.

\[ p_1 = \text{proportion of eReader users between 16-29} \]  
\[ p_2 = \text{proportion of eReader users over 29} \]

Two independent samples
Assume \( \alpha = 0.05 \)

\[ H_0: p_1 = p_2 \]  
\[ H_a: p_1 \neq p_2 \]

\[ \bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{44 + 254}{628 + 2309} = 0.1015 \]

\[ x_1 = \hat{p}_1 \cdot n_1 = 0.07(628) \approx 44 \]

\[ x_2 = \hat{p}_2 \cdot n_2 = 0.11(2309) \approx 254 \]

\[ Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}} = \frac{0.07 - 0.11}{\sqrt{\frac{(0.1015)(0.8985)}{628} + \frac{(0.1015)(0.8985)}{2309}}} \]

\[ = -2.94 \]

Reject Ho
Example #4

A golf instructor is interested in determining if her new technique for improving players' golf scores is effective. She takes four new students. She records their 18-hole scores before learning the technique and then after having taken her class. The data are as follows:

<table>
<thead>
<tr>
<th>Player</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean score before class</td>
<td>83</td>
<td>78</td>
<td>93</td>
<td>87</td>
</tr>
<tr>
<td>Mean score after class</td>
<td>80</td>
<td>80</td>
<td>86</td>
<td>86</td>
</tr>
</tbody>
</table>

Difference = after score - before score

H₀: \( \mu_d \geq 0 \)  
Hₐ: \( \mu_d < 0 \)

\( \bar{d} = \frac{-3 + 2 - 7 - 1}{4} = \frac{-9}{4} = -2.25 \)

\[ S_d = \sqrt{(-3-(-2.25))^2 + (2-(-2.25))^2 + (-7-(-2.25))^2 + (-1-(-2.25))^2} = 3.77 \]

Test statistic \( t = \frac{\bar{d} - \mu_d}{S_d / \sqrt{n}} = \frac{-2.25 - 0}{3.77 / \sqrt{4}} = \frac{-2.25}{1.88} = -1.21 \)

\( df = n-1 = 4-1 = 3 \)

\( \alpha = 0.05 \)

Fail to reject H₀ and conclude that the new technique for improving players' golf scores is ineffective.