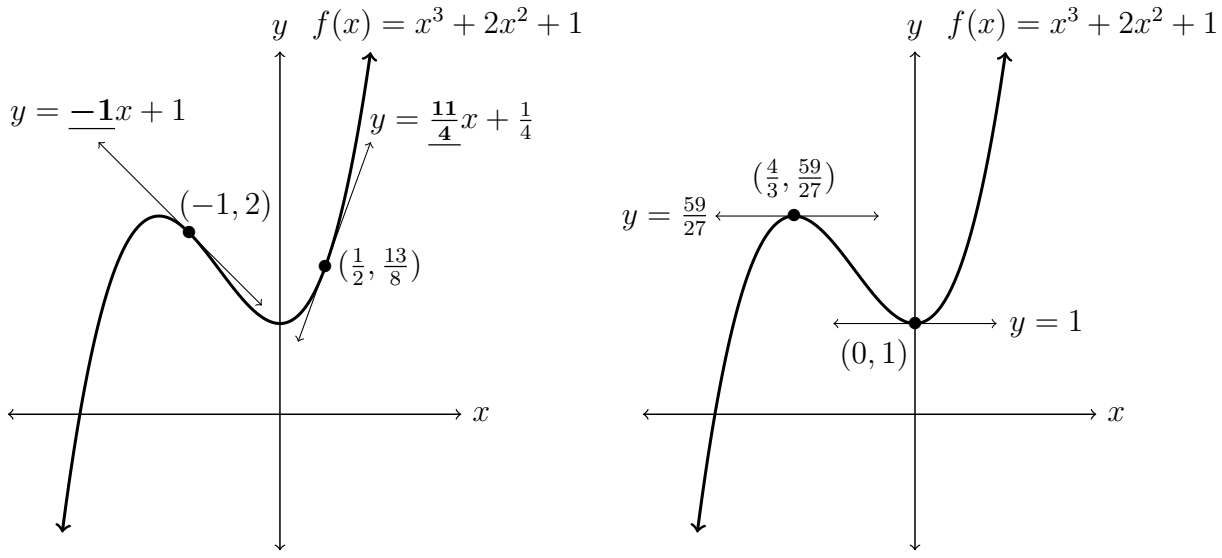
# An Introduction to Derivatives

A derivative of a function $f(x)$ can be thought of as a separate, special function which describes change in or "slope" of the original function. The derivative of $f(x)$ is referred to as $\boldsymbol{f'(x)}$ (pronounced "f prime of x").

For any point of $f(x)$, there is a "tangent line" which barely touches $f(x)$ at that point. The slopes of these tangent lines at each point give the value of the derivative function $\boldsymbol{f'(x)}$ at that point.




Notice: When the graph is "smooth" and has a maximum or a minimum point, the tangent line is horizontal, so the slope is 0. **When $\boldsymbol{f'(x) = 0}$, the corresponding point in $\boldsymbol{f(x)}$ is a potential maximum or minimum point.**

# Trig Function Project

**Assignment:** You may work with a partner. Answers must be in copmlete sentences when appropriate.

You will be working with three functions.

$$g(x) = 3\sin\left(2x + \frac{\pi}{2}\right)$$

$$h(x) = 2\cos\left(4x + \pi\right)$$

$$k(x) = 3\sin\left(2x + \frac{\pi}{2}\right) + 2\cos\left(4x + \pi\right) + 10$$

Notice: $k(x) = g(x) + h(x) + 10$

## Part 1 — For both $g(x)$ and $h(x)$:

1. Draw an accurate graph of the function on graph paper. Include at least two periods.
2. Find the domain and range.
3. Find the amplitude and period.
4. Find the phase shift.
5. Use **desmos.com** or another graphing utility to find the maximum and minimum points for **one period**. Then give general solutions for when these occur.

## Part 2 — For $k(x)$:

1. Draw an accurate graph of the function on graph paper. Include at least two periods.
2. Find the period of $k(x)$. Explain your reasoning. How does the period of $k(x)$ compare to the periods of $g(x)$ and $h(x)$?
3. Use your graph to estimate the maximum and minimum points of $k(x)$ for **one period**.
4. The *derivative* of $k(x)$ is

   $$\boldsymbol{k'(x)} = 6\cos\left(2x + \frac{\pi}{2}\right) - 8\sin\left(4x + \pi\right)$$

   Use the derivative function $\boldsymbol{k'(x)}$ to find general solutions for the maximum and minimum points. *(See first page)*
5. Find the domain and range of $k(x)$. Between the domain and range, which is related to the maximum and minimum points and how?

**Part 3 — Questions:**

1. Why is $k(0)$ not a maximum or minimum point even though $\boldsymbol{k'(0) = 0}$?
   *(Hint: Look at the graph.)*

2. Suppose a graph with the same shape as $k(x)$ but with a period of 12 months is a model for umbrella sales (where the output is in thousands). Explain one possible reason why the graph looks the way it does.

3. How might the derivative $\boldsymbol{k'(x)}$ help with the manufacture and sale of the umbrellas?

**Grading:**

- **10 points** - Part 1.
- **10 points** - Part 2.
- **6 points** - Part 3.
- **4 points** - You will be graded on clarity. Make sure that your work is **clearly laid out** and comprehensible. If your work is very difficult to read, your grade for the **entire project** may be reduced.

Each group may submit a **single** assignment.

This project is worth a total of **30 points**.

**Due date:** Your completed assignment is due on **Friday, March 15th**.