MATH 41 MPS

COURSE PACK

Instructor: Cheryl Jaeger Balm

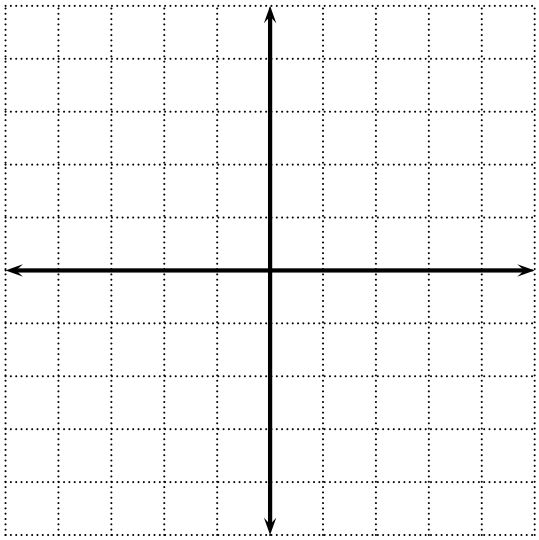
Chapter 1

Lines, functions and composition

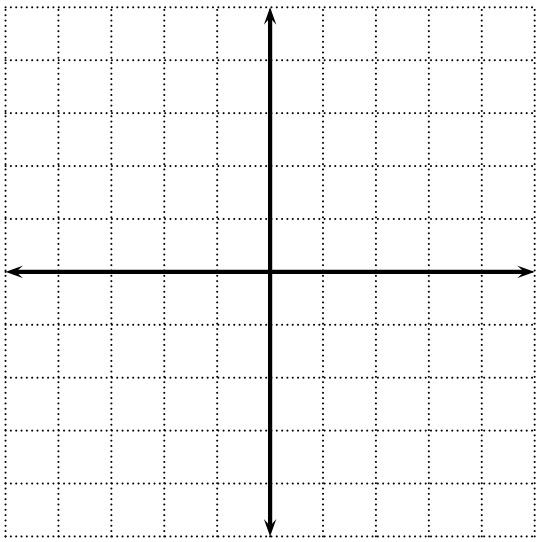
1.1 1.3A: Linear equations

Example 1. Graph the following linear equations.

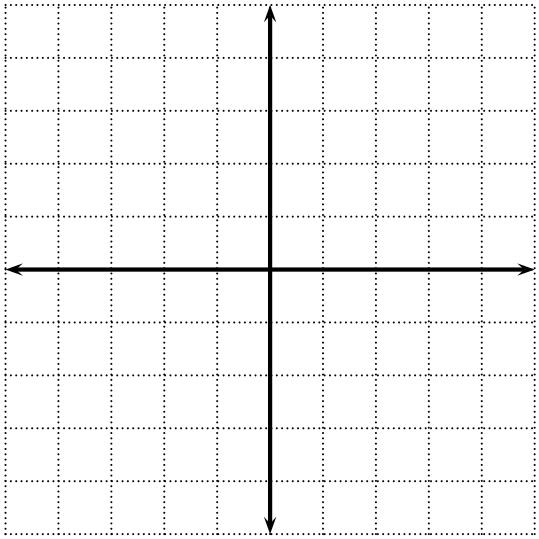
1. $y = -3x + 2$



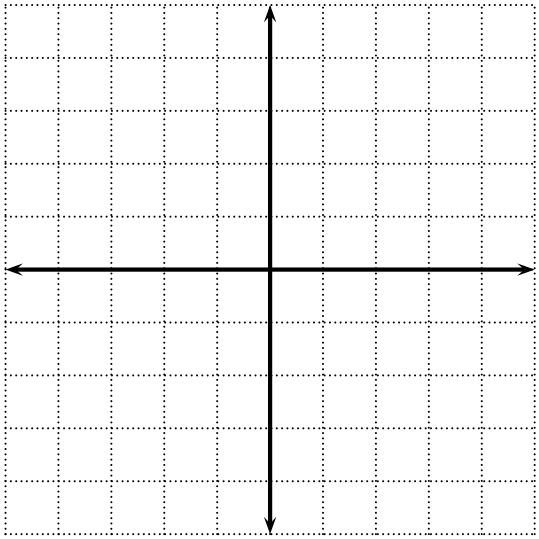
2. $y = -3$

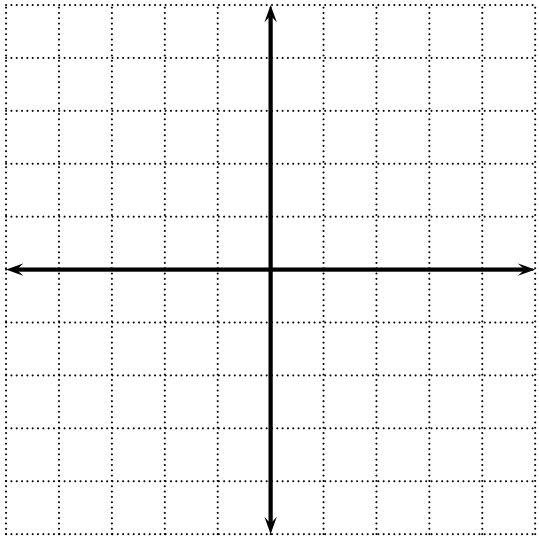


3. $x = 3$

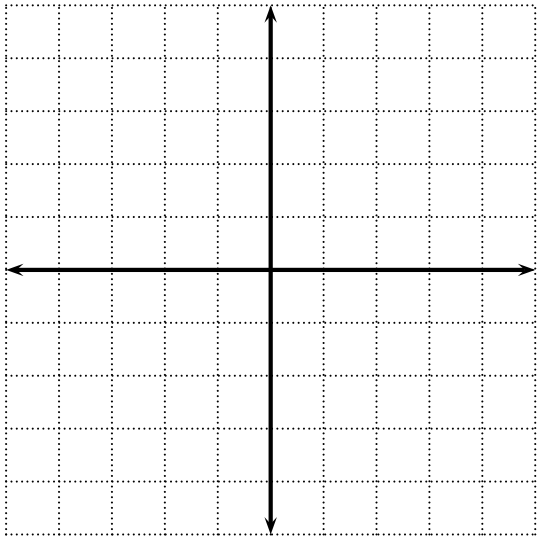


4. $x + 4y = 8$

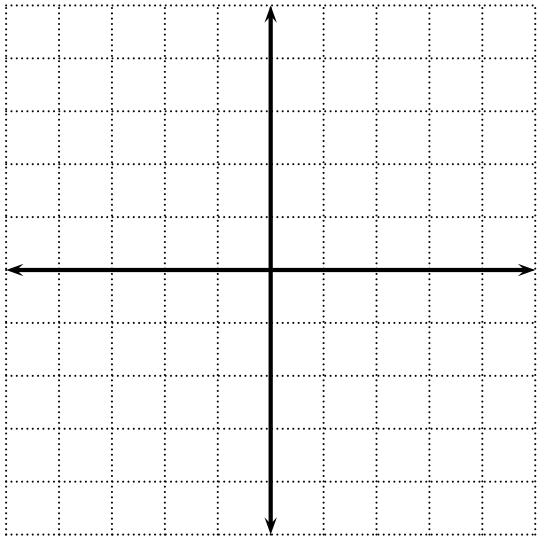




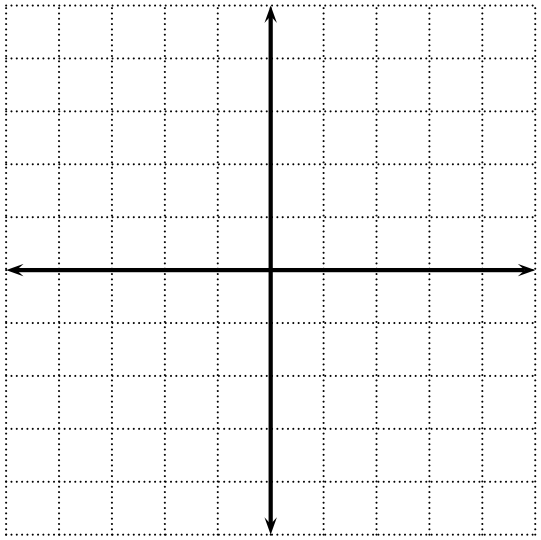
Example 2. Graph and find an equation for the line through the points $(-5, -6)$ and $(2, 8)$.



Example 3. Graph and find an equation for the line through the points $(4, 2)$ and $(2, 5)$.



Example 4. Graph and find an equation for the line through the points $(0, 0)$ and $(0, -6)$.



1.2 1.3B: More lines

Example 5. Find equations for the lines parallel and perpendicular to $5x - 3y = 8$ passing through the point $(-4, 1)$.

Example 6. Solve for x:

1. $7 - 2x = 15$

2. $7x - 9 = 5x + 7$

1.3 1.4A: Functions

Example 7. For each of the following, determine whether y is a function of x. If it is a function, find its domain and range. If it is not function, explain why not.

1.

x	0	1	2	3	4
y	-4	-2	0	-2	4

2.

x	0	1	1	-1
y	1	3	5	7

3.

x	0	1	1	-1
y	1	3	3	5

4. $y = x^2$

5. $y = \sqrt{x}$

6. $x^2 + y^2 = 4$

7. $x + y = 4$

8. $x^2 + y = 4$

9. $x + y^2 = 4$

Example 8. Find the following for the function $f(x) = 10 - 3x^2$

1. Evaluate $f(-4)$

2. Evaluate $f(x - 1)$

3. Solve $f(x) = 0$

Example 9. Solve $g(x) = x^2 - 5x + 6$ when $g(x) = 0$

Example 10. Given $f(x) = x^2 + 6x - 24$ and $g(x) = 4x - x^2$, find when $f(x) = g(x)$.

1.4 1.4B: More functions

Example 11. Evaluate each of the following for the piecewise function

$$f(x) = \begin{cases} x^2 + 1 & \text{, if } x < 1 \\ x - 1 & \text{, if } x \geq 1 \end{cases}$$

1. $f(-2)$

2. $f(2)$

3. $f(-3)$

4. $f(1)$

1.5 1.8: Composition and combining functions

Example 12. Find each of the following for $f(x) = x^2$ and $g(x) = 1 - x$

1. $(f + g)(x)$

2. $(f + g)(2)$

3. $(f - g)(x)$

4. $(g - f)(x)$

5. $(f - g)(2)$

6. $(g - f)(2)$

7. $(fg)(x)$

8. $(gf)(x)$

9. $\left(\frac{f}{g}\right)(x)$

10. $\left(\frac{g}{f}\right)(x)$

11. Find the domain and range of $\left(\frac{f}{g}\right)(x)$

12. Find the domain and range of $\left(\frac{g}{f}\right)(x)$

Example 13. Find each of the following for $f(x) = \sqrt{2x+5}$ and $g(x) = 4x^2 + 1$

1. $(f \circ g)(x)$

2. $(g \circ f)(x)$

3. $(f \circ g)\left(-\frac{1}{2}\right)$

4. $(g \circ f)\left(-\frac{1}{2}\right)$

5. Find the domain of $f(x),\ g(x),\ (f \circ g)(x)$ and $(g \circ f)(x)$

Chapter 2

Inverses, factoring and graphs

2.1 1.9A: Inverse functions

Example 14. Find the inverse of each function

1. $f(x) = \sqrt{2x-3}$

2. $g(x) = \dfrac{5x-3}{2x+5}$

3. $h(x) = \sqrt[3]{10+x}$

Example 15. Find the inverse of each function

1.

x	$f(x)$
0	5
1	7
2	9
3	11

2.

x	$f(x)$
0	1
1	0
2	1
3	2

PRACTICE ON INVERSE FUNCTIONS
Group work: Due Friday

For each function $f(x)$, find the inverse function $f^{-1}(x)$. Then find the domain and range for both $f(x)$ and $f^{-1}(x)$. For some functions, the domain has been restricted to make them invertible.

1. $f(x) = 3 - 4x$

   $f^{-1}(x) =$ _____

   Domain of $f(x)$: _____ Domain of $f^{-1}(x)$: _____

   Range of $f(x)$: _____ Range of $f^{-1}(x)$: _____

2. $f(x) = \dfrac{x^3}{8}$

   $f^{-1}(x) =$ _____

   Domain of $f(x)$: _____ Domain of $f^{-1}(x)$: _____

   Range of $f(x)$: _____ Range of $f^{-1}(x)$: _____

3. $f(x) = \dfrac{1}{x}$

   $f^{-1}(x) =$ _____

   Domain of $f(x)$: _____ Domain of $f^{-1}(x)$: _____

   Range of $f(x)$: _____ Range of $f^{-1}(x)$: _____

4. $f(x) = \sqrt{x-4}$

   $f^{-1}(x) =$ _____

   Domain of $f(x)$: _____ Domain of $f^{-1}(x)$: _____

   Range of $f(x)$: _____ Range of $f^{-1}(x)$: _____

5. $f(x) = 1 - x^3$

$f^{-1}(x) =$ _____

Domain of $f(x)$: _____ Domain of $f^{-1}(x)$: _____

Range of $f(x)$: _____ Range of $f^{-1}(x)$: _____

6. $f(x) = \dfrac{1}{1+x}$

$f^{-1}(x) =$ _____

Domain of $f(x)$: _____ Domain of $f^{-1}(x)$: _____

Range of $f(x)$: _____ Range of $f^{-1}(x)$: _____

7. $f(x) = \dfrac{x-1}{x+5}$

$f^{-1}(x) =$ _____

Domain of $f(x)$: _____ Domain of $f^{-1}(x)$: _____

Range of $f(x)$: _____ Range of $f^{-1}(x)$: _____

2.2 A3(A): Factoring GCF and Special Forms

Example 16. For each expression, factor out the GCF.

1. $5x^3 - 15x^2$

2. $-3 + 6x - 12x^3$

3. $(x+1)(2x) - (x+1)(2)$

4. $(x-2)(2x) + 3(x-2)$

Example 17. Factor $100 - 4y^2$

Example 18. Factor $(x-1)^2 - 9y^4$

Example 19. Factor $x^2 - 10x + 25$

Example 20. Factor $9x^2 + 30x + 25$

Example 21. Factor $4y^2 - 12y + 9$

Example 22. Factor $x^3 + 216$

Example 23. Factor $5y^3 - 135$

2.3 A3(B): Factoring Trinomials (Big X)

Example 24. Factor $12x^2 + 7x + 1$

Example 25. Factor $x^2 + x - 6$

PRACTICE ON FACTORING TRINOMIALS
Group work: Due Friday

1. A student submitted the following **incorrect** work to simplify an expression. Find where the student made their error(s), then redo the problem correctly.

$$\begin{aligned} 6x^2 - 5x - 4 &= 6x^2 + 3x - 8x - 4 \\ &= 3x(2x + 1) - 4(2x - 1) \\ &= (2x + 1)(2x - 1)(3x - 4) \end{aligned}$$

Factor each trinomial with leading coefficient $a \neq 1$.

2. $2x^2 + 9x + 7$

3. $5y^2 - 16y + 3$

4. $8x^2 - 18x + 9$

5. $6x^2 - 5x - 6$

Completely factor each trinomial. Make sure you factor out the GCF first if there is one.

6. $5p^2 - 5p - 60$

7. $3b^2 - 10b - 8$

8. $-x^2 - 2x + 35$

9. $2t^3 - 12t^2 - 32t$

10. $3b^2 - 2b - 6$

11. $60y - 40y^2 + 5y^3$

2.4 A3(C): Factor by Grouping

Example 26. Factor $x^3 + x^2 - 9x - 9$

Example 27. Factor $2x^3 + 6x^2 - x - 3$

Example 28. Factor $4 + 2y - 2y^2 - y^3$

PRACTICE ON FACTORING BY GROUPING
Group work: Due Friday

Factor by grouping. (The first one has been started already.)

1. $3x - 3 + 7x^3 - 7x^2 = 3(x-1) + 7x^2(x-1)$

2. $2 - 6n + 7n^2 - 21n^3$

3. $4x + x^3 + 8 + 2x^2$

4. $6y^3 + 3y^2 - 8y - 4$

5. $6 - y + 6y^2 - y^3$

2.5 1.2: Graphs of equations

Example 29. Answer the following for the equation

$$y^2 = 6 - x$$

- Is this equation symmetric with respect to the x-axis?

- Is this equation symmetric with respect to the y-axis?

- Is this equation symmetric with respect to the origin?

- Use symmetry and a well-chosen table of values to sketch a graph of this equation.

- Is y a function of x?

Example 30. Answer the following for the equation

$$y = x^3$$

- Is this equation symmetric with respect to the x-axis?

- Is this equation symmetric with respect to the y-axis?

- Is this equation symmetric with respect to the origin?

- Use symmetry and a well-chosen table of values to sketch a graph of this equation.

- Is y a function of x?

Example 31. Answer the following for the equation

$$y = |x - 2|$$

- Is this equation symmetric with respect to the x-axis?

- Is this equation symmetric with respect to the y-axis?

- Is this equation symmetric with respect to the origin?

- Use symmetry and a well-chosen table of values to sketch a graph of this equation.

- Is y a function of x?

Chapter 3

Graphing functions

3.1 1.5: Graphs of functions

Example 32. For each function, find all roots as well as the domain and range.

1. $f(x) = 2x^2 + 13x - 24$

2. $g(x) = \sqrt{x - 25}$

3. $h(x) = \dfrac{x^2 - 2}{x - 1}$

Example 33. Given $f(x) = x^2 + 2x$, find the average rate of change between the given values for x_1 and x_2

1. $x_1 = -2,\ x_2 = 0$

2. $x_1 = -3,\ x_2 = -2$

3.2 1.9B: Graphs of inverse functions

Example 34. Find two (maximal interval) ways to restrict the domain of $f(x) = |x - 3|$ so that $f(x)$ is invertible. Then find $f^{-1}(x)$ for each restriction.

Exam 1 is Friday!

Chapter 4

Graph transformations and rational functions

4.1 1.6A: Power functions

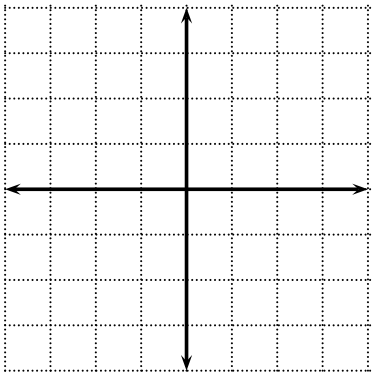
PRACTICE ON POWER FUNCTIONS
Group work: Due Friday

1. **Linear function**: $f(x) = x$

   (a) Complete the table of values

x	-3	-2	-1	0	1	2	3
$f(x)$							

   (b) Use your table of values to sketch a graph of $f(x)$



   (c) Domain:

   (d) Range:

   (e) Symmetries:

   (f) Vertical intercept:

   (g) Horizontal intercept(s):

   (h) Increasing and decreasing intervals:

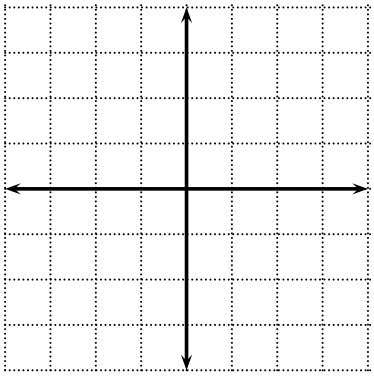
   (i) Maxima and minima:

2. **Square (or squaring) function**: $f(x) = x^2$

   (a) Complete the table of values

x	-3	-2	-1	0	1	2	3
$f(x)$							

   (b) Use your table of values to sketch a graph of $f(x)$



   (c) Domain:

   (d) Range:

   (e) Symmetries:

   (f) Vertical intercept:

   (g) Horizontal intercept(s):

   (h) Increasing and decreasing intervals:

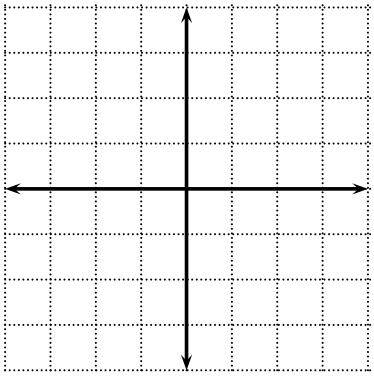
   (i) Maxima and minima:

3. **Cubic function**: $f(x) = x^3$

   (a) Complete the table of values

x	-3	-2	-1	0	1	2	3
$f(x)$							

   (b) Use your table of values to sketch a graph of $f(x)$



   (c) Domain:

   (d) Range:

   (e) Symmetries:

   (f) Vertical intercept:

   (g) Horizontal intercept(s):

   (h) Increasing and decreasing intervals:

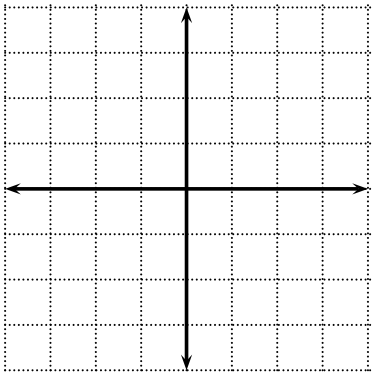
   (i) Maxima and minima:

4. **Square root function**: $f(x) = \sqrt{x} = x^{1/2}$

   (a) Complete the table of values

x	-3	-2	-1	0	1	2	3
$f(x)$							

   (b) Use your table of values to sketch a graph of $f(x)$



   (c) Domain:

   (d) Range:

   (e) Symmetries:

   (f) Vertical intercept:

   (g) Horizontal intercept(s):

   (h) Increasing and decreasing intervals:

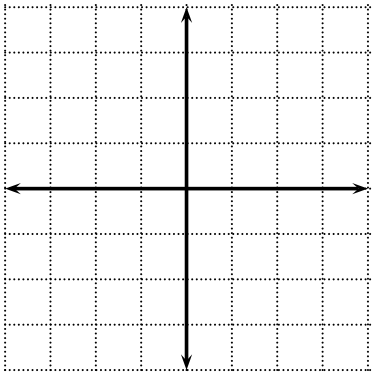
   (i) Maxima and minima:

5. **Reciprocal function**: $f(x) = \dfrac{1}{x} = x^{-1}$

   (a) Complete the table of values

x	-3	-2	-1	0	1	2	3
$f(x)$							

   (b) Use your table of values to sketch a graph of $f(x)$



   (c) Domain:

   (d) Range:

   (e) Symmetries:

   (f) Vertical intercept:

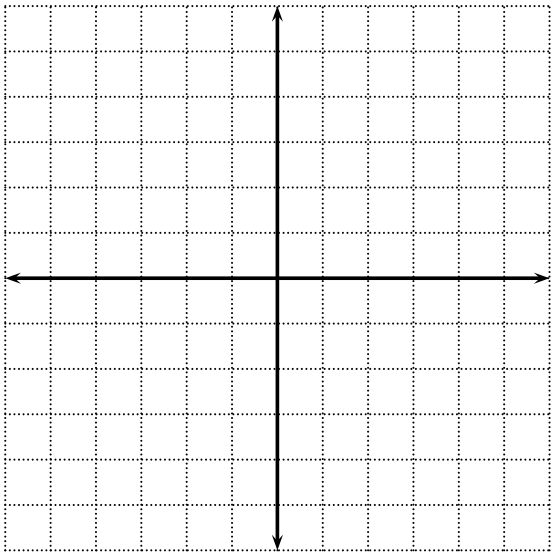
   (g) Horizontal intercept(s):

   (h) Increasing and decreasing intervals:

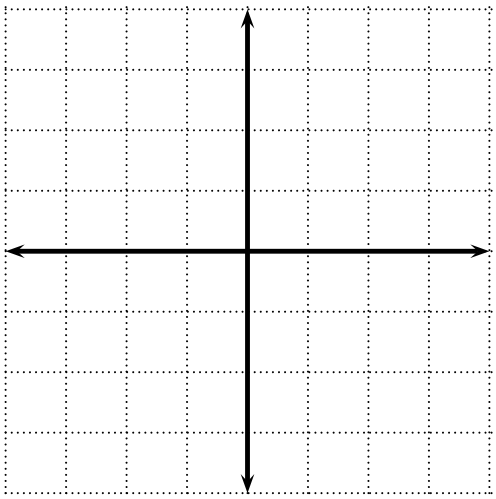
   (i) Maxima and minima:

4.2 1.6B: Piecewise and Absolute Value Functions

Example 35. Graph $f(x) = \begin{cases} -\frac{1}{2}x - 6, & x \leq -4 \\ x + 5, & x > -4 \end{cases}$



Example 36. Graph $f(x) = \begin{cases} 2x + 1, & x \leq -1 \\ 2x^2 - 1, & -1 < x \leq 1 \\ 1 - x^2, & x > 1 \end{cases}$

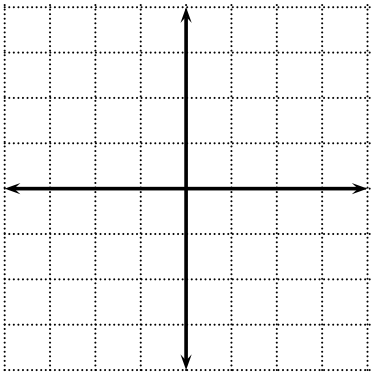


Example 37. **Absolute value function**: $f(x) = |x|$

1. Complete the table of values

x	-3	-2	-1	0	1	2	3
$f(x)$	3	2	1	0	1	2	3

2. Use your table of values to sketch a graph of $f(x)$



3. Domain:

4. Range:

5. Symmetries:

6. Vertical intercept:

7. Horizontal intercept(s):

8. Increasing and decreasing intervals:

9. Maxima and minima:

Example 38. Solve: $|x^2 - 3x| = -4x + 6$

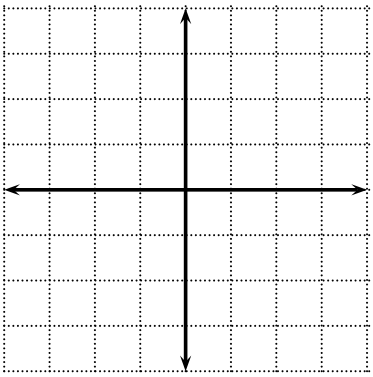
Example 39. Solve: $|x^2 + 4x| = 5x + 12$

4.3 1.7A: Graph Transformations

Example 40. Sketch the following functions on the same set of axes.

$$f(x) = x^3$$

$$g(x) = x^3 - 1$$

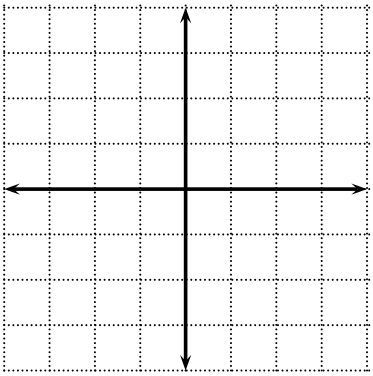


Example 41. Sketch the following functions on the same set of axes.

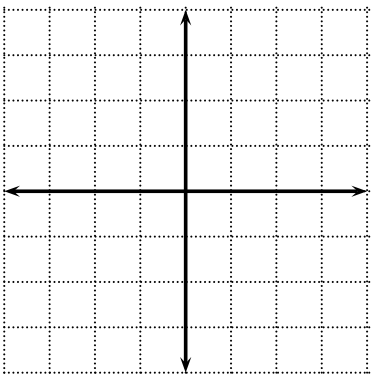
$$f(x) = x^3$$

$$g(x) = (x-2)^3$$

$$h(x) = (x-2)^3 + 1$$



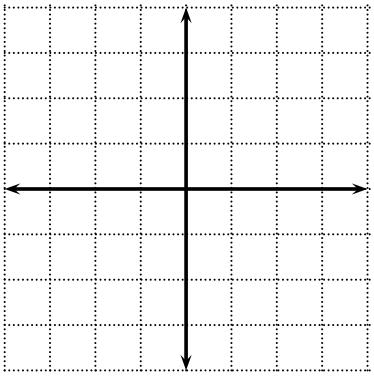
Example 42. Sketch the graph of $g(x) = (x+3)^3 + 2$ by first sketching its parent function. You may want to list your transformations and/or sketch multiple transformation graphs.



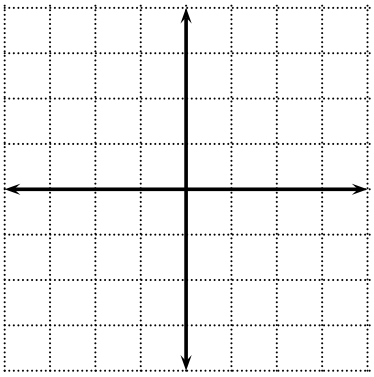
4.4 1.7B: More Graph Transformations

Example 43. Sketch the graph of each of the following functions by first sketching its parent function. You may want to list your transformations and/or sketch multiple transformation graphs.

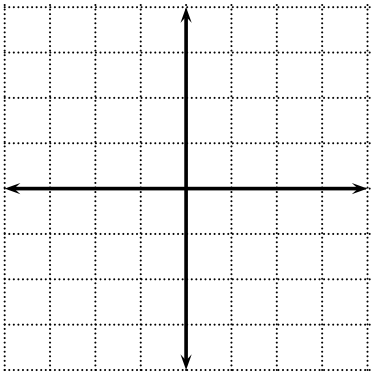
1. $f(x) = \sqrt{x-1}$



2. $f(x) = -\sqrt{x-1}$



3. $f(x) = \sqrt{-x-1}$



Example 44. Graph $g(x) = 2 - 8x^3$

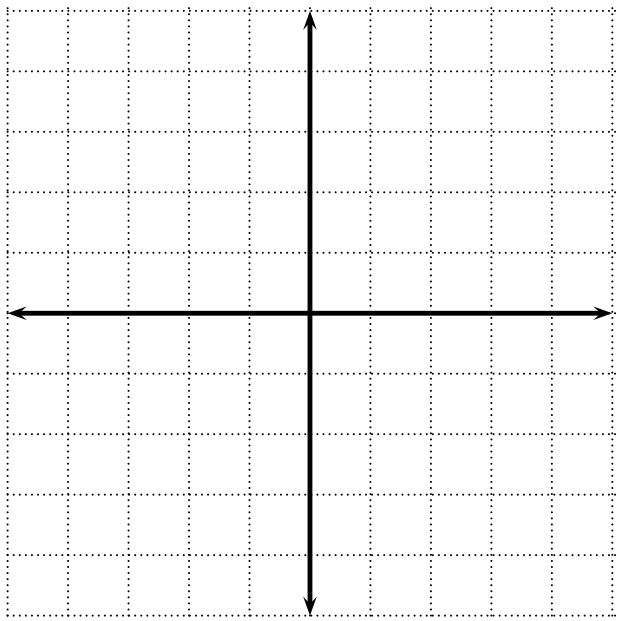
Step 1: Identify the parent function

Step 2: Identify the transformations, in order.

Method 1	Method 2

Step 3: Sketch the graph, including any intermediate graphs you find helpful. **Clearly identify the final graph of the function.**

(Hint: Choose an appropriate scale for the axes.)

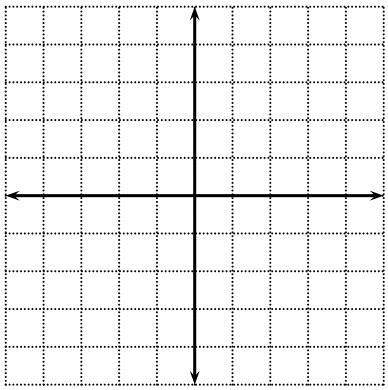


Example 45. Graph $g(x) = \sqrt{3x} + 1$

Step 1: Identify the parent function

Step 2: Identify the transformations, in order.

Step 3: Sketch the graph, including any intermediate graphs you find helpful. **Clearly identify the final graph of the function.**

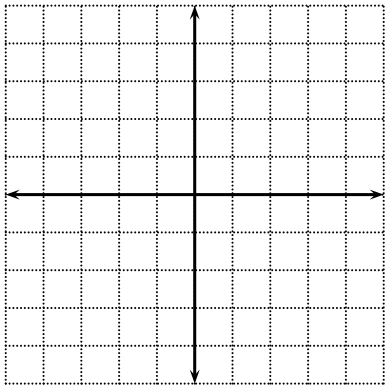


Example 46. Graph $g(x) = \frac{1}{2}|2 - x| - 3$

Step 1: Identify the parent function

Step 2: Identify the transformations, in order.

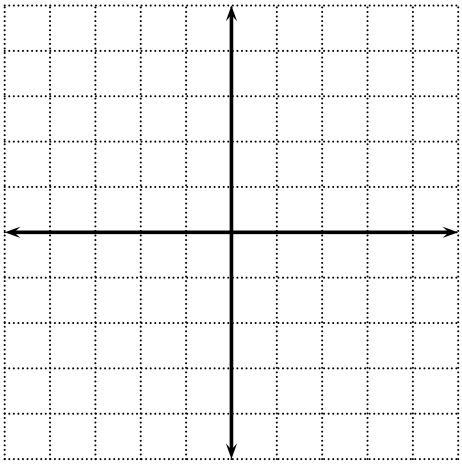
Step 3: Sketch the graph, including any intermediate graphs you find helpful. **Clearly identify the final graph of the function.**



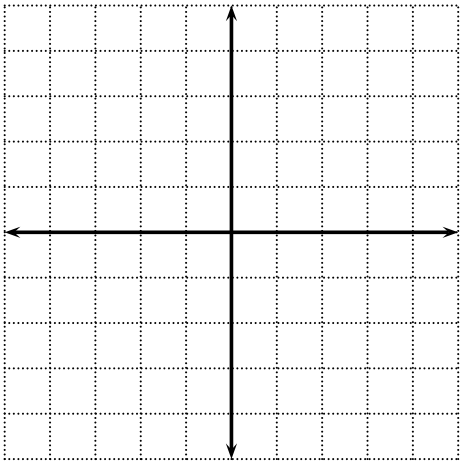
PRACTICE ON GRAPH TRANSLATIONS
Group work: Due Friday

For each of the following functions, identify the parent function, identify the necessary transformations, in order, and sketch a graph of the function. You may include any intermediate graphs you find helpful, but clearly identify the final graph of the function.

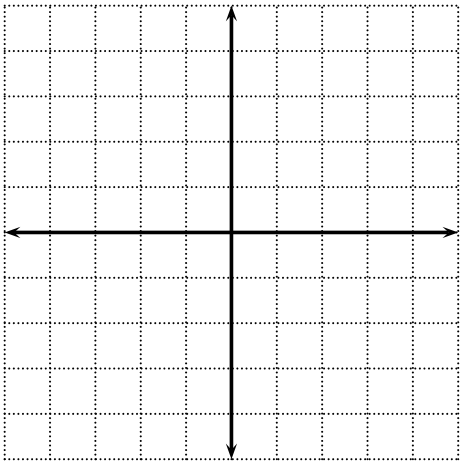
1. $g(x) = 2(x-7)^2$



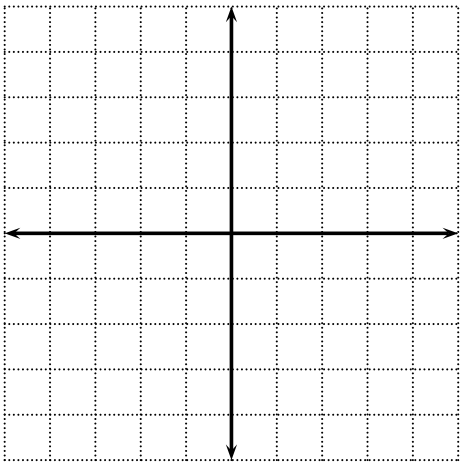
2. $g(x) = -\frac{1}{4}(x+2)^2 - 2$



3. $g(x) = 1 - \sqrt{\frac{1}{4}x}$



4. $g(x) = 6 - |-x + 3|$



4.5 A4: Rational Functions and Expressions

Example 47. Find the domain of $f(x) = \dfrac{3x}{x-1}$

Example 48. Find the domain of $f(x) = \dfrac{2x+1}{x^2-4}$

Example 49. Simplify $\dfrac{4x+12}{x^2-3x-18}$

Example 50. Simplify $\dfrac{3x^2-x-2}{5-4x-x^2}$

Example 51. Multiply by first simplifying each expression: $\dfrac{15x^2+5x}{x^3-3x^2-18x} \cdot \dfrac{x^2-2x-15}{2x^2-8x-3}$

Example 52. Divide by first simplifying each expression: $\dfrac{x^3-1}{x^2-1} \div \dfrac{x^2+x+1}{x^2+2x+1}$

Example 53. Subtract $\dfrac{x}{x-3} - \dfrac{2}{3x+4}$

Example 54. Add $\dfrac{3}{x-1} + \dfrac{x+3}{x^2-1}$

Example 55. Subtract $\dfrac{-x-5}{x^2-4} - \dfrac{4}{x+2}$

Example 56. Add $\dfrac{x}{2x-1} + \dfrac{1}{x+2}$

Chapter 5

Rational Functions

5.1 2.6A: Graphing Rational Functions

Notes on **horizontal asymptotes**:

- If deg N = deg D:

- If deg N < deg D:

- If deg N > deg D:

Example 57. For each of the following functions, find any vertical and horizontal asymptotes:

1. $f(x) = \dfrac{2x+1}{x+1}$

2. $g(x) = \dfrac{4}{x^2+1}$

3. $h(x) = \dfrac{2}{(x-1)^2}$

4. $j(x) = \dfrac{x^2+x-2}{x^2-x-6}$

5. $k(x) = \dfrac{2x^2}{x^2+1}$

Six Steps for Graphing Rational Functions

Example 58. Follow the Six Steps to graph $j(x) = \dfrac{x^2 + x - 2}{x^2 - x - 6}$

1. Simplify:

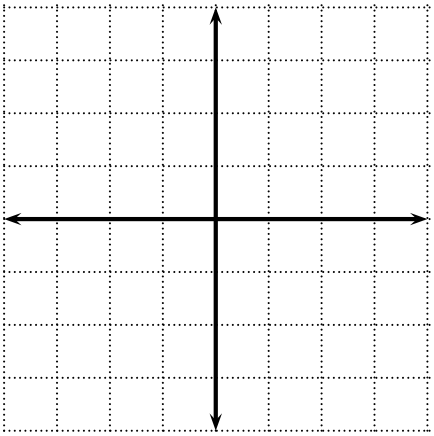
2. Vertical asymptote(s) and hole(s):

3. Horizontal asymptote:

4. Vertical intercept:

5. Horizontal intercept(s):

6. Connect the dots



Example 59. Follow the Six Steps to graph $f(x) = \dfrac{3x}{x^2 + x - 2}$

1. Simplify:

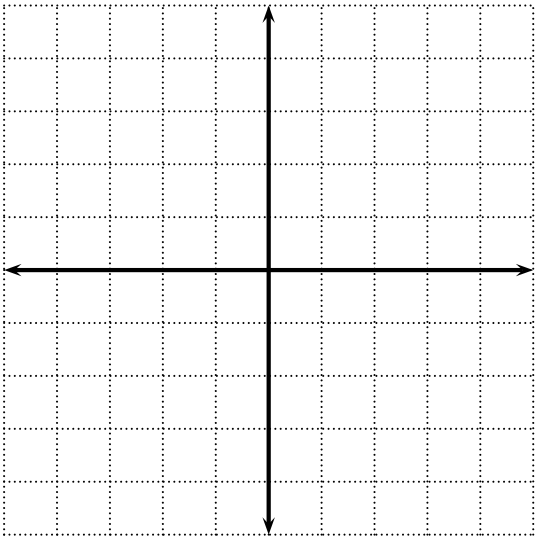
2. Vertical asymptote(s) and hole(s):

3. Horizontal asymptote:

4. Vertical intercept:

5. Horizontal intercept(s):

6. Connect the dots



PRACTICE GRAPHING RATIONAL FUNCTIONS
Group work: Due Friday

Example 60. Follow the Six Steps to graph $f(x) = \dfrac{3+2x}{1+x}$

1. Simplify:

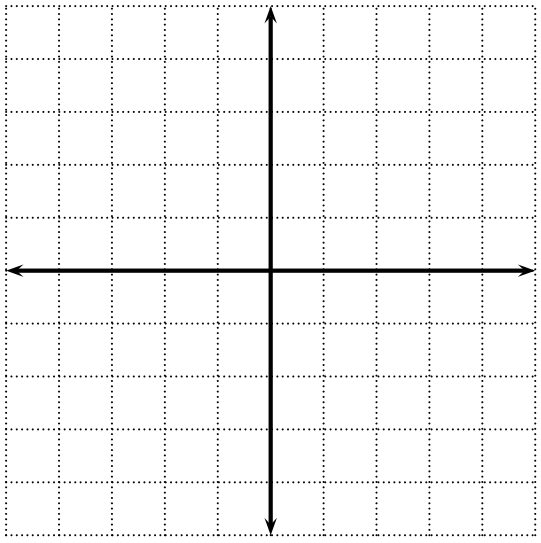
2. Vertical asymptote(s) and hole(s):

3. Horizontal asymptote:

4. Vertical intercept:

5. Horizontal intercept(s):

6. Connect the dots



Example 61. Follow the Six Steps to graph $f(x) = \dfrac{x^2 - 4}{x^2 - x - 6}$

1. Simplify:

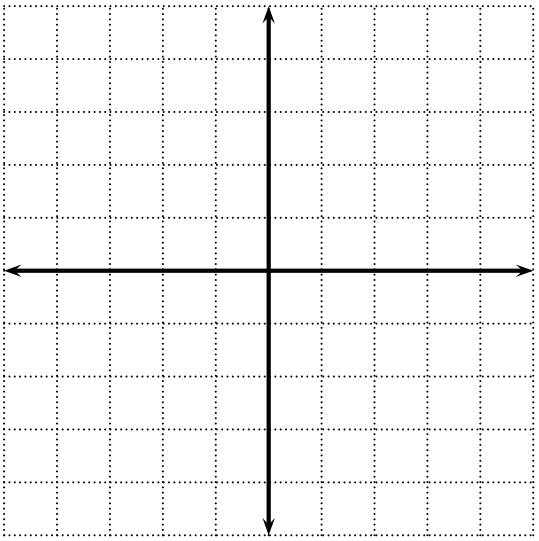
2. Vertical asymptote(s) and hole(s):

3. Horizontal asymptote:

4. Vertical intercept:

5. Horizontal intercept(s):

6. Connect the dots



5.2 2.3A: Polynomial Division

Example 62. $f(x) = \dfrac{3x^2 + 19x + 28}{x + 4}$

Example 63. $f(x) = \dfrac{x^2 + 3x + 5}{x + 1}$

Example 64. Divide $(x^3 - 2x^2 - 9) \div (x - 3)$

Example 65. Divide $(-x^3 + 9x + 6x^4 - x^2 - 3) \div (1 + 3x)$

Example 66. Simplify $f(x) = \dfrac{5x^3 + 8x^2 - x + 6}{x + 2}$

PRACTICE ON POLYNOMIAL LONG DIVISION
Group work: Due Friday

Use long division to simplify the following rational functions.

1. $f(x) = \dfrac{x^5 - 13x^4 - 120x + 80}{x + 3}$

2. $f(x) = \dfrac{x^3 - 729}{x - 9}$

3. $f(x) = \dfrac{-3x^4}{x + 2}$

4. $f(x) = \dfrac{5 - 3x + 2x^2 - x^3}{x + 1}$

5.3 2.6B: Slant Asymptotes

Example 67. Follow the Six Steps to graph $f(x) = \dfrac{x^2 - x}{x + 1}$

1. Simplify:

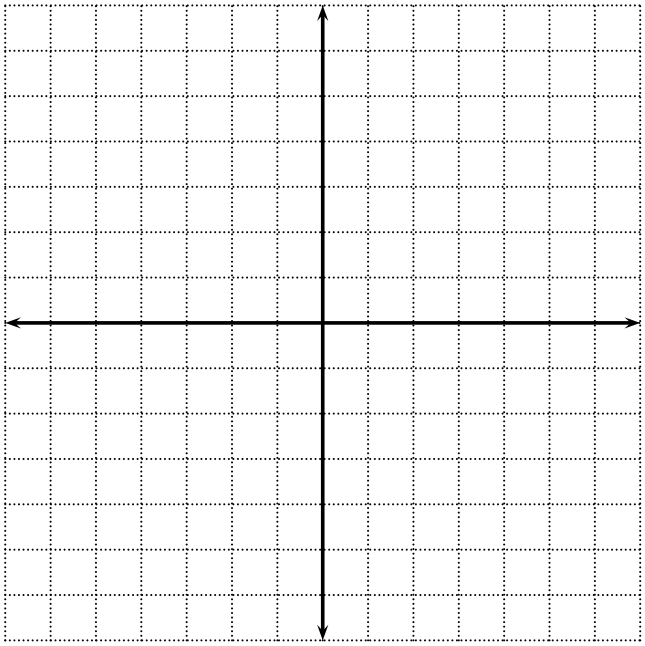
2. Vertical asymptote(s) and hole(s):

3. Horizontal (???) asymptote:

4. Vertical intercept:

5. Horizontal intercept(s):

6. Connect the dots



Example 68. Follow the Six Steps to graph $f(x) = \dfrac{3x^2 + 1}{x}$

1. Simplify:

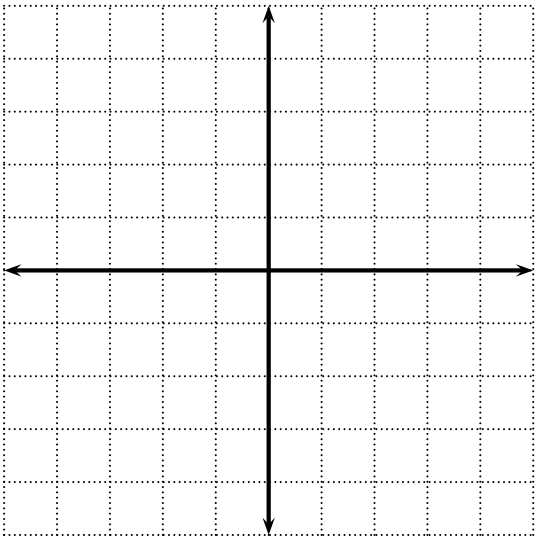
2. Vertical asymptote(s) and hole(s):

3. Horizontal or slant asymptote:

4. Vertical intercept:

5. Horizontal intercept(s):

6. Connect the dots



Example 69. Follow the Six Steps to graph $f(x) = \dfrac{2x^2 - 5x + 5}{x - 2}$. *Hint: Scale your vertical axis by 2's.*

1. Simplify:

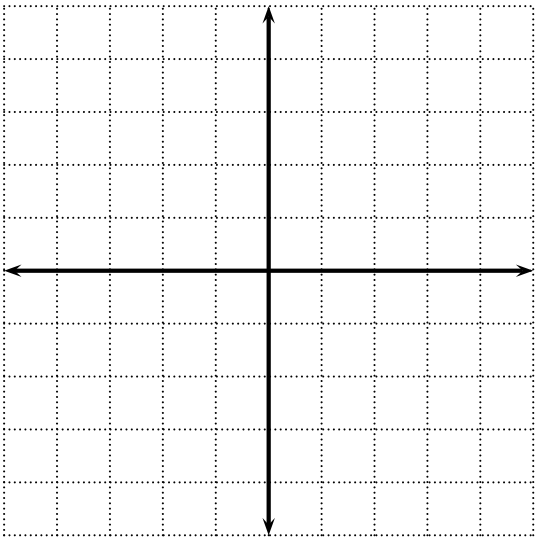
2. Vertical asymptote(s) and hole(s):

3. Horizontal or slant asymptote:

4. Vertical intercept:

5. Horizontal intercept(s):

6. Connect the dots



MORE PRACTICE GRAPHING RATIONAL FUNCTIONS
Group work: Due Friday

Example 70. Follow the Six Steps to graph $f(x) = \dfrac{x^3}{2x^2 - 8}$.

1. Simplify:

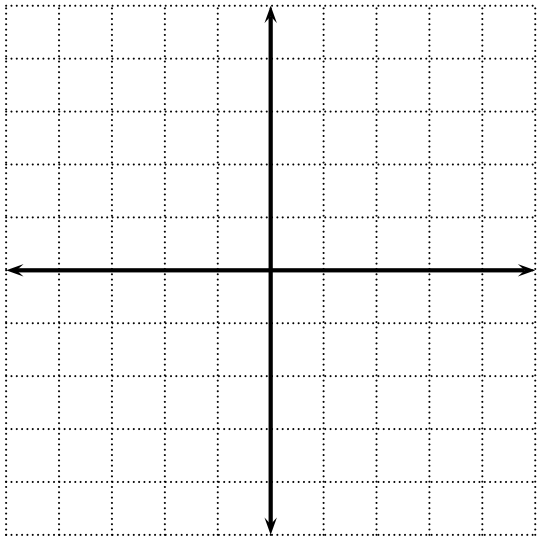
2. Vertical asymptote(s) and hole(s):

3. Horizontal or slant asymptote:

4. Vertical intercept:

5. Horizontal intercept(s):

6. Connect the dots



Example 71. Follow the Six Steps to graph $f(x) = \dfrac{2x^3 + x^2 - 8x - 4}{x^2 - 3x + 2}$. *Hint: Scale your vertical axis by 4's or 5's.*

1. Simplify:

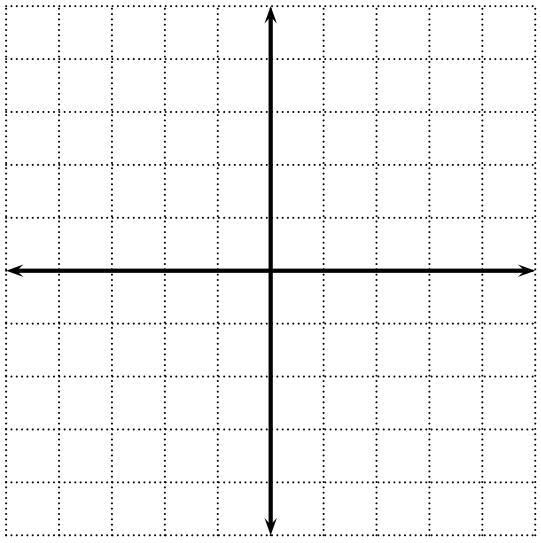
2. Vertical asymptote(s) and hole(s):

3. Horizontal or slant asymptote:

4. Vertical intercept:

5. Horizontal intercept(s):

6. Connect the dots



Example 72. Follow the Six Steps to graph $f(x) = \dfrac{6x^2 - 11x + 3}{6x^2 - 3x - 3}$.

1. Simplify:

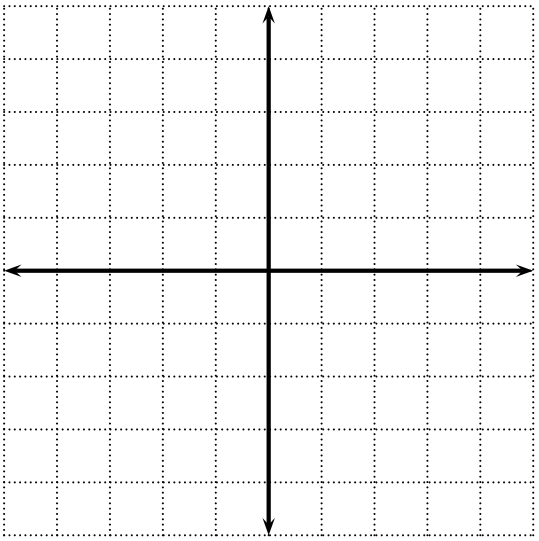
2. Vertical asymptote(s) and hole(s):

3. Horizontal or slant asymptote:

4. Vertical intercept:

5. Horizontal intercept(s):

6. Connect the dots



5.4 2.6C, 1.10: Applications of Rational Functions

Example 73. Solve $\frac{4x}{9} - \frac{1}{3} = x + \frac{5}{3}$

Example 74. Solve $\frac{1}{x-2} = \frac{3}{x+2} - \frac{6x}{x^2-4}$

Example 75. Solve $\frac{3x}{x-4} = 5 + \frac{12}{x-4}$

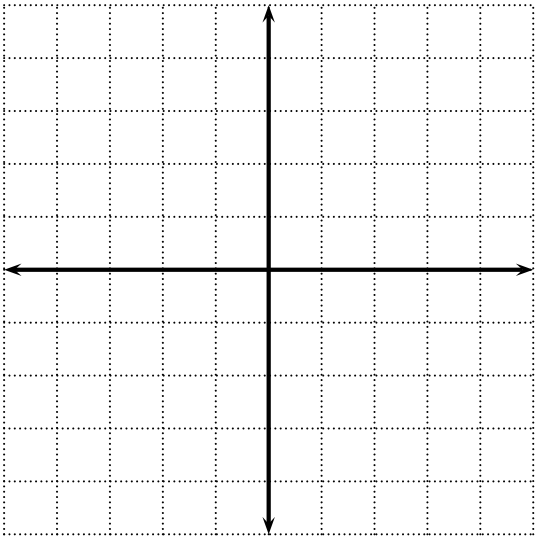
Exam 2 is Monday!

Chapter 6

Quadratic functions

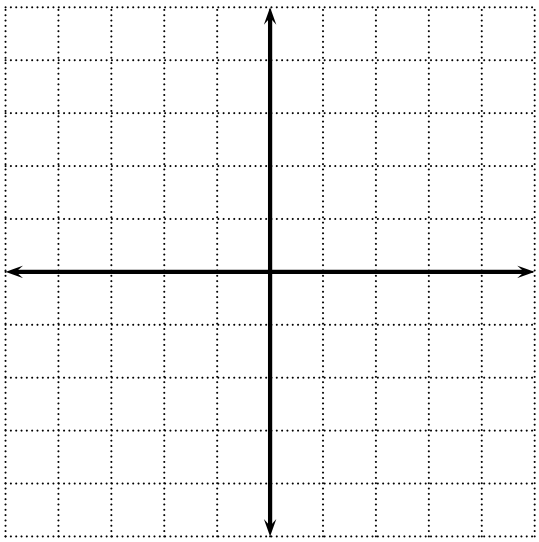
6.1 2.1A: Parabolas

Example 76. Sketch the graphs of $y = x^2,\ y = 2x^2,\ y = \frac{1}{2}x^2$ and $y = -2x^2$.

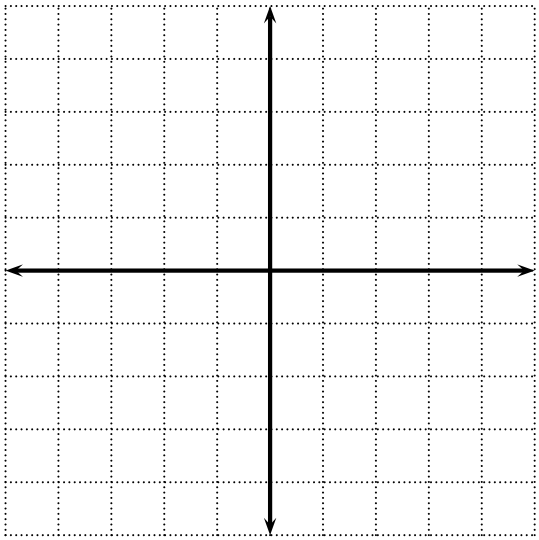


Example 77. Graph the following quadratic functions by translating the parent function $y = x^2$.

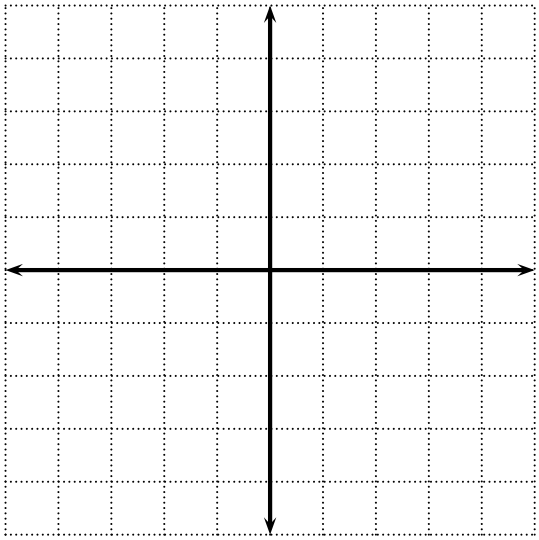
1. $f(x) = -x^2 + 1$



2. $g(x) = (x+2)^2 - 3$



3. $h(x) = 2(x+2)^2 - 1$



6.2 2.1B: Vertex form and completing the square

Example 78. Write $f(x) = x^2 + 6x - 5$ in vertex form by first completing the square.

Example 79. Write $f(x) = x^2 - 4x - 1$ in vertex form.

Example 80. Write $f(x) = 3x^2 - 4x - 5$ in vertex form.

Example 81. Write $f(x) = 2x^2 - 5x - 1$ in vertex form.

Example 82. Write $f(x) = 3x^2 - 10x - 2$ in vertex form.

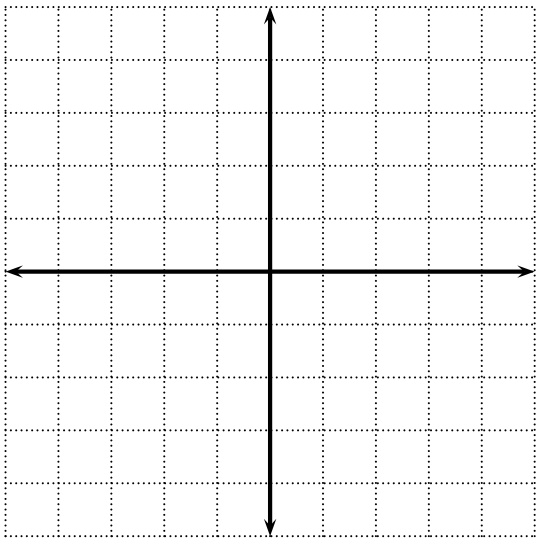
Example 83. Graph $f(x) = x^2 - 4x + 3$ by following the steps below.

1. Write $f(x)$ in vertex form. Find the vertex of the parabola and decide if it's a maximum or minimum.

2. Find the vertical intercept.

3. Decide if there are horizontal intercepts, and find them if they exist.

4. Graph the function and label the vertex, intercepts and axis of symmetry.



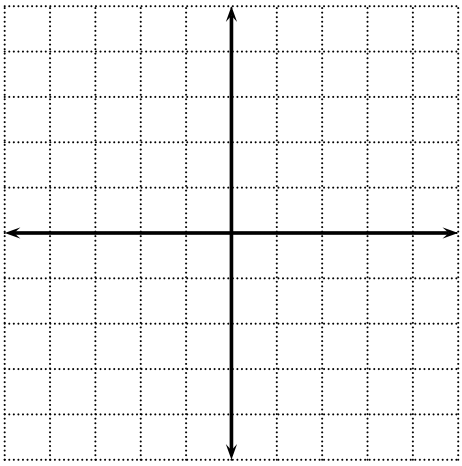
Example 84. Follow the steps to graph $f(x) = 3x^2 - 6x + 4$.

1. Vertex:

2. Vertical intercept:

3. Horizontal intercepts:

4. Graph the function and label the vertex, intercepts and axis of symmetry.



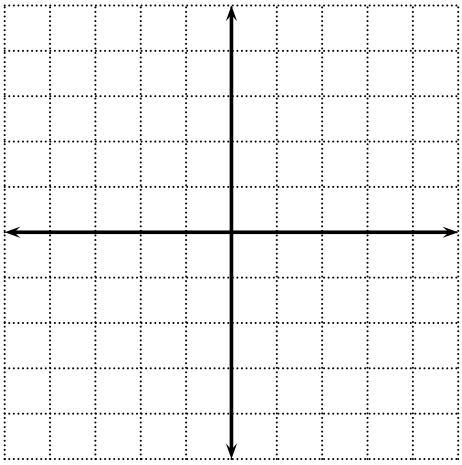
Example 85. Follow the steps to graph $f(x) = -x^2 - 4x + 1$.

1. Vertex:

2. Vertical intercept:

3. Horizontal intercepts:

4. Graph the function and label the vertex, intercepts and axis of symmetry.



PRACTICE COMPLETING THE SQUARE
Group work: Due Friday

Write the following quadratic functions in vertex form by completing the square.

1. $f(x) = x^2 + 4x - 32$

2. $f(x) = x^2 + 6x + 2$

3. $f(x) = 9x^2 - 18x + 3$

4. $f(x) = 2x^2 + 5x - 8$

5. $f(x) = x^2 + 8x + 14$

6. $f(x) = -x^2 + 2x + 7$

6.3 2.1C: Applications of quadratic functions

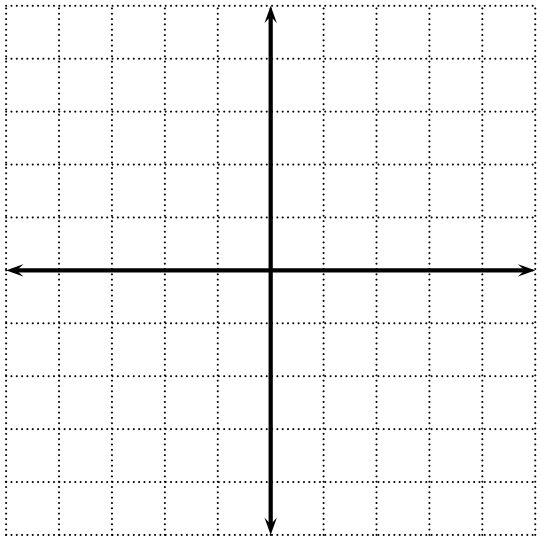
To find a maximum or minimum value:

To find when something hits the ground or lands:

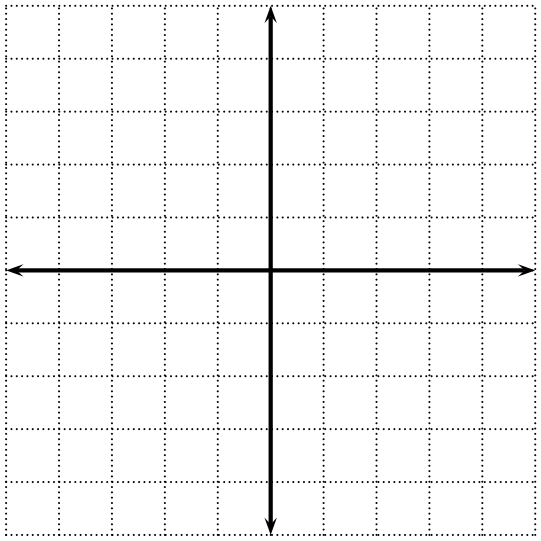
To find the value something starts at:

6.4 2.2: Polynomials

Example 86. Graph the function $f(x) = 2x^3 - 6x^2$ by first determining the end behavior and horizontal and vertical intercepts.



Example 87. Graph the function $f(x) = -\frac{1}{4}x^4 + \frac{3}{2}x^3 - \frac{9}{4}x^2$ by first determining the end behavior and horizontal and vertical intercepts.



Chapter 7

Polynomials

7.1 2.3B: Polynomial Factor Theorem

Homework notes and instructions: 59, 61, 67-73 odd, 87-90 all, 98 (see notes in course pack)

- For #59-62, use polynomial long division (not synthetic division).
- For #61 and 62, keep in mind that if $\left(x - \frac{1}{2}\right)$ is a factor of $f(x)$, then so is $(2x - 1)$.
- For #67-73 odd:
  - (a) You may verify the factors by plugging in, or by using polynomial long division.
  - (e) Use the tools we have practiced to sketch a graph of $f(x)$ instead of verifying your answers on a graphing utility.
- For #89, *improper* means it does not divide without a remainder.

PRACTICE ON POLYNOMIAL FACTORING
Group work: Due tomorrow

Example 88. Answer the following for the polynomial function $f(x) = x^4 + x^3 - 28x^2 + 20x + 48$

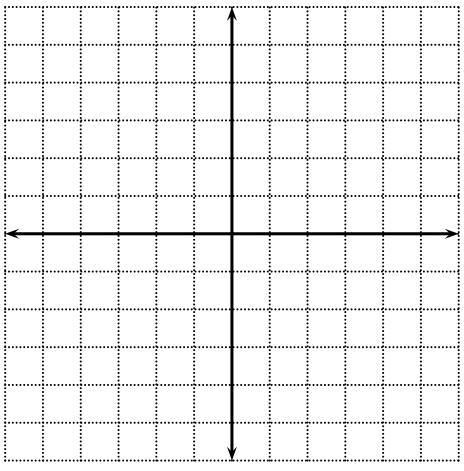
1. Verify that $x = 4$ is a root of the function by evaluating $f(4)$.

2. Verify that $x = 4$ is a root of the function using polynomial long division.

3. Factor $f(x)$ completely.

4. List the horizontal intercepts for $f(x)$.

5. Graph $f(x)$ using end behavior and horizontal and vertical intercepts. (Be sure to choose an appropriate scale for the vertical axis.)



Example 89. Answer the following for the polynomial function $f(x) = x^3 + 2x^2 - 2x - 4$

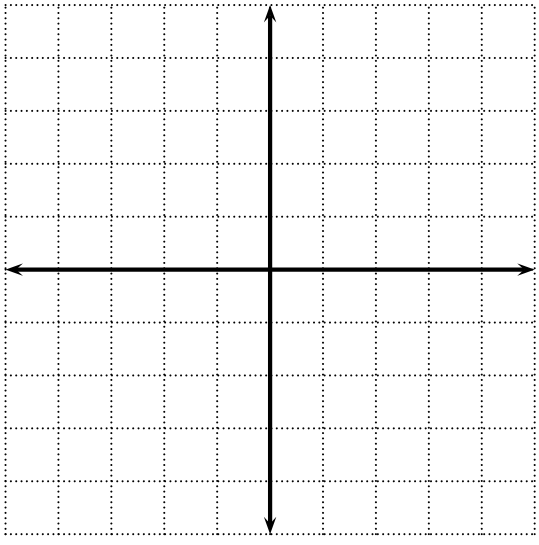
1. Verify that $x = -2$ is a root of the function by evaluating $f(-2)$.

2. Verify that $x = -2$ is a root of the function using polynomial long division.

3. Factor $f(x)$ completely.

4. List the horizontal intercepts for $f(x)$.

5. Graph $f(x)$ using end behavior and horizontal and vertical intercepts.



Example 90. Answer the following for the polynomial function $f(x) = x^3 - x^2 + x - 1$

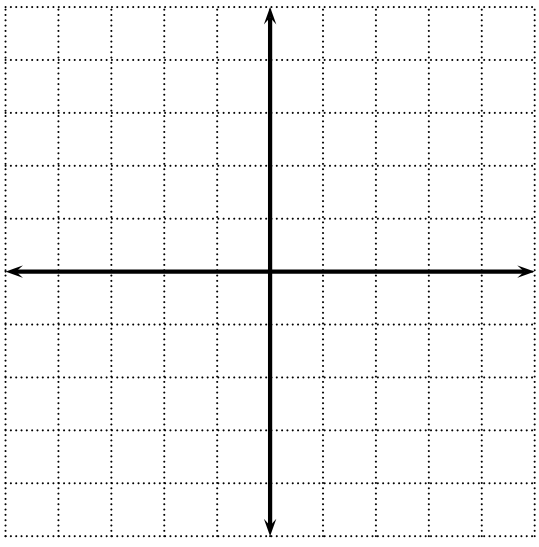
1. Verify that $x = 1$ is a root of the function by evaluating $f(1)$.

2. Verify that $x = 1$ is a root of the function using polynomial long division.

3. Factor $f(x)$ completely.

4. List the horizontal intercepts for $f(x)$.

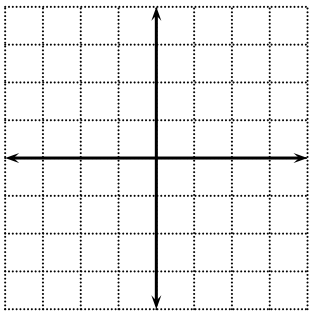
5. Graph $f(x)$ using end behavior and horizontal and vertical intercepts.



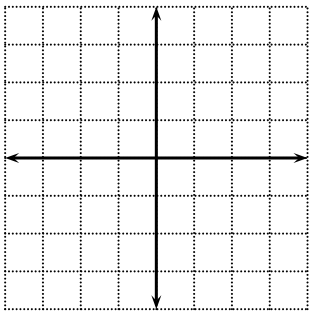
7.2 2.5A: Fundamental Theorem of Algebra

Example 91. Graph the following *quadratic* functions and find all real and complex roots, with multiplicity.

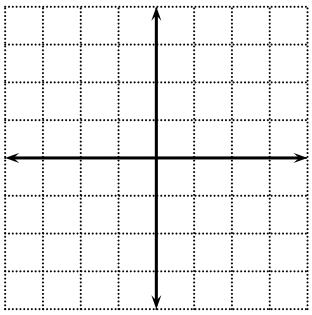
1. $f(x) = x^2 - 1$



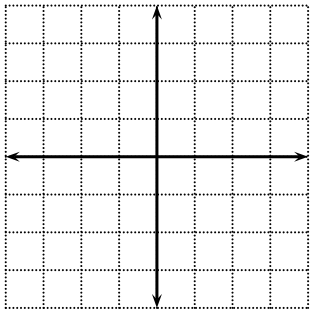
2. $f(x) = x^2 + 2x + 1$



3. $f(x) = x^2 + 1$

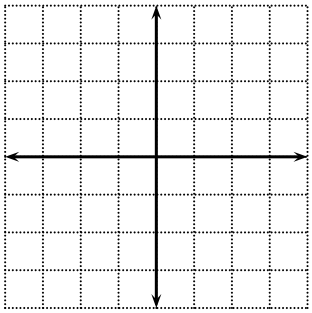


Example 92. Graph and find all roots for the function $f(x) = x^4 - 1$

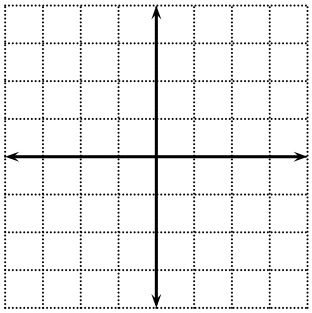


Example 93. Graph the following *cubic* functions and find all real and complex roots, with multiplicity.

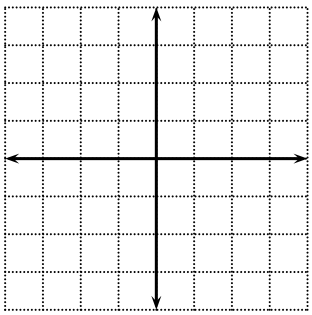
1. $f(x) = x^3$



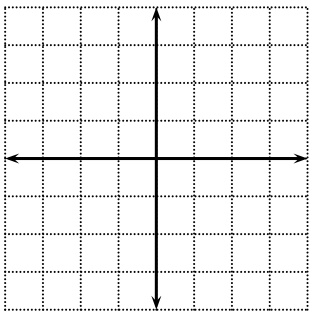
2. $f(x) = x^3 + 1$



3. $f(x) = -x^3 - 2x^2 - x - 2$ *(Hint:* $(x = -2)$ *is one root.)*



4. $f(x) = x^3 - x^2 - 8x + 12$ *(Hint:* $(x = -3)$ *is one root.)*



7.3 2.5B: Roots of polynomials

Example 94. Solve $9x^4 = 12x^2$

Example 95. Solve $x^3 - 5x^2 - 2x + 10 = 0$

Example 96. Find all *rational* roots for $f(x) = x^3 - 5x^2 + 2x + 8$

Example 97. Find *all* roots for $f(x) = x^3 - 3x^2 + 2x - 6$

Example 98. Answer the following for $f(x) = x^3 + 2x^2 + 6x - 4$

- Use the Rational Root Theorem to list all possible *rational* roots for $f(x)$.

- Sketch a graph of $f(x)$ and use it to eliminate some of the possible rational roots.

- Use polynomial long division to test the remaining possible rational root(s).

- How may rational, real (not rational) and complex (nonreal) roots does $f(x)$ have?

Example 99. Answer the following for $f(x) = 2x^3 - 3x^2 + 2x - 6$

- Use the Rational Root Theorem to list all possible *rational* roots for $f(x)$.

- Sketch a graph of $f(x)$ and use it list all rational root(s) for $f(x)$.

- Write $f(x)$ in its full factored form.

- How may rational, real (not rational) and complex (nonreal) roots does $f(x)$ have?

Example 100. Find *all* solutions to $10x^3 - 15x^2 - 16x + 12 = 0$ and identify them as rational, real (not rational) and complex (nonreal).

7.4 A6: Absolute Value Inequalities

Example 101. Solve $7x - 3 \leq 2x + 7$. Answer in both inequality and interval notation.

Example 102. Solve the **compound inequality** $1 < 2x + 7 \leq 11$. Answer in both inequality and interval notation.

Example 103. Solve $-1 \leq -2(x - 4) < 7$. Answer in both inequality and interval notation.

Example 104. Solve $|x - 20| \leq 4$. Answer in both inequality and interval notation.

Example 105. Solve $|14 - x| + 3 > 7$. Answer in both inequality and interval notation.

Example 106. Solve $\left|1 + \frac{2}{3}x\right| \geq 1$. Answer in both inequality and interval notation.

Example 107. Solve $3|4 - 5x| \leq 9$. Answer in both inequality and interval notation.

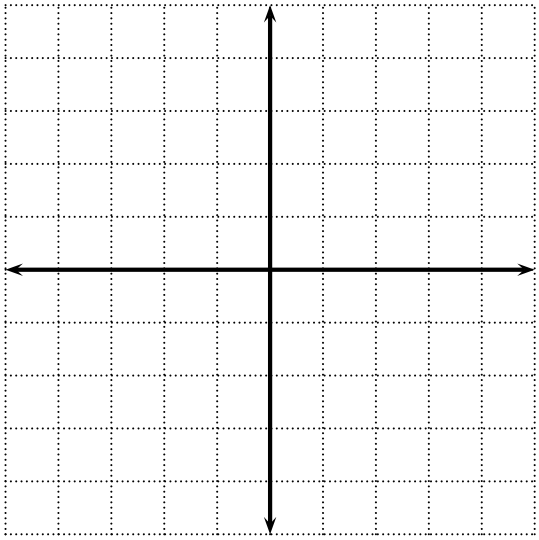
Chapter 8

Nonlinear Inequalities and Conic Sections

8.1 2.7: Nonlinear Inequalities

Example 108. Solve $x^2 - x - 20 < 0$:

- Graphically:



- Algebraically:

Example 109. Solve $3x^3 - x^2 - 12x > -4$

Example 110. Solve $2x^2 + 3x \leq 5$

Example 111. Solve $\dfrac{2x-7}{x-5} \leq 3$

Example 112. Solve $\dfrac{x-2}{x-3} \geq -3$

Example 113. Solve $\dfrac{5}{x-6} > \dfrac{3}{x+2}$

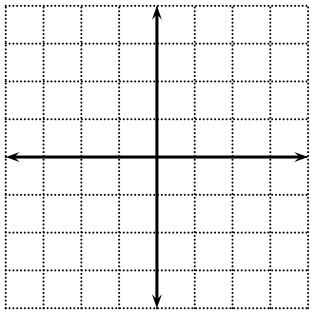
8.2 10.2: Conic Sections and Parabolas

The two **conic equations** for a parabola are:

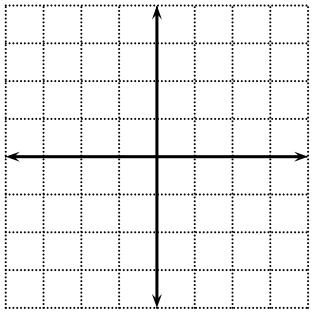
Example 114. Find an equation for the parabola with vertex $(0,0)$ and focus $\left(0, \frac{3}{8}\right)$.

Example 115. Find an equation for the parabola with vertex $(2, -3)$ and focus $(4, -3)$.

Example 116. For the parabola $x = \frac{1}{4}y^2 + \frac{3}{2}y + \frac{13}{4}$, find the vertex, focus and directrix and sketch a graph.



Example 117. For the parabola $y^2 - 4y + 4x = 0$, find the vertex, focus and directrix and sketch a graph.



8.3 10.3A: Ellipses

EXPLORATION OF ELLIPSES
Group work

Circles

1. Use a loop of string, a pushpin and a pen to draw a circle.

2. Use your technique to draw a circle with radius of 3 inches.

   (a) How long does the string need to be to draw this circle (ignore the "tails" leftover from tying the knot)?

   (b) In general, how long would the string need to be to draw a circle with radius r inches?

3. On graph paper, use your technique to draw a circle with radius 5 units and center $(3, -2)$. Where did you place your pushpin?

Ellipses

1. Use a loop of string, two pushpins and a pen to draw an ellipse.

2. On graph paper, use your technique to draw an ellipse centered at $(0, 0)$ so that the longest part of the ellipse is along the horizontal axis. *(Tip: Your work will be easier later if your string length is an even number of graph paper units.)*

   (a) Where did you place your pushpins?

   (b) Can you draw another ellipse centered at $(0, 0)$ with the longest part of the ellipse is along the horizontal axis? Where did you place the pushpins for this ellipse?

   (c) Draw one more ellipse centered at $(0, 0)$ with the longest part of the ellipse is along the horizontal axis. Where did you place the pushpins this time?

   (d) The points on the ellipse that are farthest from the center are called **vertices**. What are the vertices of each of the three ellipses your just drew? (Each ellipse has two vertices, and for these three ellipses they should be on the horizontal axis.)

Ellipse (a) vertices	Ellipse (b) vertices	Ellipse (c) vertices

(e) What was the length of string you used for each of the three ellipses your just drew? (Even if they are all the same length, record them below.)

Ellipse (a) string	Ellipse (b) string	Ellipse (c) string

(f) The location of the pushpins are the **foci** of the ellipse. What pattern do you notice between the foci, vertices and string length for each of the three ellipses your just drew?

3. On graph paper, use your technique to draw an ellipse centered at $(0, 0)$ with one vertex at $(0, 5)$.

   (a) Where is the other vertex?

   (b) Where are the foci?

   (c) What is your string length?

   (d) Can you draw another ellipse centered at $(0, 0)$ with one vertex at $(0, 5)$? Where are the foci? What is the string length?

4. On graph paper, use your technique to draw an ellipse with one focus at $(3, 2)$ and one vertex at $(7, 2)$.

   (a) Where is the other focus?

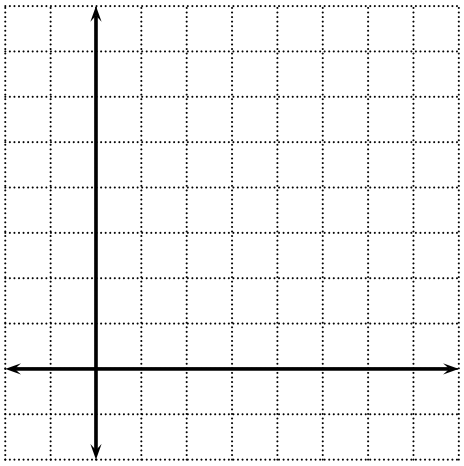
   (b) Where is the center?

   (c) What is your string length?

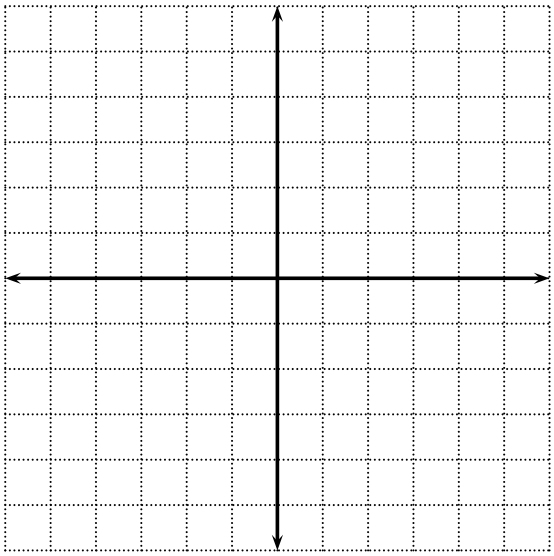
   (d) If the center of your ellipse is between $(3, 2)$ and $(7, 2)$, draw another ellipse with one focus at $(3, 2)$ and one vertex at $(7, 2)$ but with the center not between $(3, 2)$ and $(7, 2)$. Likewise, if the center of your ellipse is not between $(3, 2)$ and $(7, 2)$, draw another ellipse with one focus at $(3, 2)$ and one vertex at $(7, 2)$ but with the center between $(3, 2)$ and $(7, 2)$. List the center, other focus and string length for this ellipse.

The **standard conic equation** of an ellipse is:

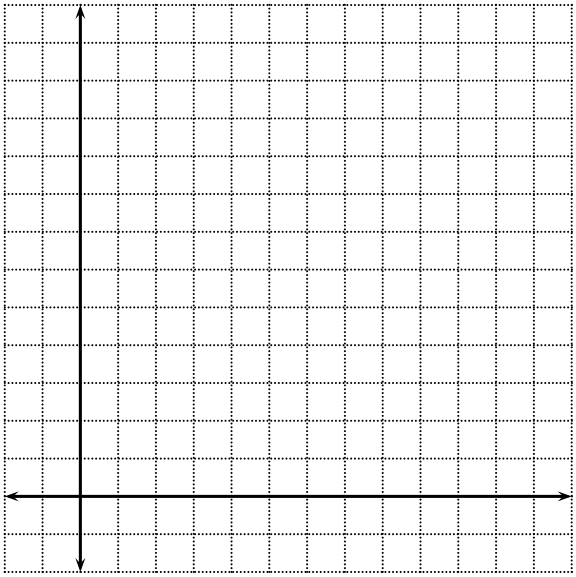
Example 118. Find an equation for the ellipse with foci $(2, 0)$ and $(2, 6)$ and major axis length 8.



Example 119. Find an equation for the ellipse with foci $(2, 0)$ and $(-2, 0)$ and major axis length 10.

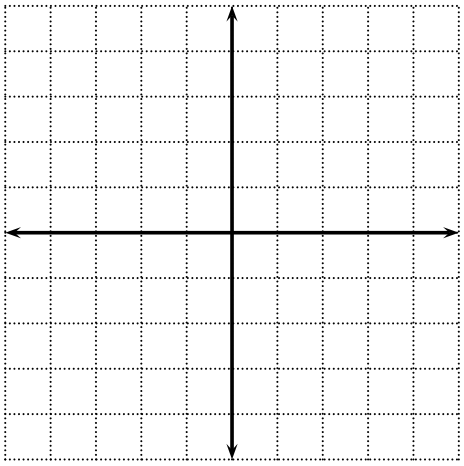


Example 120. Find an equation for the ellipse with vertices $(5, 0)$ and $(5, 12)$ and minor axis endpoints $(1, 6)$ and $(9, 6)$.

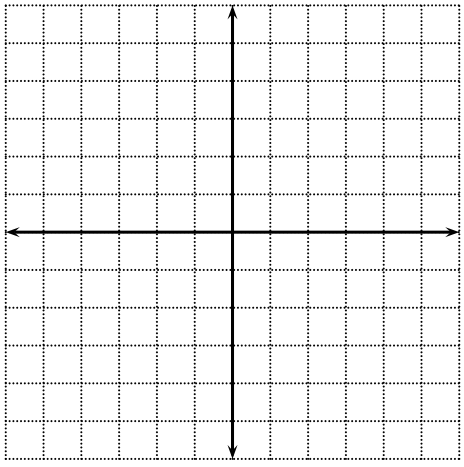


8.4 10.3B, 10.4A: Ellipses and Hyperbolas

Example 121. Find the center, vertex and foci and sketch the graph of $9x^2 + 4y^2 + 36x - 8y + 4 = 0$.



Example 122. Find the center, vertex and foci and sketch the graph of $5x^2 + 9y^2 + 10x - 54y + 41 = 0$.

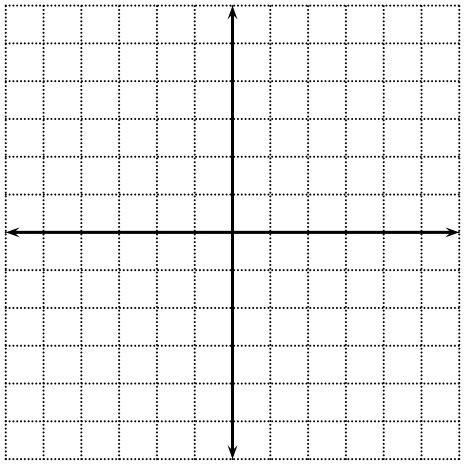


The **standard conic equations** for a hyperbola are:

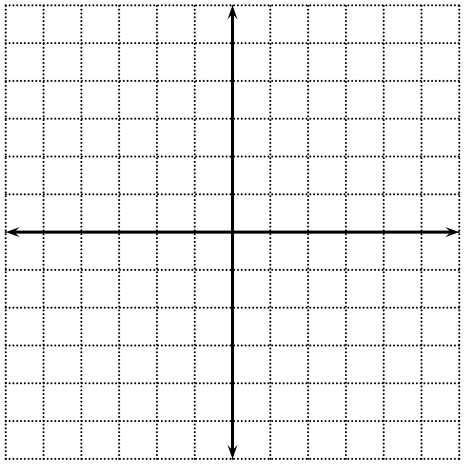
Example 123. Find an equation for the hyperbola with foci $(2, -5)$ and $(2, 3)$ and vertices $(2, -4)$ and $(2, 2)$.

Example 124. Find an equation for the hyperbola with vertices $(12, 1)$ and $(2, 1)$ and foci $(-3, 1)$ and $(3, 1)$.

Example 125. Graph $4y^2 - 9x^2 = 36$. Label the center, vertices, foci and asymptotes of the hyperbola



Example 126. Graph $9x^2 - 4y^2 + 8y - 40 = 0$. Label the center, vertices, foci and asymptotes of the hyperbola



Exam 3 is Wednesday!

Chapter 9

Happy Thanksgiving

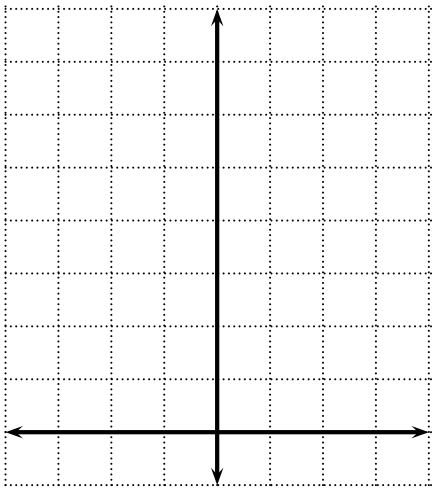
Chapter 10

Exponential and Logarithmic Functions

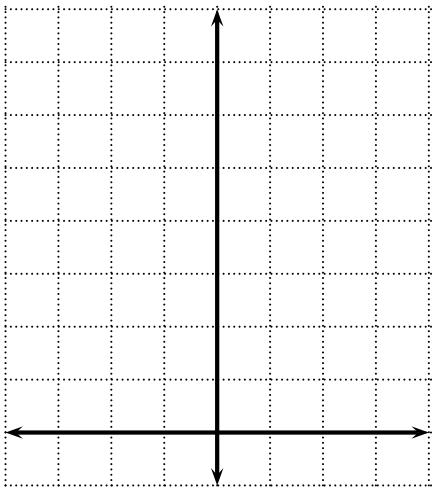
10.1 3.1: Exponential Functions

Example 127. Graph the following functions.

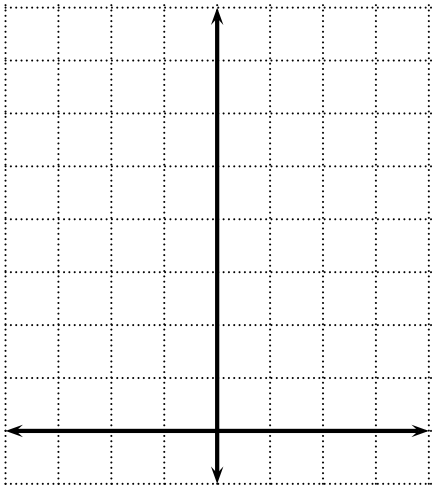
1. $f(x) = 2^x$ (You may want to create a table of values to help you.)



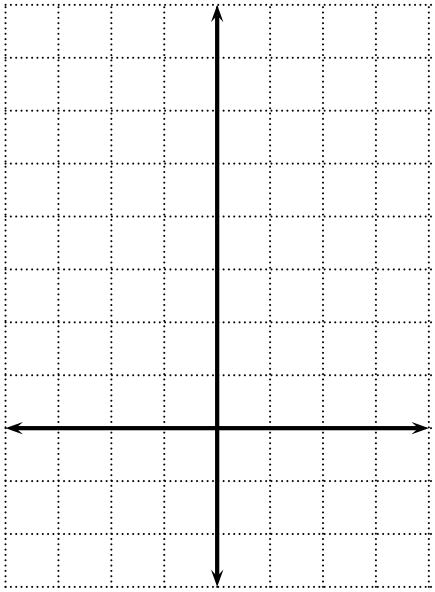
2. Sketch $f(x) = 2^x$ again and use it to graph $g(x) = 2^{x-2}$.



3. Sketch $f(x) = 2^x$ again and use it to graph $g(x) = 2^x + 3$.



4. Sketch $f(x) = 2^x$ again and use it to graph $g(x) = 2^{-x} - 3$.



Example 128. Use your calculator to evaluate the following expressions.

1. $e^{0.23}$

2. $e^{-1.2}$

Example 129. Solve the following exponential equations *without* using a calculator

1. $2^{2x-1} = 8$

2. $\left(\frac{1}{2}\right)^{x} = 4$

3. $27 = 3^{2x+1}$

4. $\left(\frac{1}{3}\right)^{-x} = 81$

10.2 A2: Exponent Properties

Example 130. Simplify the following expressions.

1. -3^4

2. $(-3)^4$

3. $3^2 \cdot 3^4$

4. $3^2 \cdot 3$

5. $\dfrac{3^5}{3^8}$

6. Evaluate $-x^{-2}$ for $x = 4$

7. Evaluate $\dfrac{1}{4}(-x)^4$ for $x = 4$

PRACTICE WITH EXPONENT PROPERTIES
Group work

Completely simplify the following expressions. Your final answers should not include negative exponents.

1. $(2x^{-2}y^3)(-x^4y)$

2. $(4a^2b^3)^0$

3. $(-5z)^3(z^2)$

4. $\left(\dfrac{3x^4}{x^2y^2}\right)^2$

5. $2x^{-2}$

6. $\left(\frac{x}{10}\right)^{-3}$

7. $\frac{3a^{-3}b^4}{15ab^{-1}}$

8. $(-2x^2)^3(4x^3)^{-1}$

10.3 3.2A: Logarithmic Functions

$y = \log_a x$ is the same as

Example 131. Evaluate the following logarithmic expressions *without* a calculator

1. $\log_{10}(10,000)$

2. $\log_6 1$

3. $\log_5\left(\frac{1}{125}\right)$

The **common logarithm** is

Example 132. Evaluate the following common logarithms using a calculator

1. $\log(275)$

2. $\log\left(\frac{1}{2}\right)$

3. $\log\left(-\frac{1}{2}\right)$

Basic Properties of Logarithms and Exponents

Example 133. Solve for x: $\log_3 x = \log_3 12$

Example 134. Solve for x: $\log(2x + 1) = \log(3x)$

Example 135. Solve each of the following for x.

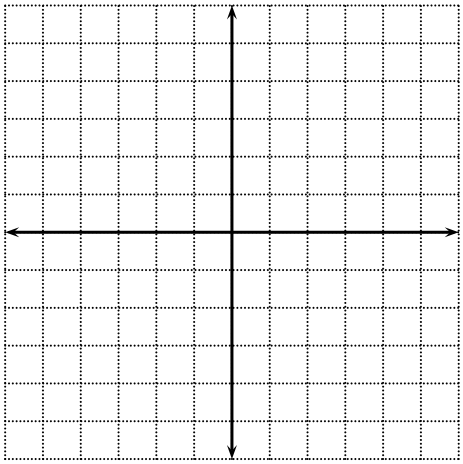
1. $x = \log_{\sqrt{3}}(1)$

2. $x = \log_9 9$

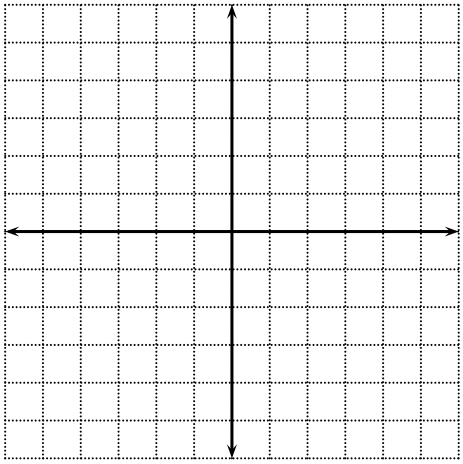
3. $\log_5(x^2+3) = \log_5 12$

Example 136. Graph each of the following by first graphing $y = 3^x$ and $y = \log_3 x$. State the domain and range of the function, and find an equation for its vertical asymptote.

1. $f(x) = -1 + \log_3 x$



2. $g(x) = \log_3(x+3)$



10.4 3.2B, 3.3A: The Natural Logarithm and Change of Base Formula

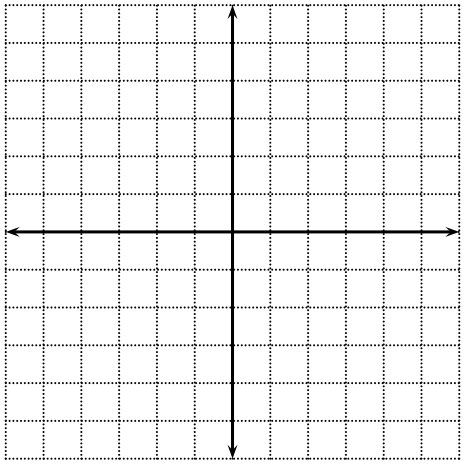
Example 137. Use a calculator to evaluate the following.

1. $\ln(0.01)$

2. $\ln(\sqrt{3}-2)$

3. $\ln(e)$

Example 138. Sketch a graph of $f(x) = \ln(x+3)$. State the domain and range of the function, and find an equation for its vertical asymptote.



Change of Base Formula

Example 139. Approximate $\log_2 12$ rounded to two decimal places.

10.5 3.3B: Properties of Logarithms

Example 140. Evaluate the following without using a calculator.

1. $\log_5 \sqrt[3]{5}$

2. $\ln(e^6) - \ln(e^2)$

Example 141. Fully expand the logarithmic expressions

1. $\log_3 \left(\dfrac{4x^2}{\sqrt{y}} \right)$

2. $\log_2(16x^2y^4)$

3. $\ln\left(\dfrac{\sqrt{x+1}}{xy^2}\right)$

Example 142. Condense each expression into a single logarithm with leading coefficient 1.

1. $2(\log(x+3) - 2\log(x-2))$

2. $-4\log_6(2x)$

3. $\frac{1}{2}(\log_4(x+1) + 2\log_4(x-1)) + 6\log_4 x)$