

Formula Sheet

$$\sinh(x) = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh(x) = \frac{1}{2}(e^x + e^{-x})$$

$$\sinh^{-1}(x) = \ln(x + \sqrt{x^2 + 1})$$

$$\cosh^{-1}(x) = \ln(x + \sqrt{x + x^2 - 1}) \text{ for } x > 1$$

$$\tanh^{-1}(x) = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \text{ for } -1 < x < 1$$

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$

$$\frac{d}{dx} (\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int \frac{1}{\sqrt{1+x^2}} dx = \sinh^{-1}(x) + C$$

$$\int \frac{1}{\sqrt{x^2-1}} dx = \cosh^{-1}(x) + C$$

$$\int \frac{1}{1-x^2} dx = \tanh^{-1}(x) + C$$

$$\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + C$$

$$\int \tan(x) dx = \ln |\sec(x)| + C$$

$$\int \cot(x) dx = \ln |\sin(x)| + C$$

$$\int \sec(x) dx = \ln |\sec(x) + \tan(x)| + C$$

$$\int \csc(x) dx = \ln |\csc(x) - \cot(x)| + C$$

$$\int \ln(x) dx = x \ln(x) - x + C$$

Logistic population growth:

$$\frac{dP}{dt} = kP \left(1 - \frac{P}{M} \right)$$

$$P(t) = \frac{M}{1 + \left(\frac{M-P_0}{P_0} \right) e^{-kt}}$$