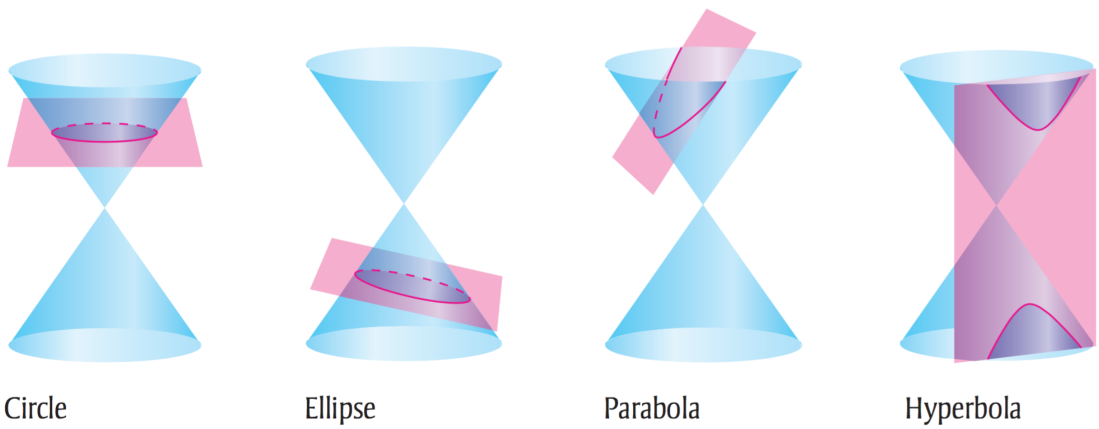
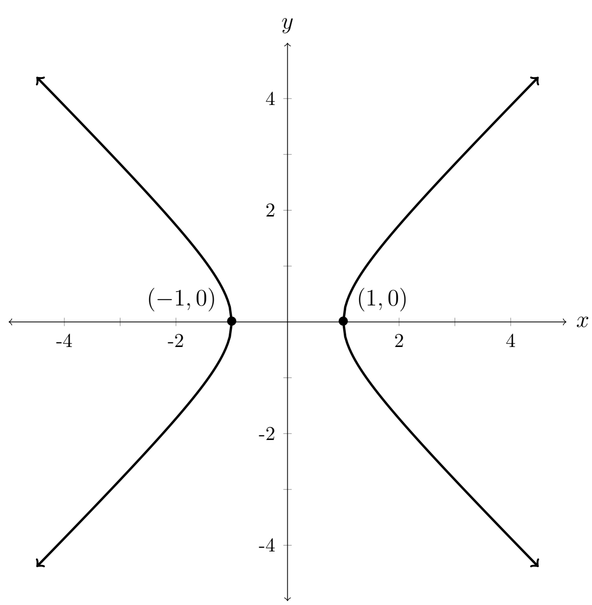
Hyperbolic Function Project

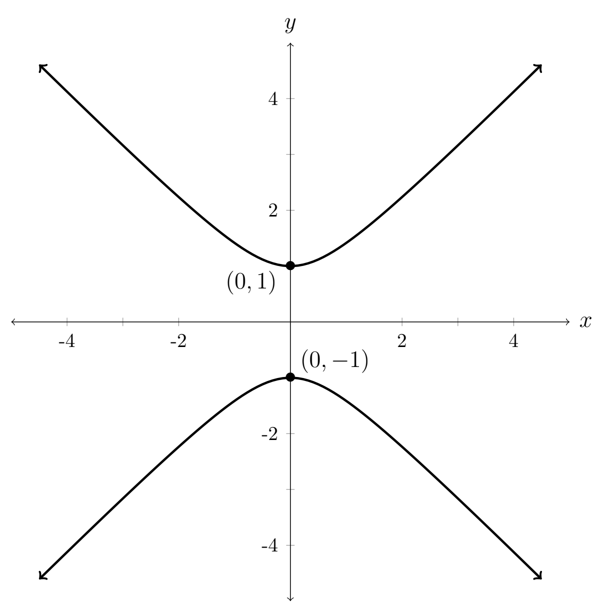
Circles are part of a family of curves called **conics**. The various conic sections can be derived by slicing a plane through a double cone.



A **hyperbola** is a conic with two basic forms:



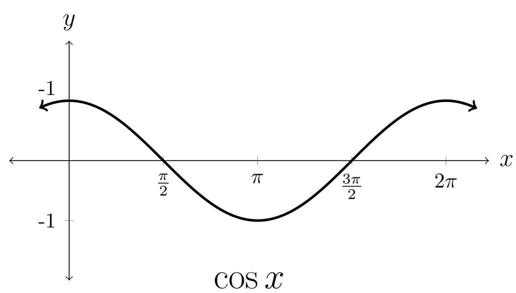
$x^2 - y^2 = 1$



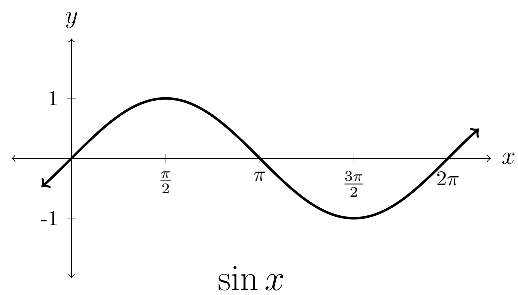
$y^2 - x^2 = 1$

$x^2 - y^2 = 1$ is the **unit hyperbola**.

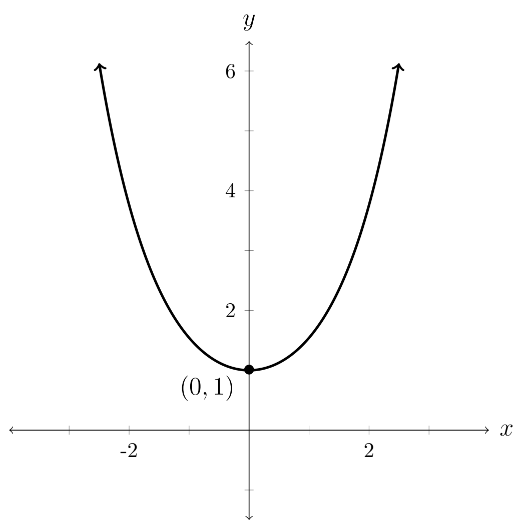
Hyperbolic Functions: Similar to how the trigonometric functions, cosine and sine, correspond to the x and y values of the unit circle ($x^2 + y^2 = 1$), there are hyperbolic functions, hyperbolic cosine (cosh) and hyperbolic sine (sinh), which correspond to the x and y values of the **right side** of the **unit hyperbola** ($x^2 - y^2 = 1$).



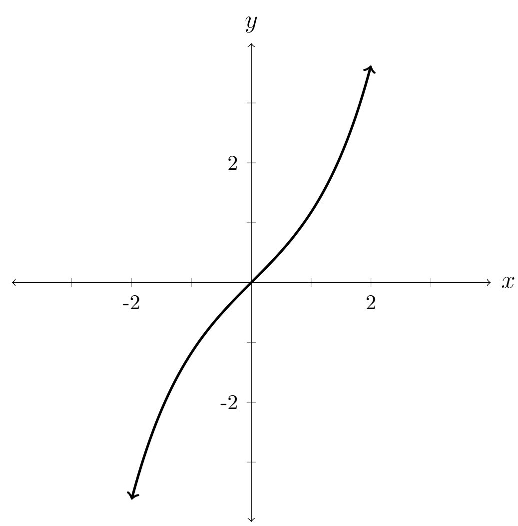
$\cos x$



$\sin x$



$\cosh x$

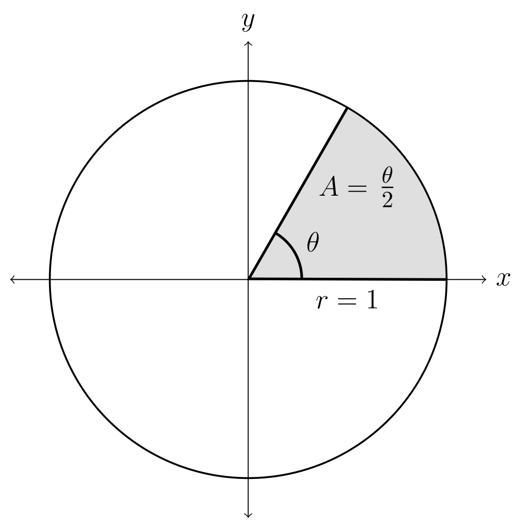


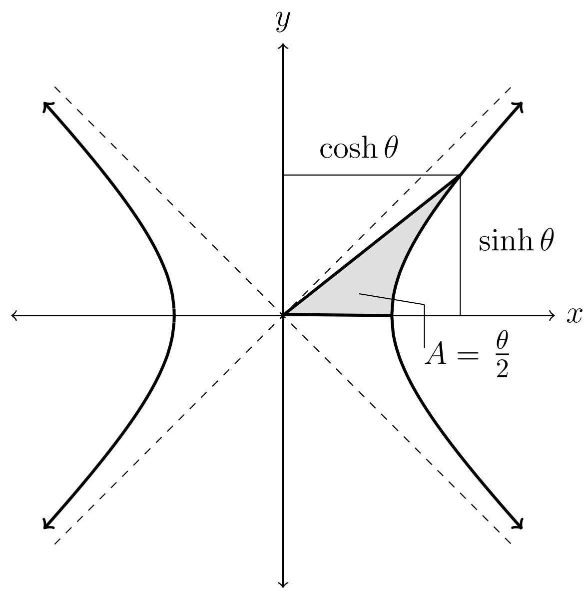
$\sinh x$

Hyperbolic Angle: Just like how the *argument* for the trigonometric functions is an *angle*, the *argument* for the hyperbolic functions is something called a **hyperbolic angle**. Instead of being defined by arc length, the hyperbolic angle is defined by **area**.

If that seems confusing, consider the area of the circular sector of the unit circle. The equation for the area of a circular sector is $Area = \frac{r^2\theta}{2}$, so because the radius of the unit circle is 1, any sector of the unit circle will have an area equal to half of the sector's central angle, $A = \frac{\theta}{2}$. We could even *define* the angle measure to be twice the area of the sector it creates, $\theta = 2A$.

This is how we define the hyperbolic angle. **The hyperbolic angle is equal to twice the area of the hyperbolic sector.**





Note that for a negative hyperbolic angle, the area of the hyperbolic sector will be below the x-axis and can be considered negative for the purpose of finding the angle.

Grading: This project is worth a total of **35 points**. You may work with a partner if you choose to. Make sure your work is clear and comprehensible. Remember to **show your work** and write your name(s) on the front page. All problems should be done in order and **not on this packet**. Your completed assignment is due on **Wednesday, May 1** at the **beginning** of class. Late assignments will not be accepted for full credit.

In Exercises 1–6, evaluate sinh and cosh for the real number. Round to 1 decimal place. Then draw the hyperbolic angles in standard position.

1. 1 **2.** $-\frac{5}{2}$ **3.** 7 **4.** $\ln 2$

5. $\frac{3}{4}$ **6.** $-\frac{8}{5}$

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7. Compare the graphs of $\cosh x$ and x^2. What similarities and differences do you notice? Evaluate both functions with a number greater than 10. Which seems to be "getting bigger" faster?

8. Compare the graphs of $\sinh x$, x^3, and $\tan x$. What similarities and differences do you notice? Evaluate $\sinh x$ and x^3 with a number greater than 10. Which seems to be "getting bigger" faster?

9. Are $\cosh x$ and $\sinh x$ odd, even, or neither? Justify your answer.

10. Are $\cosh x$ and $\sinh x$ periodic?

11. How do we define $\tanh x$?

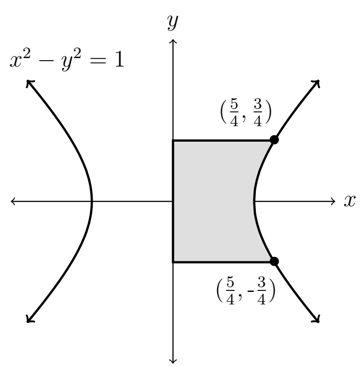
12. The Pythagorean trigonometric identity is $\sin^2 x + \cos^2 x = 1$. Is there an analogous identity for the hyperbolic functions? If so, what is it?
(Hint: How does $\sin^2 x + \cos^2 x = 1$ *relate to the equation of the unit circle?)*

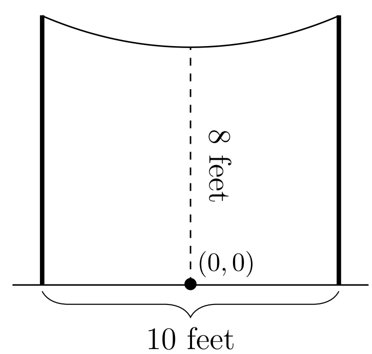
13. (a) Graph $\cosh x + \sinh x$. What other function does this look like?

(b) Recall $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$ and $\sinh(x) = \dfrac{e^x - e^{-x}}{2}$.

Add $\dfrac{e^x + e^{-x}}{2} + \dfrac{e^x - e^{-x}}{2}$ and simplify. What is the result?

In Exercises 14–15, refer to the figures below.





14. Find the exact area of the shaded region in the graph on the left.
(Hint: The region contains two right triangles and two hyperbolic sectors.)

15. The picture on the right depicts a wire hanging from two poles which can be modeled by a function called a **catenary**. A catenary is a curve used to model hanging ropes, wires, chains, etc., which takes the form $y = a\cosh\frac{x}{a} + b$.
For parts (b) and (c), round your answer to two decimal places.

(a) Given that $b = -4$, find the equation of the catenary in the picture.

(b) Find the height of the two poles the wire is hanging from.

(c) Find the distance from the center of the ground, $(0, 0)$, to the tops of the poles.